SINGULYAR INTEGRALLARNI TAQRIBIY HISOBLASH .

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Abstract: *Jahon miqyosida olib borilayotgan ko'plab ilmiy-amaliy tadqiqotlar natijasida vujudga keladigan muammolarning yechimlari integral va differensial tenglamalarga keltiriladi. Ushbu maqolada bugungi kunda, kubatur, kvadratur formulalarni qurish va ularni tatbiq etish bo'yicha bir qator, jumladan: differensiallanuvchi funksiyalarning gilbert fazolarida kvadratur, kubatur formulalarning xatolik funksionallari ekstremal funksiyalarini topish.*

Kirish

Mustaqillik yillarida mamlakatimizda amaliy tatbiqqa ega bo'lgan dolzarb yo'nalishlarga e'tibor kuchaytirildi, xususan, hisoblash matematikasining kubatur formulalar nazariyasi bo'yicha yuqori algebraik aniqlik darajasiga ega bo'lib, biror bir muntazam ko'pyoqning aylanishlar gruppasi akslantirishlariga nisbatan invariant hamda ortogonal ko'phadlar nazariyasiga asoslangan Gauss tipidagi kubatur formulalarni qurishga alohida e'tibor qaratildi. Hosilasi kvadrati bilan integrallanuvchi davriy va davriy emas, bir va ko'p o'zgaruvchili funksiyalarning Sobolev fazolarida panjarali optimal kubatur formulalar qurish sezilarli natijalarga erishild.Ushbu ishda to'g'ri kasr tartibining integral tenglamasini taxminiy echish uchun kvadratik formulalarning maqbul koeffitsientlari topildi, bundan tashqari, Sobolev fazosidagi kvadratik formulalarning xatoligi uchun funktsional normaning kvadrati hisoblab chiqildi.

Kalit so'zlar : Singulyar integral, Gilbert almashtirishlar, Furye koeffitsientlari, kvadratur formula, trigonometrik ko'phad,

$$f(x)=\frac{\varphi(x)}{|x-c|^{\alpha}} \quad (0<\alpha<1).$$

Biz ushbu ko'rinishdagi maxsuslikka ega bo'lgan integrallarni hisoblash masalasini ko'rganmiz .

Ko'p tatbiqiy masalalarida , jumladan aerodinamikada , shunday integrallar uchraydiki , ularda $\alpha=1$ bo'ladi . Bunday holda integrallarni Koshi bo'yicha bosh qiymat ma'nosida tushunish kerak .

Agar $f(x)$ funksiya $[a,b]$oraliqning c nuqtasi atrofida chegaralanmagan bo'lib ,

$$\lim_{\varepsilon\to\infty}\left[\int_a^{c-\varepsilon} f(t)dt+\int_{c+\varepsilon}^{b} f(t)dt\right]$$

limit mavjud bo'lsa , bu limit $[a,b]$ oraliq bo'yicha $f(x)$ funksiyadan olingan xosmas integrallarning Koshi bo'yicha bosh qiymati deyiladi va

$$V.p.\int_a^b f(x)dx, yoki, \int_a^b f(x)dx$$

kabi belgilanadi .(Bu yerda $V.p.$ <<valeur principale>> so'zlaning bosh harflari bo'lib , fransuzcha <<bosh qiymat>> ni bildiradi).

Bosh qiymat ma'nosidagi integrallarni ko'pincha maxsus yoki singulyar integallar deb atashadi.

Misol. Faraz qilaylik , $f(x)=\dfrac{1}{x-c}$, $c\in(a,b)$ bo'lsin.U holda

$$\int_a^{c-\varepsilon_1}\frac{dx}{x-c}+\int_{c+\varepsilon_2}^b\frac{dx}{x-c}=\ln\frac{b-c}{c-a}+\ln\frac{\varepsilon_1}{\varepsilon_2}. \qquad (1.1)$$

Ko'rinib turibdiki , ε_1 va ε_2 ixtiyoriy ravishda nolga intilsa , bu yig'indining limiti mavjud bo'lmaydi , ya'ni $\int_a^b\dfrac{dx}{x-c}$ hosmas integrallarni mavjud bo'maydi.

Bu yerda $\varepsilon_1=\varepsilon_2=\varepsilon$ deb olamiz . U holda $\varepsilon\to 0$ da (1.1) ifodaning limiti mavjud bo'lib ta'rifga ko'ra $\int_a^b\dfrac{dx}{x-c}$ integrallarning bosh qiymatini beradi:

$$V.p.\int_a^b\frac{dx}{x-c}=\ln\frac{b-c}{c-a}. \qquad (1.2)$$

Ta'rif. Agar ixtiyoriy $x_1,x_2\in[a,b]$ nuqtalar uchun

$$|f(x_1)-f(x_2)|\le L|x_1-x_2|^\alpha$$

tengsizlik o'rinli bo'lsa , u holda $f(x)$ funksiya $[a,b]$ oraliqda Gelder shartini qanoatlantiradi deyiladi , bu yerda L va α qandaydir musbat miqdorlar. Agar $\alpha=1$ bo'lsa , u holda $f(x)$ Lipshits shartini qanoatlantiradi deyiladi . Biz doim $0<\alpha\le 1$ deb olamiz. Ko'rinib turibdiki, $[a,b]$ da Gelder shartini qanoatlantiradigan funksiya shu oraliqda uzluksizdir .

Faraz qilaylik , $y\in(a,b)$ ixtiyoriy nuqta bo'lsin , $\int_a^b\dfrac{f(x)}{x-y}dx$ integralda $K(x,y)=\dfrac{1}{x-y}$ Koshi yadrosi deyiladi va integralning o'zi Koshining sigulyar integrali deyiladi .

1-Teorema. Agar $f(x)$ funksiya $[a,b]$ oraliqda Gelder shartini qanoatlantirsa, u holda Koshining singulyar integrali bosh qiymat ma'nosida mavjuddir .

Isbot . Xaqiqatan ham ,

$$\int_a^b \frac{f(x)}{x-y}dx = \int_a^b \frac{f(x)-f(y)}{x-y}dx + f(y)\int_a^b \frac{dx}{x-y} \qquad (1.3)$$

Gelder shartiga ko'ra $\left|\frac{f(x)-f(y)}{x-y}\right| \le \frac{L}{|x-y|^{1-\alpha}}$ $(\alpha > 0)$, shuning uchun ham (1.3) ning o'ng tomonidagi integral xosmas , integral sifatida mavjud va (1.2) formulaga ko'ra ikkinchi integral ham mavjuddir. Bundan esa $\int_a^b \frac{f(x)dx}{x-y}$ integralning bosh qiymat ma'nosida mavjudligi kelib chiqadi :

$$\int_a^b \frac{f(x)}{x-y}dx = \int_a^b \frac{f(x)-f(x)}{x-y}dx + f(y)\ln\frac{b-y}{y-a}.$$

Yana singulyar integralga misol sifatida Gilbert almashtirishlarini olishimiz mumkin.

$$\psi(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \varphi(x) ctg\frac{x-y}{2}dx,$$

$$\varphi(y) = -\frac{1}{2\pi}\int_{-\pi}^{\pi} \psi(x) ctg\frac{x-y}{2}dx,$$

Bu yerda $\varphi(x)$ va $\psi(x)$ funksiyalar $[-\pi, \pi]$da Gelder shartini qanoatlantiradi va shu bilan birga :

$$\int_{-\pi}^{\pi} \varphi(x)dx = \int_{-\pi}^{\pi} \psi(x)dx = 0$$

Ko'rsatish mumkin , sinkx va coskx barcha k=1,2, ... uchun gilbert almashtirishlari bo'ladi :

$$\left.\begin{aligned} \cos ky &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin kx\, ctg\frac{x-y}{2}dx, \\ \sin ky &= -\frac{1}{2\pi}\int_{-\pi}^{\pi} \cos kx\, ctg\frac{x-y}{2}dx. \end{aligned}\right\} \qquad (1.4)$$

2.Gilbert yadroli singulyar integrallarni taqribiy hisoblash.

Qulaylik uchun Gilbert integralini almashtirish yordamida quyidagi ko'rinishida yozib olamiz:

$$If(y) = \int_0^1 f(x) ctg\pi(x-y)dx. \qquad (1.5)$$

Odatda Gilbert integralida $f(x)$ funksiyasini Gelder shartini qanoatlantirishidan tashqari , u davriy funksiya deb qaraladi . Biz bu yerda $f(x)$ funksiyasining Furye koeffitsientlari

$$f(x) \sim \sum_{n=-\infty}^{\infty} c_n e^{2\pi i n x} \qquad (1.6)$$

Quyidagi

$$|C_n| \le \frac{c}{|n|\alpha} \; (n \ne 0) \qquad (1.7)$$

Shartini qanoatlantiradi va $\alpha < 1$ deb faraz qilib , (1.5) integral uchun kvadratur formula tuzamiz.

Buning uchun trigonometrik ko'phadni quyidagicha kiritamiz:

$$P_N(x) = \sum_{m=-N+1}^{N-1} \tilde{C}_m e^{2\pi i m x}, \qquad (1.8)$$

$$\tilde{C}_m = \frac{1}{2N} \sum_{k=1}^{2N} f\left(\frac{k}{2N}\right) e^{-2\pi i \frac{km}{2N}} \qquad (1.9)$$

Endi (1.9) ni (1.8) ga qo'ysak,

$$P_N(x) = \frac{1}{2N} \sum_{k=1}^{2N} f\left(\frac{k}{2N}\right) \psi_k(x)$$

ga ega bo'lamiz , bu yerda

$$\psi_k(x) = \sum_{m=-N+1}^{N-1} e^{2\pi i m\left(x - \frac{k}{2N}\right)} = 1 + 2\sum_{m=1}^{N-1} \cos 2\pi m\left(x - \frac{k}{2N}\right).$$

(1.4) formula yordamida

$$I\psi_k(y) = -2\sum_{m=1}^{N-1} \sin 2\pi m\left(y - \frac{k}{2N}\right) = \varphi_k(y)$$

ni hosil qilamiz . Endi

$$f(x) = P_N(x) + r_N(x) \qquad (1.10)$$

deb olib , buni (1.5) integralga qo'ysak ,

$$If(y) = \frac{1}{2N} \sum_{k=1}^{2N} f\left(\frac{k}{2N}\right) \varphi_k(y) + R_N(y) \qquad (1.11)$$

kvadratur formulani hosil qilamiz .

2-Teorema. Agar $f(x)$ funksiyasining Furye koeffitsientlari (1.7) shartni qanoatlantirsa va $\alpha < 1$ bo'lsa , u holda (1.11) kvadratur formulaning qoldiq hadi uchun .

$$\max_{0 \le y \le 1} |R_N(y)| \le \frac{4c}{\alpha - 1} \cdot \frac{1}{N^{\alpha - 1}} \qquad (1.12)$$

baho o'rinlidir . Bu yerda C o'zgarmas son.

Isbot. Shartga ko'ra $\alpha < 1$, shuning uchun ham (1.7) dan ko'ramizki,

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{2\pi i n y} \qquad (1.13)$$

Furye qatori absolyut yaqinlashadi. Faraz qilaylik, $R_N(y)$ ning Furye yoyilmasi

$$R_N(y) = \sum_{n=-\infty}^{\infty} C_n e^{2\pi i n x} \qquad (1.14)$$

bo'lsin . (1.14) formulalardan

$$Ie^{2\pi i n m y} = i \cdot signm \cdot e^{2\pi i n m y} \qquad (1.15)$$

ekanligi ravshan . Endi (1.9),(1.10),(1.13) va (1.15)dan koeffitsientlarning quyidagiga tengligini ko'ramiz:

$$\overset{\square}{c}_m + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} c_{n+m} \left(\frac{1}{2N} \sum_{k=1}^{2N} e^{2\pi i \frac{nk}{2N}} \right) = c_m + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} c_{2nN+m}$$

$$C_m = \begin{cases} (c_m - \overset{\square}{c}_m) isignm, |m| < N \quad bo'lsa, \\ c_m \square signm, |m| \geq N \;\; bo'lsa. \end{cases} \qquad (1.16)$$

Ushbu bevosita ko'rinib turgan

$$\frac{1}{2N} \sum_{k=1}^{2N} e^{2\pi i \frac{nk}{2N}} = \begin{cases} 1, agar \; n \;\; 2N \;\; ga \; bo'linsa, \\ 0, agar \; n \; 2N \;\; ga \; bo'linmasa, \end{cases}$$

Tengliklar va

$$f(x) e^{-2\pi i m x} = \sum_{n=-\infty}^{\infty} c_n e^{2\pi i (n-m)x} = \sum_{n=-\infty}^{\infty} c_{n+m} e^{2\pi i n x}$$

dan foydalanib , $\overset{\square}{c}_m$ uchun quyidagiga ega bo'lamiz :

$$\overset{\square}{c}_m = \frac{1}{2N} \sum_{k=1}^{2N} \sum_{n=-\infty}^{\infty} c_{n+m} e^{2\pi i \frac{nk}{2N}} =$$

$$\overset{\square}{c}_m + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} c_{n+m} \left(\frac{1}{2N} \sum_{k=1}^{2N} e^{2\pi i \frac{nk}{2N}} \right) = c_m + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} c_{2nN+m}$$

Demak ,

$$\left| \overset{\square}{c}_m - c_m \right| = \left| \sum_{n=-\infty, n\neq 0}^{\infty} C_{2nN+m} \right|. \qquad (1.17)$$

Endi (1.4) , (1.15) va (1.17) dan quyidagini hosil qilamiz:

$$|R_N(y)| \leq \sum_{m=-\infty}^{\infty} |c_m| = \sum_{|m|<N} \left| c_m - \overset{\square}{c}_m \right| + \sum_{|m|\geq N} |c_m| =$$

$$= \sum_{|m|<N} \left| \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} C_{2nN+m} \right| + \sum_{|m|\geq N} |c_m| = 4 \sum_{m=N}^{\infty} |c_m| - 2 \sum_{m=1}^{\infty} \left| c_{(2m+1)N} \right|$$

Demak ,

$$\left|R_N(y)\right| \le 4\sum_{m=N}^{\infty}\left|c_m\right|$$

Bundan va (1.7) dan teoremaning tasdig'i kelib chiqadi.

XULOSA.

Ushbu ishda eksponent optimal kvadratura formulasi Sobolev fazosida o'lchov bilan qurilgan . Dastlab, ushbu kvadratura formulasi uchun funktsional xato normasini hisoblash uchun ekstremal funktsiya topildi. Keyin, ushbu ekstremal funktsiyadan foydalanib, xato funktsional normasi aniqlandi. Eksponensial optimal kvadratura formulasining mavjudligi va yaqinlashishi namoyish etildi. Optimal kvadratura formulasining koeffitsientlari uchun chiziqli tenglamalar tizimi olindi. Furye qatorining yoyib chiqib va Koshi teoremasidan foydalangan holda Optimal kvadratur formula qurildi va bir nechta hisoblashlar orqali o'z tasdig'ini topdi . Nihoyat, optimal farq topildi.

ЛИТЕРАТУРА:

1. *Шадиметов Х.М., Далиев Б.С.* Экстремальная функция квадратурных формул для приближенного решения обобщенного интегрального уравнения Абеля. // Проблемы вычислительной и прикладной математики. є2(20), 2019, с. 88-95.

2. *Соболев С.Л.* Введение в теорию кубатурных формул. М. 1974. 808с

3. *Шадиметов Х.М., Далиев Б.С.* Коэффициенты оптимальных квадратурных формул для приближенного решения общего интегрального уравнения Абеля. // Проблемы вычислительной и прикладной математики. є2(26) 2020, с. 24-32.

4. *Kh.Shadimetov, B.Daliyev* Composite optimal formulas for approximate integration of weight integrals // AIP Conference Proceedings. 2020, 20p.

5. Israilov M.I., Shamsiev E.A. Nekotorыe kubaturnыe formulы dlya shestimernogo prostranstva // Voprosы vыchisl. i prikl. matematiki: Sb. nauch. tr. – Tashkent: IK AN RUz, 1983. - vыp. 71. - S. 3-14.

6. Shamsiddinovich, M. R., & Obidjonovich, Z. N. (2021). Advantages and Improvements of ETextbook Teaching of Computer Science in General Secondary Educatin. CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES, 2(12), 71-74.

7. Shushbaev S.Sh., Shamsiev E.A. Postroenie kubaturnых formul, invariantnых otnositelno grupp selochislennых avtomorfizmov sovershennых kvadratichnых form // Dokladы AN Uzbekistana. – Tashkent, 1982. - № 3. - S.

8. Begaliyevich, N. C., Obidjonovich, N. Z., & Bahodir og'li, A. O. (2022). SOLVING MULTIDIMENSIONAL PROBLEMS WITH A WEAK APPROXIMATION METHOD. Galaxy International Interdisciplinary Research Journal, 10(5), 949-955.

9. Ismatullaev G.P. O postroenii kubaturnых formul dlya gipershara i poverxnosti sferы // Voprosы vычisl. i prikl. matematiki: Sb. nauch. tr. – Tashkent: IK AN RUz, 1975. - vыp. 32. - S. 60-68.

10. Nurmatov, Z., To'rayev, D., & Pirimova, F. (2023). TIXONOV BO 'YICHA KORREKTLIK VA KORREKTLIK TO 'PLAMI. NORMAL YECHIMNI TOPISHNING REGULYARLASHTIRISH USULI. Евразийский журнал академических исследований, 3(5 Part 3), 28-37.