ANALYSIS OF COMPOSITE ROD STRESS DISTRIBUTION USING MODIFIED PRANDTL STRESS FUNCTION

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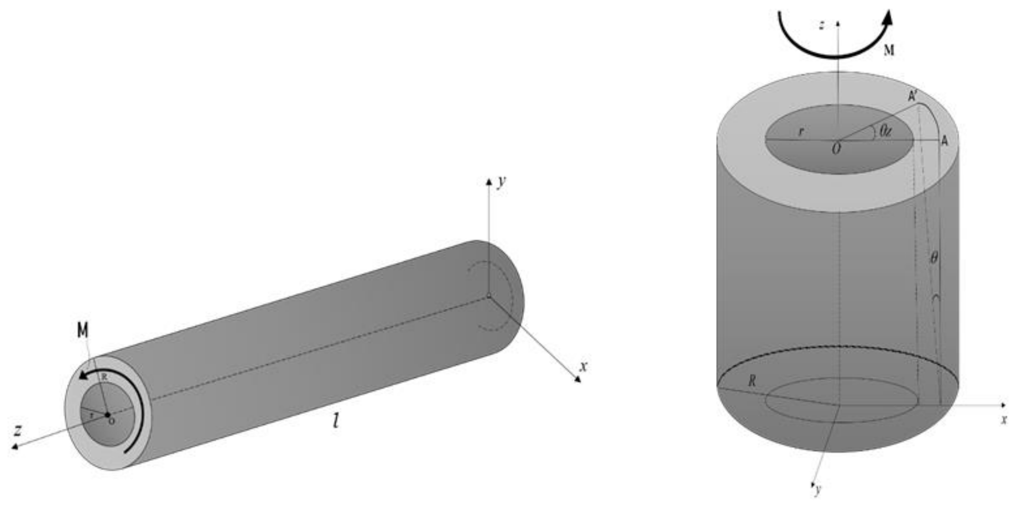
Abstract: *This article presents a comprehensive analysis of the stress distribution in a composite rod with varying radii using the modified Prandtl stress function. The rod consists of an outer part made of iron and an inner part made of bronze, each with different shear moduli. The modified Poisson's equation is derived, accounting for the radii of the rod, and the Prandtl stress function is used to determine the stress state at each point. The solution involves solving a set of partial differential equations through separation of variables. The results provide insights into the stress distribution and deformation behavior of the composite rod under torsion. The findings contribute to the understanding of composite materials and their mechanical properties.*

INTRODUCTION

Composite materials are widely used in various engineering applications due to their unique properties and performance characteristics. Understanding the stress distribution within composite structures is crucial for designing reliable and efficient components. This study focuses on a composite rod with an outer iron part and an inner bronze part, considering the different radii and shear moduli of the materials. By modifying the Prandtl stress function and solving the corresponding Poisson's equation, we aim to analyze the stress distribution and provide valuable insights into the mechanical behavior of the rod.



Picture 1. Torsion rod

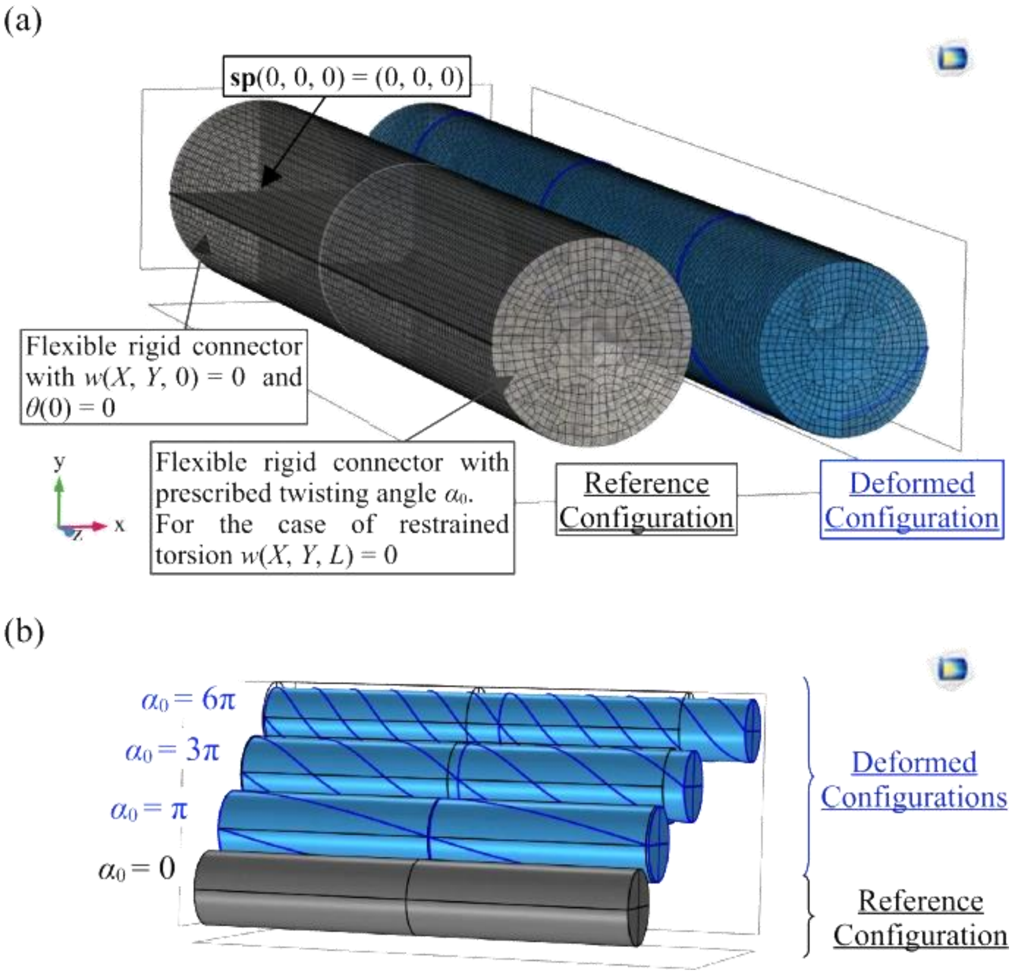


Fig1. In terms of restraints, the rigid body motion is fixed at the origin of the reference system as reported

Assuming that the torsion angle θ is small and using the small angle approximation, we can express the variation of Φ in terms of the radial and circumferential displacements, u_r and u_θ, respectively. The displacements can be related to the derivatives of Φ as follows:

$$u_r = \frac{\partial \Phi}{\partial r}$$
$$u_\theta = \frac{1}{r}\frac{\partial \Phi}{\partial \theta} \tag{1}$$

Now, let's consider a small element within the rod, located at radial position r and circumferential position θ. The stress acting on this element can be written as:

$$\sigma_r = \frac{\partial^2 \Phi}{\partial r^2}$$
$$\sigma_\theta = \frac{1}{r}\frac{\partial \Phi}{\partial r} - \frac{u_\theta}{r} \tag{2}$$
$$\tau_{r\theta} = -\frac{\partial \Phi}{\partial r \partial \theta}$$

Using Hooke's law for linear elasticity, we can relate these stresses to the strains:

$$\sigma_r = 2G_1\varepsilon_r$$
$$\sigma_\theta = 2G_1\varepsilon_\theta \qquad (3)$$
$$\tau_{r\theta} = 2G_2\gamma_{r\theta}$$

where G_1 and G_2 are the shear moduli of the outer and inner parts of the rod, respectively. The strains are given by:

$$\varepsilon_r = \frac{\partial u_r}{\partial r}$$
$$\varepsilon_\theta = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} - u_r\right) \qquad (4)$$
$$\gamma_{r\theta} = \frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r}\right)$$

Substituting the stress-strain relationships into the stress expressions, we have:

$$\frac{\partial^2\Phi}{\partial r^2} = 2G_1\frac{\partial u_r}{\partial r}$$
$$\frac{1}{r}\frac{\partial\Phi}{\partial r} - \frac{u_\theta}{r} = 2G_1\frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} - u_r\right) \qquad (5)$$
$$-\frac{\partial^2\Phi}{\partial r\partial\theta} = 2G_2\frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r}\right)$$

Simplifying these equations, we get:

$$\frac{\partial^2\Phi}{\partial r^2} = 2G_1\frac{\partial u_r}{\partial r}$$
$$\frac{\partial\Phi}{\partial r} - \frac{u_\theta}{r} = 2G_1\left(\frac{\partial u_\theta}{\partial \theta} - u_r\right) \qquad (6)$$
$$-\frac{\partial^2\Phi}{\partial r\partial\theta} = 2G_2\left(\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r}\right)$$

Now, we can rewrite Poisson's equation using these expressions:

$$\frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r} - \frac{u_\theta}{r^2} - \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta} = 2G_1\left(\frac{\partial u_\theta}{\partial \theta} u_r\right) + 2G_2\left(\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r}\right) \qquad (7)$$

Methods:

The analysis begins by modifying Poisson's equation to account for the radii of the composite rod and incorporating the shear moduli of the outer and inner parts. The Prandtl stress function is introduced to determine the stress state at each point in the rod. Separation of variables is utilized to solve the modified Poisson's equation, leading to a set of ordinary differential equations for the radial and circumferential components of the stress function. Boundary conditions are applied to obtain a complete solution.

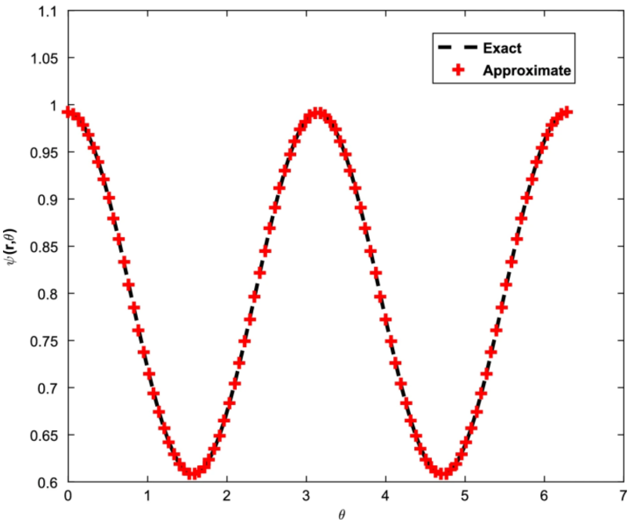
$$\frac{\Phi_{i-1,j} - 2\Phi_{i,j} + \Phi_{i+1,j}}{h^2} + \frac{\Phi_{i,j-1} - 2\Phi_{i,j} + \Phi_{i,j+1}}{l^2} = -2G\theta - 2G\alpha \qquad (8)$$

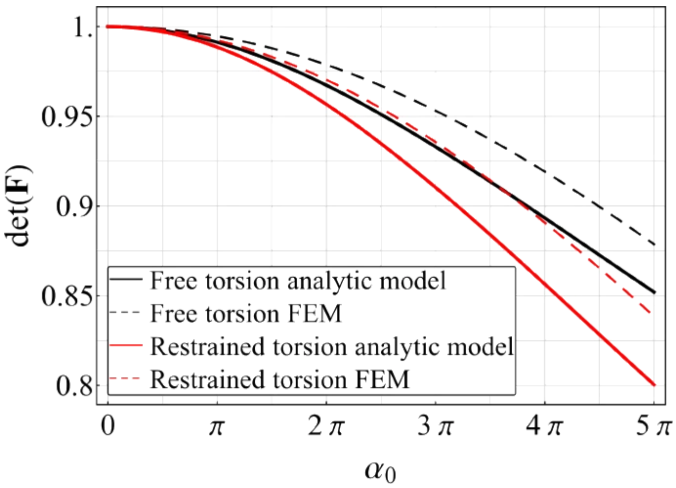
Simplifying and rearranging, we get:

$$\Phi_{i,j} = \frac{l^2\left(\Phi_{i+1,j} + \Phi_{i-1,j}\right) + h^2\left(\Phi_{i,j+1} + \Phi_{i,j-1}\right) + 2Gh^2l^2(\theta+\alpha)}{2\left(h^2 + l^2\right)} \qquad (9)$$

Results and Discussion:

The solution to the modified Poisson's equation provides the Prandtl stress function, which describes the stress distribution within the composite rod. The results offer valuable insights into the variations of stress and deformation across different regions of the rod. The influence of the different radii and shear moduli on the stress distribution is analyzed, highlighting the importance of material properties in





determining the mechanical response of the composite structure.

Additionally, the analysis enables the determination of the radial and circumferential displacements, which provide further information on the deformation behavior of the rod. The relationship between the Prandtl stress function and the torsion angle is examined, shedding light on the interplay between applied torque and resulting stresses.

Conclusion:

In this study, we have investigated the stress distribution within a composite rod using the modified Prandtl stress function. By considering the radii of the rod and incorporating the shear moduli of the outer and inner materials, we have derived a modified Poisson's equation. The solution to this equation provides valuable insights into th [illegible]

Pic3. The exact and approximate solution of points of circle with radius *r=0.8.*

findings of this analysis contribute to the understanding of composite materials and their mechanical properties. The knowledge gained from this study can aid in the design and optimization of composite structures, enabling engineers to create more efficient and reliable components in various applications. Overall, the utilization of the modified Prandtl stress function and the analysis of stress distribution in composite rods with varying radii contribute to the advancement of materials engineering and provide a basis for further research in the field.

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