TETRAEDR HAJMINI ARALASH KO'PAYTMA ORQALI TOPISHNING ISBOTI .

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Anatatsiya: *Ushbu mavzuda biz aralash ko'paytma yordamida tetraedr hajmini topishning isbotini o'rganamiz. Shu formulalar orqali misollar yechib mavzuni to'liqroq yoritib beramiz*

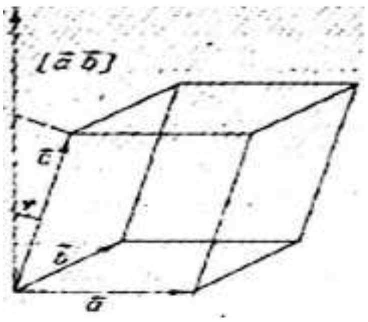
Kalit so'zlar: *Vektorlar, aralash ko'paytma, parallelepipedning hajmi, tetraedr hajmi.*

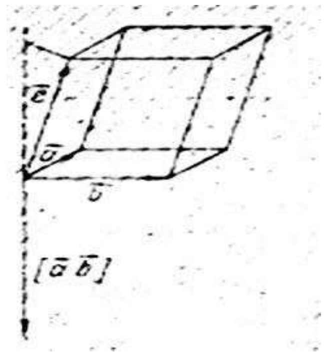
Ma'ruza qismi: Uchta $\vec{a}, \vec{b}$ va $\vec{c}$ vektor berilgan bo'lsin.

Ta'rif: Birinchi ikki vektorning vektor ko'paytmasidan iborat vektorni uchinchi vektorga skalyar ko'paytirishdan hosil qilingan son shu uch vektorning aralash ko'paytmasi deb ataladi, ya'ni $[\vec{a}, \vec{b}], \vec{c}$ bu ko'paytma $(\vec{a}, \vec{b}, \vec{c})$ ko'rinishida belgilanadi.

Avvalo aralash ko'paytmaning geometrik ma'nosi bilan tanishaylik. $\vec{a}, \vec{b}, \vec{c}$ bir nuqtadan qo'yilgan bo'lib,komplanar bo'lmasin hamda o`ng uchlikni hosil qilsin. Qirralari shu berilgan vektorlardan iborat parallelepipedni yasasak, $\left|[\vec{a}, \vec{b}]\right|$ miqdor shu parallelepiped asosining yuzini bildiradi.

Aralash ko'paytma ta'rifiga asosan $([\vec{a}, \vec{b}], \vec{c}) = \left|[\vec{a}, \vec{b}]\right| |\vec{c}| \cos\varphi$, bu yerda φ, $[\vec{a}, \vec{b}]$ va $\vec{c}$ vektorlar orasidagi burchak bo'lib, $|\vec{c}|\cos\varphi$ miqdor $\vec{c}$ vektorning $[\vec{a}, \vec{b}]$ vektor yo'nalishidagi to'g'ri chiziqdagi proyeksiyasiga teng bo'lib, parallelepipedning balandligidir. ($|\vec{c}| \cos\varphi = h$).


a)


b)

U holda $[\vec{a}, \vec{b}], \vec{c} = S_{ACB} h = V$ bu son esa parallelepipedning hajmini aniqlaydi.

$\vec{a}, \vec{b}, \vec{c}$ lar chap uchlikdan iborat bo'lsa,$[\vec{a}, \vec{b}]$ vektorlar bilan $\vec{c}$ orasidagi burchak $\varphi \geq \pi/2$ $(\cos\varphi \leq 0)$ bo'ladi. U holda $[\vec{a}, \vec{b}], \vec{c} = -V$ demak,

$$|[\vec{a}, \vec{b}]||\vec{c}| = V \qquad (1)$$

Biz quyidagini isbot qildik,uch vektorning aralash ko'paytmasidan iborat sonning absolyut qiymati qirralari shu vektordan iborat parallelepiped hajmiga tengdir.

Endi aralash ko'paytmaning xossalari bilan tanishaylik.

1^0. $(\vec{a}\,\vec{b}\,\vec{c})=(\vec{b}\,\vec{c}\,\vec{a})$.

Haqiqatan ham, bu uch vektorga qurilgan parallelepiped hajmlarining absolyut qiymatlari teng, undan tashqari $\vec{a}, \vec{b}, \vec{c}$ uchlik bilan $\vec{b}, \vec{c}, \vec{a}$ uchlikning orientatsiyalari bir xil. Shuning singari $(\vec{a}\,\vec{b}\,\vec{c})=(\vec{b}\,\vec{c}\,\vec{a})=(\vec{c}\,\vec{a}\,\vec{b})$.

2^0. $(\vec{a}\,\vec{b}\,\vec{c})=-(\vec{b}\,\vec{a}\,\vec{c})$, chunki $(\vec{a}\,\vec{b}\,\vec{c})=[\vec{a}\,\vec{b}]\vec{c}=-[\vec{b}\,\vec{a}]\vec{c}=-(\vec{b}\,\vec{a}\,\vec{c})$

demak, $(\vec{a}\,\vec{b}\,\vec{c})=-(\vec{b}\,\vec{a}\,\vec{c})$, $(\vec{b}\,\vec{c}\,\vec{a})=-(\vec{c}\,\vec{b}\,\vec{a})$, $(\vec{c}\,\vec{a}\,\vec{b})=(\vec{a}\,\vec{c}\,\vec{b})$

3^0. $((\vec{a}+\vec{b})\vec{c}\,\vec{d})=(\vec{a}\,\vec{c}\,\vec{d})+(\vec{b}\,\vec{c}\,\vec{d})$, chunki $((\vec{a}+\vec{b})\vec{c}\,\vec{d})=[\vec{a}+\vec{b}\quad\vec{c}]\vec{d}=$
$=([\vec{a}\;\vec{c}]+[\vec{b}\;\vec{c}]\vec{d})=[\vec{a}\;\vec{c}]\vec{d}+[\vec{b}\;\vec{c}]\vec{d}=(\vec{a}\,\vec{c}\,\vec{d})+(\vec{b}\,\vec{c}\,\vec{d})$.

4^0. $\forall\lambda\in R$ uchun

$(\lambda\,\vec{a}\,\vec{b}\,\vec{c})=[\lambda\,\vec{a}\,\vec{b}\,]\vec{c}==\lambda[\vec{a}\;\vec{b}]\vec{c}=\lambda(\vec{a}\,\vec{b}\,\vec{c})\,(\lambda\,\vec{a}\,\vec{b}\,\vec{c})=\lambda(\vec{a}\,\vec{b}\,\vec{c})$, chunki

5^0 $\vec{a}, \vec{b}, \vec{c}$ komplanar bo'lsa, ularning aralash ko'paytmasi nolga teng, chunki ularga qurilgan parallelepiped tekislikda joylashib qoladi, bunday parallelepipedning balandligi nolga tengligidan hajmi ham nolga teng; aksincha $(\vec{a}\,\vec{b}\,\vec{c})=0\Rightarrow\vec{a}, \vec{b}, \vec{c}$ vektorlar komplanar. Haqiqatan ham $(\vec{a}\,\vec{b}\,\vec{c})=0\Rightarrow[\vec{a}\;\vec{b}]\vec{c}=0$ yoki $[\vec{a}\;\vec{b}]\perp\vec{c}$. Lekin vektor ko'paytmaning ta'rifiga asosan $[\vec{a}\;\vec{b}]\perp\vec{a}$, $[\vec{a}\;\vec{b}]\perp\vec{b}$, bundan $[\vec{a}\;\vec{b}]$ vektorning $\vec{a}, \vec{b}, \vec{c}$ ning har biriga perpendikularligi kelib chiqadi, demak, $\vec{a}, \vec{b}, \vec{c}$ komplanar.

Endi koordinatalari bilan berilgan uchta vektorning aralash ko'paytmasini topaylik.

$\vec{a}=x_1\vec{i}+y_1\vec{j}+z_1\vec{k}$, $\vec{b}=x_2\vec{i}+y_2\vec{j}+z_2\vec{k}$, $\vec{c}=x_3\vec{i}+y_3\vec{j}+z_3\vec{k}$

$[\vec{a},\vec{b}]$ bilan $\vec{c}$ vektorning skalyar ko'paytmasi mos koordinatalari ko'paytmalarining yig'indisiga teng.

$$[\vec{a},\vec{b}],\vec{c}=\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} x_3+\begin{vmatrix} x_1 & z_1 \\ x_3 & z_3 \end{vmatrix} y_3+\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} z_3$$

$$\text{Demak, } (\vec{a},\vec{b},\vec{c})=\begin{vmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{vmatrix} \qquad (2)$$

Bu formulaning tarkibi sifatida uchlarning koordinatalari bo'yicha tetraedr hajmini hisoblash formulasini keltirib chiqaraylik.

$\vec{a}=\{X_1; Y_1; Z_1\}$, $\vec{b}=\{X_2; Y_2; Z_2\}$, $\vec{c}=\{X_3; Y_3; Z_3\}$ $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2,)$, $C(x_3, y_3, z_3)$, $D(x_4, y_4, z_4)$,

Nuqtalar tetraedrning uchlari bo'lsin.

$\overrightarrow{AB}=(x_2-x_1, y_2-y_1, z_2-z_1)$,

$\overrightarrow{AC}$=(x_3-x_1, y_3-y_1, z_3-z_1),

$\overrightarrow{AD}$=(x_4-x_1, y_4-y_1, z_4-z_1)

Tetraedrning hajmi tetraedrning bir uchidan chiqqan uchta qirrasiga qurilgan parallelepiped hajmining oltidan bir qismiga teng bo'lganligi uchun hamda (2)formulaga asosan $V_{tet}=\frac{1}{6}\left|(\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD})\right|=\pm\frac{1}{6}\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \\ x_4-x_1 & y_4-y_1 & z_4-z_1 \end{vmatrix}$

(3) Formulani ba'zan undan ko'ra qulayroq quyidagi holda yozish maquldir.

$$V_{tet}=\pm\frac{1}{6}\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}$$

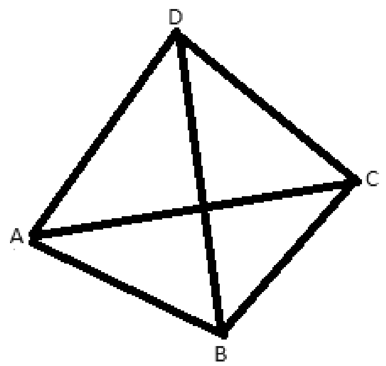
(4) Yoki (3) formula izlangan formuladir.

1-misol . $\overrightarrow{AB}$ (2,0.0), $\overrightarrow{AC}$ (3,4,0), $\overrightarrow{AD}$ (3,4,2) vektorlarga qurilgan tetraedr

berilgan . Quyidagilar topilsin : a) tetraedrning hajmi , b) ABC yoqning yuzi , v) D uchdan

tushurilgan bandlik , g) AB va BC qirralar orasidagi φ1 burchak kosinusi , d) ABC va ADC

yoqlar orasidagi φ2 burchak kosinusi.



$\overrightarrow{AB}$ (2,0.0), $\overrightarrow{AC}$ (3,4,0), $\overrightarrow{AD}$ (3,4,2)

a) Tetraedr hajmi .

b) ABC yoqning yuzi .

d) D uchidan tushirilgan balandlik .

a) Formulaga kora $V=\pm\frac{1}{6}\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD}=\pm\frac{1}{6}\begin{bmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \\ 3 & 4 & 2 \end{bmatrix}=$

$=\pm\frac{1}{6}(16+0+0-0-0)=\pm\frac{1}{6}\cdot 16=\frac{8}{3}$ kub birlik

Maktab geometriyasidan malumki

$V=\frac{1}{3}S_a\cdot h \Rightarrow h=\frac{3\cdot V}{S_{ACB}}$

b)$S_{ACB}=\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{AB}\right|$

$\overrightarrow{AC}$ x $\overrightarrow{AB}$=$\begin{bmatrix} i & j & k \\ 3 & 4 & 0 \\ 2 & 0 & 0 \end{bmatrix}$ = (0+0+0-8k-0-0)=8k

$S_{ACB} = \frac{1}{2} \cdot 8 = 4$

v) $h = \frac{3V}{S_{ACB}} = \frac{\frac{8}{3} \cdot 3}{4} = \frac{8}{4} = 2$ h=2

2) $\overrightarrow{AB} = \bar{B} - \bar{A}$ $\overrightarrow{AC} - \overrightarrow{AB} = \bar{C} - \bar{B} = \overrightarrow{BC}$

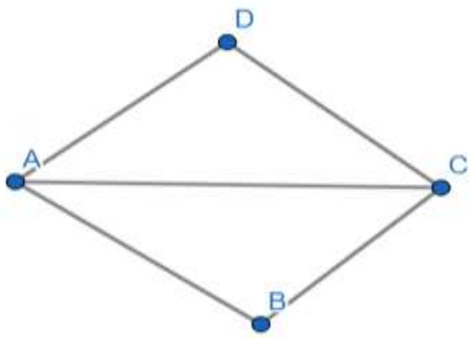
AC=$\bar{C} - \bar{A}$

$\overrightarrow{AB}$ va $\overrightarrow{BC}$ qirralar orasidagi burchak

$\cos\varphi = \frac{\bar{a}}{|a|} \cdot \frac{\bar{b}}{|b|}$ $\overrightarrow{AB}$ (2,0.0) BC(1,4,0)

$\cos\varphi = \frac{\overline{AB}}{|AB|} \cdot \frac{\overline{BC}}{|BC|} = \frac{2+0+0}{\sqrt{4} \cdot \sqrt{1+16}} = \frac{2}{2\sqrt{17}} = \frac{1}{\sqrt{17}}$

ABC va ADC yoqlar orasidagi φ burchak kosinusi



φ -?

1^0 Buning uchun Δ ABC dan $\overrightarrow{AB}$ va $\overrightarrow{AC}$ yo'naltiruvchisini va Δ ADC dan esa $\overrightarrow{AD}$ va $\overrightarrow{AC}$

ni yo'naltiruvchisini topamiz…

$\overrightarrow{AB}$ va $\overrightarrow{AC}$ to'g'ri chiziq tenglamasiga ko'ra

$\frac{x-2}{3-2} = \frac{y-4}{4-0} = \frac{z-0}{0-0}$ $\frac{x-2}{1} = \frac{y}{4} = \frac{z}{0}$ yo'naltiruvchisi $\vec{n}$(1;4;0)

2^0 $\overrightarrow{AD}$ va $\overrightarrow{AC}$ dan esa:

$\frac{x-3}{0} = \frac{y-4}{0} = \frac{z-2}{2-0}$ $\frac{x-3}{0} = \frac{y-4}{0} = \frac{z-2}{2}$ yo'naltiruvchisi esa $\vec{l}$(0;0;2)

$\cos(n \wedge l) = \frac{0}{\sqrt{17}} \cdot \frac{1}{\sqrt{4}} = 0$ $\varphi = 90^0$

2-misol

1) $\vec{a} = 3i + 4j$ $\vec{b} = -3j + k$ $\vec{c} = 2j + 3k$ vektorga yasalgan paralelopipetning hajmini toping.

$\vec{a}(3;4;0)$ $\vec{b}(0;-3;1)$ $\vec{c}(0;2;5)$ $V = |\vec{a}\cdot\vec{b}\cdot\vec{c}|$ ushbu formula orqali paralelopipetning hajmi hisoblaniladi.

$$V = \begin{vmatrix} 3 & 4 & 0 \\ 0 & -3 & 1 \\ 0 & 2 & 5 \end{vmatrix} = |-45+0+0+0-6-0| = 51$$

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