MENELAUS' THEOREMS ON TRIANGLES AND THEIR NEW PROOFS

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Abstract: *This paper presents new and easy-to-learn proofs of Menalay's theorems. This theorem is the key to many complex problems. In schools, lyceums and specialized boarding schools, studying mathematics is necessary and has a very important place in preparing a student for the Olympiad. Triangular similarity and the theorem of sines were used to prove Menelaus' theorem.*

Key words: *Menelaus' theorem, sine of an angle, similarity of a triangle, ratio of sections.*

Theorem 1 (Theorem of Menelaus):

A straight line l intersects the sides AB and BC the continuation of the side AC of the given triangle ABC at the points C_1, A_1 and B_1, respectively.

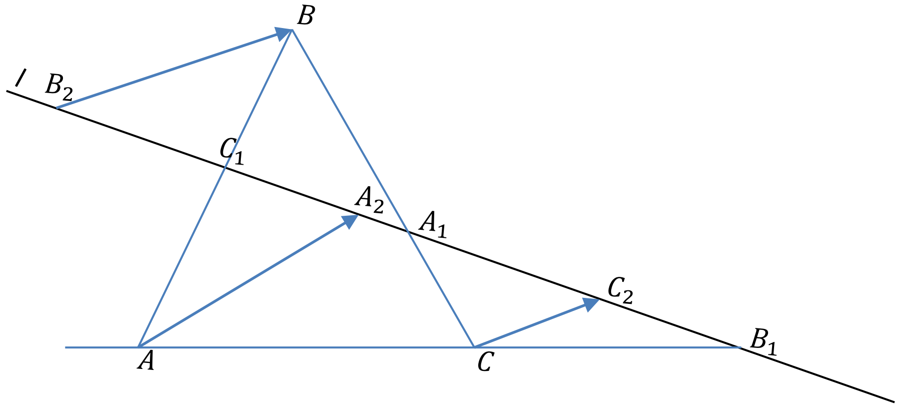
In this

$$\frac{CB_1}{AB_1}\cdot\frac{AC_1}{BC_1}\cdot\frac{BA_1}{CA_1}=1$$

equality is appropriate.

Proof:

Let there be three sections parallel to each other from points A, B and C . Here $BB_2||AA_2||CC_2$, $B_2\in l, A_2\in l$ and $C_2\in l$.



$AA_2||CC_2 \Rightarrow \Delta CC_2B_1 \sim \Delta AA_2B_1 \Rightarrow \frac{CB_1}{AB_1} = \frac{CC_2}{AA_2}$ (1)

$AA_2||BB_2 \Rightarrow \Delta BB_2C_1 \sim \Delta AA_2C_1 \Rightarrow \frac{AC_1}{BC_1} = \frac{AA_2}{BB_2}$ (2)

$CC_2||BB_2 \Rightarrow \Delta BB_2A_1 \sim \Delta CC_2A_1 \Rightarrow \frac{BA_1}{CA_1} = \frac{BB_2}{CC_2}$ (3)

multiplying (1), (2) and (3) above, we get the following:

$$\frac{CB_1}{AB_1} \cdot \frac{AC_1}{BC_1} \cdot \frac{BA_1}{CA_1} = \frac{CC_2}{AA_2} \cdot \frac{AA_2}{BB_2} \cdot \frac{BB_2}{CC_2} = 1$$

the theorem is proved.

Theorem 2 (Searching sines):

A straight line l intersects the sides AB and BC and the continuation of the side AC of the given triangle ΔABC at the points C_1, A_1 and B_1, respectively.

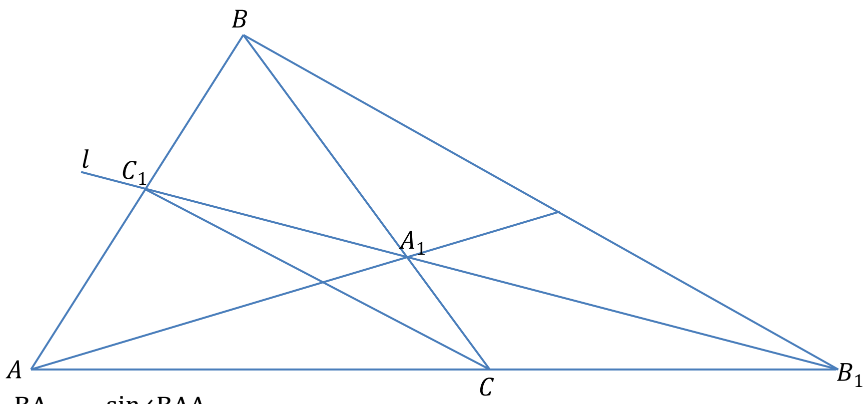
In this

$$\frac{\sin\angle BAA_1}{\sin\angle CAA_1} \cdot \frac{\sin\angle ACC_1}{\sin\angle BCC_1} \cdot \frac{\sin\angle CBB_1}{\sin\angle ABB_1} = 1$$

equality is appropriate.

Proof:

We use Menelaus' theorem to prove the Menelaus sine



ΔABA_1 da $\frac{BA_1}{AA_1} = \frac{\sin\angle BAA_1}{\sin\angle ABC}$ (1)

ΔACA_1 da $\frac{AA_1}{CA_1} = \frac{\sin\angle ABC}{\sin\angle CAA_1}$ (2)

ΔACC_1 da $\frac{AC_1}{CC_1} = \frac{\sin\angle ACC_1}{\sin\angle CAB}$ (3)

ΔBCC_1 da $\frac{CC_1}{BC_1} = \frac{\sin\angle ABC}{\sin\angle BCC_1}$ (4)

ΔBCB_1 da $\frac{CB_1}{BB_1} = \frac{\sin\angle CBB_1}{\sin\angle ACB}$ (5)

ΔABB_1 da $\frac{BB_1}{AB_1} = \frac{\sin\angle BAC}{\sin\angle ABB_1}$ (6)

We multiply all the above equations.

It follows that

$$\frac{\sin\angle BAA_1}{\sin\angle CAA_1}\cdot\frac{\sin\angle ACC_1}{\sin\angle BCC_1}\cdot\frac{\sin\angle CBB_1}{\sin\angle ABB_1}=\frac{BA_1}{CA_1}\cdot\frac{AC_1}{BC_1}\cdot\frac{CB_1}{AB_1}=1$$

he theorem is proved.

Theorem 3 (Converse Theorem of Menelaus):

For the points C_1, A_1 and B_1 taken on the sides AB and BC and along the side AC of the given triangle ABC

$$\frac{CB_1}{AB_1}\cdot\frac{AC_1}{BC_1}\cdot\frac{BA_1}{CA_1}=1$$

if the equality holds, the points A_1, B_1 and C_1 lie on the same straight line.

Proof:

Let's define the surfaces as in the drawing below. So that the points A_1, B_1 and C_1 lie on the same straight line

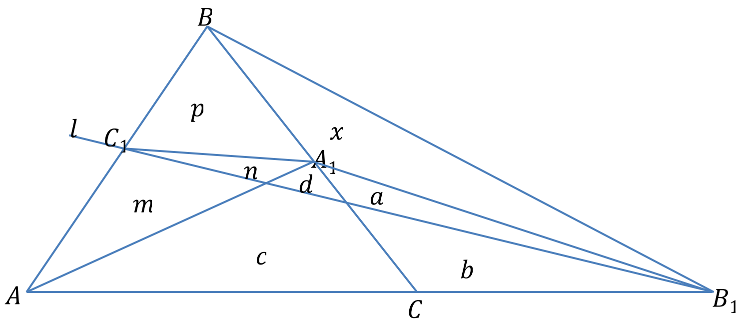
$$S_{A_1B_1C_1}=n+d+a=0$$

it is enough to prove that.

Lemma: For the point D drawn by BC of the triangle ΔABC

$$\frac{S_{ABD}}{S_{ACD}}=\frac{BD}{CD}$$

equality is appropriate.



Lemmas are:

$$\frac{CB_1}{AB_1}=\frac{a+b}{a+b+c+d}$$

$$\frac{AC_1}{BC_1}=\frac{m+n}{p}$$

$$\frac{BA_1}{A_1C}=\frac{m+n+p}{c+d}$$

We multiply the equations and using the condition of the theorem

$$\frac{a+b}{a+b+c+d}\cdot\frac{m+n}{p}\cdot\frac{m+n+p}{c+d}=1\ (*)$$

we will achieve equality.

Lemmas are:

$$\frac{BC_1}{AC_1}=\frac{x+p+d+n+a}{m+c+b}=\frac{p}{m+n}\Rightarrow x=\frac{p(m+c+b)-(m+n)(p+d+n+a)}{m+n}\ (1)$$

Lemmas are:

$\frac{BA_1}{A_1C} = \frac{x}{a+b} = \frac{p+n+m}{c+d} => x = \frac{(p+n+m)(a+b)}{c+d}$ (2)

It follows from (1) and (2).

$\frac{p(m+c+b)-(m+n)(p+d+n+a)}{m+n} = \frac{(p+n+m)(a+b)}{c+d} =>$

$\frac{c+d}{(m+n)(m+n+p)(a+b)} = \frac{1}{p(m+c+b)-(m+n)(p+d+n+a)}$ (**)

By multiplying (*) and (**).

$$\frac{1}{p(a+b+c+d)} = \frac{1}{p(m+c+b)-(m+n)(p+d+n+a)}$$
$$ap + bp + cp + dp = pm + cp + bp - mp - np - (m+n)(a+n+d)$$
$$(a+n+d)(m+n+p) = 0$$
$$S_{ABA_1} \cdot S_{A_1B_1C_1} = 0$$

This multiplication gives $S_{A_1B_1C_1} = 0$ because $S_{ABA_1} \neq 0$

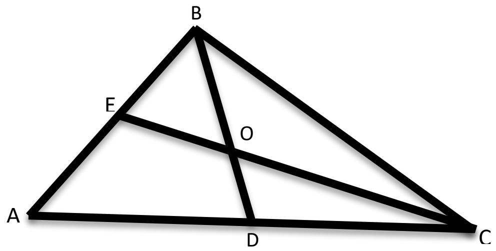
Applications of Menelaus' theorem to some problems.

Prove that at the intersection of the medians of a triangle, counting from the tip of the triangle, it divides in the ratio 2:1.

Proof: Let BD and CE be medians in ΔABC and intersect at point O.

$$BO: OD = 2: 1$$

we prove that.



It is known that AE=EB, CD=DA. Applying Menelaus' theorem to the triangle ΔABD, we get the following:

It is known that $AE = EB, CD = DA$ Applying Menelaus' theorem to the triangle ΔABD, we get the following:

$$\frac{AE}{EB} \cdot \frac{BO}{OD} \cdot \frac{CD}{CA} = 1$$

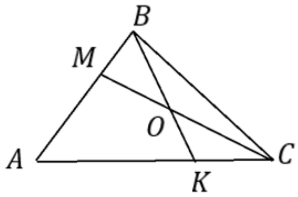
Ago

$$\frac{AE}{EB} \cdot \frac{BO}{OD} \cdot \frac{CD}{2CD} = 1$$
$$\frac{BO}{OD} = 2: 1$$

The proof is over.

Example: In triangle ABC, straight lines BK and CM intersect at point O. If $AM: MB = 2: 1$ and $AK: KC = 3: 2$, $BO: OK =?$

Solution: We can draw a diagram for the given conditions as follows:



Using Menelaus' theorem for ΔABK , we get the following.

$$\frac{AM}{MB}\cdot\frac{BO}{OK}\cdot\frac{CK}{CA}=1$$

$$\frac{2MB}{MB}\cdot\frac{BO}{OK}\cdot\frac{2CK}{5CK}=1$$

$$\frac{4BO}{5OK}=1$$

$$BO:OK=5:4$$

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