TEMPERLANGAN HILFER OPERATORI BUZILADIGAN ODDIY DIFFERENSIAL TENGLAMA UCHUN KOSHI MASALASI

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Annotatsiya: *Ushbu maqolada temperlangan Hilfer operatori buziladigan oddiy differensial tenglama uchun Koshi masalaning yechimi yaqqol ko'rinishda topilgan.*

Kalit so'zlar: *Kasr taribli operatorlar, Kaputo kasr tartibli hosila operator, Riman-Liuvill kasr tartibli hosila operatori, integral tenglama.*

ЗАДАЧА КОШИ ДЛЯ ОБЫКНОВЕННОГО ДИФФЕРЕНЦИАЦИАЛЬНОГО УРАВНЕНИЯ С ВЫРОЖДАЮЩИМСЯ УСТОЙЧИВЫМ ОПЕРАТОРОМ ХИЛЬФЕРА

Аннотация: *В статье найдено явное решение задачи Коши для обыкновенного дифференциационального уравнения с вырождающимся устойчивым оператором Хильфера.*

Ключевые слова: *Дробные операторы, оператор дробной производной Капуто, оператор дробной производной Римана-Лиувилля, интегральное уравнение.*

THE CAUCHY PROBLEM FOR AN ORDINARY DIFFERENTIAL EQUATION WHICH THE TEMPERED HILFER OPERATOR

Abstract: In *this article, the solution of the Cauchy problem for an ordinary differential equation in which the tempered Hilfer operator is found an explicit form.*

Keywords: *Fractional operators, Caputo fractional derivative operator, Riemann-Liouville fractional derivative operator, integral equation.*

Ma'lumki oxirgi yillarda kasr tartibli differensial tenglamalarga qiziqish tobora ortib bormoqda. Bu bir tomondan, matematik umumlashma sifatida oldingi natijalarni o'z ichiga olsa, boshqa tomondan ko'pgina amaliy jarayonlarning matematik modelida turli kasr tartibli differensial tenglamalar ishlatilmoqda [1,2]. Bunday differensial tenglamalarni tadqiq etishda boshlang'ich shartli masalaning yaqqol ko'rinishdagi yechimlari muhim rol o'ynaydi [3]. Bunday masalalarni tadqiq etishda Laplas almashtirishlari, operator usullar yoki integral tenglamalarga keltirib ishlash usullaridan foydalaniladi [4].

Kasr tartibli operatorlarning asosiylari Riman-Liuvill va Kaputo ma'nosidagi operatorlar bo'lsa, ularning turli umumlashmalari ko'p ishlarda tadqiq etilmoqda [5].

Ushbu ishda temperlangan Hilfer operatori buziladigan oddiy differensial tenglama uchun Koshi masalasini tadqiq etamiz.

$(0,T)$ oraliqda

$$ {}^{TH}D_{0t}^{\alpha,\beta,\gamma} y(t) + \lambda y(t) = f(t) \qquad (1) $$

tenglamani qaraylik, bu yerda ${}^{TH}D_{0t}^{\alpha,\beta,\gamma}$ – temperlangan Hilfer kasr tartibli operator bo'lib,

$$ {}^{TH}D_{0t}^{\alpha,\beta,\gamma} y(t) = e^{-\gamma t} I_{0t}^{\beta(1-\alpha)} \frac{d}{dt} I_{0t}^{(1-\beta)(1-\alpha)} \left(e^{\gamma t} y(t) \right) \qquad (2) $$

ko'rinishda aniqlangan,

$$ I_{0t}^{\varphi} g(t) = \frac{1}{\Gamma(\varphi)} \int_0^t (t-s)^{\varphi-1} g(s) ds \qquad (3) $$

Riman-Liuvill ma'nosidagi kasr tartibli integral operator, $\alpha,\beta,\gamma,\lambda \in \mathbb{R}$, $\gamma \geq 0$, $0<\alpha<1$, $0 \leq \beta \leq 1$, $f(t)$ - berilgan funksiya.

Agar $\beta = 0$ ${}^{TH}D_{0t}^{\alpha,\beta,\gamma}$ dan ${}^{THL}D_{0t}^{\alpha,\gamma}$ temperlangan Riman-Liuvill kasr tartibli hosila operator bo'ladi, agar $\beta = 1$ ${}^{TH}D_{0t}^{\alpha,\beta,\gamma}$ dan ${}^{TC}D_{0t}^{\alpha,\gamma}$ temperlangan Kaputo kasr tartibli hosila operator bo'ladi.

Koshi masalasi. (1) tenglamani va

$$ \lim_{t\to 0} I_{0t}^{(1-\beta)(1-\alpha)} \left(e^{\gamma t} y(t) \right) = A \qquad (4) $$

$t^{-(1-\beta)(1-\alpha)} y(t) \in C[0,t]$ shartlarni qanoatlantiruvchi funksiya topilsin, bu yerda A berilgan haqiqiy son.

(1) tenglamani (3) dan foydalanib,

$$ e^{-\gamma t} I_{0t}^{\beta(1-\alpha)} \frac{d}{dt} I_{0t}^{(1-\beta)(1-\alpha)} \left(e^{\gamma t} y(t) \right) + \lambda y(t) = f(t) \qquad (5) $$

ko'rinishda yozib olamiz.

(5) ni $e^{\gamma t}$ ga ko'paytiramiz,

$$ I_{0t}^{\beta(1-\alpha)} \frac{d}{dt} I_{0t}^{(1-\beta)(1-\alpha)} \left(e^{\gamma t} y(t) \right) + \lambda e^{\gamma t} y(t) = e^{\gamma t} f(t) \qquad (6) $$

(6) ni hosil qilamiz.

Quyidagi belgilashlarni kiritaylik:

$e^{\gamma t} y(t) = Y(t)$, $e^{\gamma t} f(t) = F(t)$.

U holda (6) tenglamani

$$ I_{0t}^{\beta(1-\alpha)} \frac{d}{dt} I_{0t}^{(1-\beta)(1-\alpha)} Y(t) + \lambda Y(t) = F(t) \qquad (7) $$

ko'rinishda yozib olamiz.

(7) ga $D_{0t}^{\beta(1-\alpha)}$ operatorini ta'sir qildirib, $D_{0t}^{\beta(1-\alpha)} I_{0t}^{\beta(1-\alpha)} \Phi(t) = \Phi(t)$ tenglikdan foydalanib,

$$\frac{d}{dt}I_{0t}^{(1-\beta)(1-\alpha)}Y(t)+\lambda\frac{1}{\Gamma(1-\beta(1-\alpha))}\frac{d}{dt}\int_0^t(t-s)^{-\beta(1-\alpha)}Y(s)ds=$$

$$=\frac{1}{\Gamma(1-\beta(1-\alpha))}\frac{d}{dt}\int_0^t(t-s)^{-\beta(1-\alpha)}F(s)ds \qquad (8)$$

tenglamani hosil qilamiz. (8) tenglamani $[0,t]$ da integrallab, (2) shartdan foydalanib,

$$I_{0t}^{(1-\beta)(1-\alpha)}Y(t)+\lambda I_{0t}^{1-\beta(1-\alpha)}Y(t)=I_{0t}^{1-\beta(1-\alpha)}F(t)+A \qquad (9)$$

tenglamani hosil qilamiz.

(9) ga $D_{0t}^{(1-\beta)(1-\alpha)}$ operatorni ta'sir qildirib, $D_{0t}^{(1-\beta)(1-\alpha)}I_{0t}^{(1-\beta)(1-\alpha)}\Phi(t)=\Phi(t)$ va $I_{0t}^{\alpha_1}I_{0t}^{\beta_1}=I_{0t}^{\alpha_1+\beta_1}$ tengliklardan foydalanib,

$$Y(t)+\lambda\frac{d}{dt}I_{0t}^{1-(1-\beta)(1-\alpha)+1-\beta(1-\alpha)}Y(t)=$$

$$=\frac{d}{dt}I_{0t}^{1-(1-\beta)(1-\alpha)+1-\beta(1-\alpha)}F(t)+\frac{d}{dt}I_{0t}^{1-(1-\beta)(1-\alpha)}A \qquad (10)$$

(10) ni hosil qilamiz. Ba'zi hisoblashlarni amalga oshirib (10) ni

$$Y(t)+\lambda I_{0t}^{\alpha}Y(t)=I_{0t}^{\alpha}F(t)+A\frac{t^{-(1-\beta)(1-\alpha)}}{\Gamma(1-(1-\beta)(1-\alpha))}$$

ko'rinishda yozib olamiz. $I_{0t}^{\alpha}F(t)+A\frac{t^{-(1-\beta)(1-\alpha)}}{\Gamma(1-(1-\beta)(1-\alpha))}=g(t)$ belgilashni kiritib,

$$Y(t)+\lambda\int_0^t\frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}Y(s)ds=g(t) \qquad (11)$$

integral tenglamani hosil qilamiz.

(11) integral tenglama yadrosi $K(t,s)=\frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}$ ko'rinishda bo'lib, $0<\alpha<1$ bo'lgani uchun $K(t,s)$ yadro sust maxsuslikka ega bo'ladi.

Iteratsiyalangan yadrolarni $K_i(t,s)=\int_s^t K(t,\xi)K_{i-1}(\xi,s)d\xi$, $K_1(t,s)=K(t,s)$, formula bilan hisoblaymiz. Ko'rsatish qiyin emaski iteratsiyalangan yadrolar

$$K_1(t,s)=\frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)},K_2(t,s)=\frac{(t-s)^{2\alpha-1}}{\Gamma(2\alpha)},\dots,K_n(t,s)=\frac{(t-s)^{\alpha n-1}}{\Gamma(\alpha n)} \qquad (12)$$

ko'rinishda bo'ladi. (12) dan foydalanib, (11) integral tenglamaning yechimini

$$Y(t)=g(t)+\sum_{n=1}^{+\infty}(-\lambda)^n\int_0^t\frac{(t-s)^{\alpha n-1}}{\Gamma(\alpha n)}g(s)ds \qquad (13)$$

ko'rinishda topamiz.

$$g(t)=I_{0t}^{\alpha}F(t)+A\frac{t^{-(1-\beta)(1-\alpha)}}{\Gamma\left(1-(1-\beta)(1-\alpha)\right)}$$

belgilashdan foydalanib (13) ni

$$Y(t)=I_{0t}^{\alpha}F(t)+\sum_{n=1}^{+\infty}(-\lambda)^{n}I_{0t}^{\alpha n}\left(I_{0t}^{\alpha}F(t)\right)+A\frac{t^{-(1-\beta)(1-\alpha)}}{\Gamma\left(1-(1-\beta)(1-\alpha)\right)}+$$

$$+\frac{A}{\Gamma\left(1-(1-\beta)(1-\alpha)\right)}\sum_{n=1}^{+\infty}(-\lambda)^{n}I_{0t}^{\alpha n}(t^{-(1-\beta)(1-\alpha)})$$

ko'rinishda yozib olamiz.

Ba'zi hisoblashlarni amalga oshirib, $Y(t)$ ni

$$Y(t)=At^{-(1-\beta)(1-\alpha)}E_{\alpha,1-(1-\beta)(1-\alpha)}\left[-\lambda t^{\alpha}\right]+\int_{o}^{t}(t-s)^{\alpha-1}E_{\alpha,\alpha}\left[-\lambda(t-s)^{\alpha}\right]F(s)ds$$

(14)

ko'rinishda yozib olamiz, bu yerda $E_{\alpha_1,\beta_1}(z)=\sum_{n=0}^{+\infty}\frac{z^{n}}{\Gamma(\alpha_1 n+\beta_1)}$ Mittag-Leffler funksiya.

$Y(t)=e^{\gamma t}y(t)$, $F(t)=e^{\gamma t}f(t)$ belgilashdan foydalanib (14) ni

$$e^{\gamma t}y(t)=At^{-(1-\beta)(1-\alpha)}E_{\alpha,1-(1-\beta)(1-\alpha)}\left[-\lambda t^{\alpha}\right]+\int_{0}^{t}(t-s)^{\alpha-1}E_{\alpha,\alpha}\left[-\lambda(t-s)^{\alpha}\right]e^{\gamma s}f(s)ds$$

ko'rinishda yozib olamiz.

Oxirgi tenglikni $e^{-\gamma t}$ ga ko'paytiramiz, masalaning formal yechimini

$$y(t)=Ae^{-\gamma t}t^{-(1-\beta)(1-\alpha)}E_{\alpha,1-(1-\beta)(1-\alpha)}\left[-\lambda t^{\alpha}\right]+$$

$$+e^{-\gamma t}\int_{0}^{t}(t-s)^{\alpha-1}E_{\alpha,\alpha}\left[-\lambda(t-s)^{\alpha}\right]e^{\gamma s}f(s)ds \qquad (15)$$

ko'rinishda topamiz.

Teorema. Agar $f(x)\in C^{1}\left[0,T\right]$ bo'lsa u holda (15) formula bilan aniqlangan $y(t)$ funksiya $\{(1),(2)\}$ masalaning yechimi bo'ladi.

Isbot. Dastlab (15) ni (1) tenglamani qanoatlantirishini ko'rsatamiz.

(15) ni $y(t)=y_1(t)+y_2(t)$ ko'rinishda yozib olamiz, bu yerda

$$y_1(t)=Ae^{-\gamma t}t^{-(1-\beta)(1-\alpha)}E_{\alpha,1-(1-\beta)(1-\alpha)}\left[-\lambda t^{\alpha}\right], \qquad (16)$$

$$y_2(t)=e^{-\gamma t}\int_{0}^{t}(t-s)^{\alpha-1}E_{\alpha,\alpha}\left[-\lambda(t-s)^{\alpha}\right]e^{\gamma s}f(s)ds. \qquad (17)$$

(16) dan foydalanib,

$$I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_1(t)\right)=AI_{0t}^{(1-\beta)(1-\alpha)}\left(t^{-(1-\beta)(1-\alpha)}E_{\alpha,1-(1-\beta)(1-\alpha)}\left[-\lambda t^{\alpha}\right]\right)=$$

$$=A\frac{1}{\Gamma((1-\beta)(1-\alpha))}\int_0^t(t-s)^{(1-\beta)(1-\alpha)-1}s^{-(1-\beta)(1-\alpha)}E_{\alpha,1-(1-\beta)(1-\alpha)}\left[-\lambda s^{\alpha}\right]ds=$$

$$=A\frac{1}{\Gamma((1-\beta)(1-\alpha))}\sum_{n=0}^{+\infty}\frac{(-\lambda)^n}{\Gamma\left(\alpha n+1-((1-\beta)(1-\alpha))\right)}\int_0^t(t-s)^{(1-\beta)(1-\alpha)-1}s^{\alpha n-(1-\beta)(1-\alpha)}ds$$

tenglikni yozib olamiz. Bu tenglikda $s=tz$ almashtirish bajaramiz:

$$A\frac{1}{\Gamma((1-\beta)(1-\alpha))}\sum_{n=0}^{+\infty}\frac{(-\lambda)^n t^{(1-\beta)(1-\alpha)+\alpha n-(1-\beta)(1-\alpha)}}{\Gamma\left(\alpha n+1-((1-\beta)(1-\alpha))\right)}\int_0^1(1-z)^{(1-\beta)(1-\alpha)-1}z^{\alpha n-(1-\beta)(1-\alpha)+1-1}dz=$$

$$=A\frac{1}{\Gamma((1-\beta)(1-\alpha))}\sum_{n=0}^{+\infty}\frac{(-\lambda)^n t^{\alpha n}}{\Gamma\left(\alpha n+1-((1-\beta)(1-\alpha))\right)}\times$$
$$\times B\left(\alpha n-(1-\beta)(1-\alpha)+1,(1-\beta)(1-\alpha)\right)$$

$B(a,b)=\dfrac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ formuladan foydalanib,

$$I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_1(t)\right)=AE_{\alpha,1}(-\lambda t^{\alpha})\qquad(18)$$

tenglikni hosil qilamiz.

$$\frac{d}{dt}E_{\alpha,1}\left[-\lambda t^{\alpha}\right]=-\lambda t^{\alpha-1}E_{\alpha,\alpha}\left[-\lambda t^{\alpha}\right]$$

formuladan foydalanib,

$$\frac{d}{dt}I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_1(t)\right)=-A\lambda t^{\alpha-1}E_{\alpha,\alpha}\left[-\lambda t^{\alpha}\right]\qquad(19)$$

tenglikni hosil qilamiz. (19) tenglikka $I_{0t}^{\beta(1-\alpha)}$ ni ta'sir qildiramiz,

$$I_{0t}^{\beta(1-\alpha)}\frac{d}{dt}I_{0t}^{(1-\alpha)(1-\beta)}\left(e^{\gamma t}y_1(t)\right)=$$

$$=-\lambda A\frac{1}{\Gamma\left(\beta(1-\alpha)\right)}\int_0^t(t-s)^{\beta(1-\alpha)-1}s^{\alpha-1}\sum_{n=0}^{+\infty}\frac{(-\lambda)^n s^{\alpha n}}{\Gamma(\alpha n+\alpha)}ds=$$

$$=-\lambda A\frac{1}{\Gamma\left(\beta(1-\alpha)\right)}\sum_{n=0}^{+\infty}\frac{(-\lambda)^n}{\Gamma(\alpha n+\alpha)}\int_0^t(t-s)^{\beta(1-\alpha)-1}s^{\alpha-1+\alpha n}ds.$$

Bu integralda ham $s=tz$ almashtirish bajarib,

$$I_{0t}^{\beta(1-\alpha)}\frac{d}{dt}I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_1(t)\right)=$$

$$=-\lambda A\frac{1}{\Gamma\left(\beta(1-\alpha)\right)}\sum_{n=0}^{+\infty}\frac{(-\lambda)^n t^{\beta(1-\alpha)-1+\alpha+\alpha n}}{\Gamma(\alpha n+\alpha)}\int_0^1(1-z)^{\beta(1-\alpha)-1}z^{\alpha+\alpha n-1}dz$$

tenglikni hosil qilamiz.

$B(a,b)=\dfrac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ formuladan foydalanib,

$$I_{0t}^{\beta(1-\alpha)}\frac{d}{dt}I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_1(t)\right)=-\lambda At^{-(1-\beta)(1-\alpha)}E_{\alpha,1-(1-\beta)(1-\alpha)}\left[-\lambda t^{\alpha}\right]$$

(20)

tenglikni hosil qilamiz. (20) tenglikni $e^{-\gamma t}$ ga ko'paytiramiz. ${}^{TH}D_{0t}^{\alpha,\beta,\gamma}y_1(t)$ ni

$${}^{TH}D_{0t}^{\alpha,\beta,\gamma}y_1(t)=-\lambda e^{-\gamma t}At^{-(1-\beta)(1-\alpha)}E_{\alpha,1-(1-\beta)(1-\alpha)}\left[-\lambda t^{\alpha}\right]$$

(21)

ko'rinishda topamiz, topilgan (21) va (16) lardan foydalanib, $y_1(t)$ ni ${}^{TH}D_{0t}^{\alpha,\beta,\gamma}y_1(t)+\lambda y_1(t)=0$ tenglama qanoatlanishi kelib chiqadi.

Endi $y_2(t)$ ni (1) tenglamani qanoatlantirishini ko'rsatamiz.(17) dan foydalanib,

$$I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_2(t)\right)=\frac{1}{\Gamma\left((1-\beta)(1-\alpha)\right)}\int_0^t(t-z)^{(1-\beta)(1-\alpha)-1}dz\int_0^z(z-s)^{\alpha-1}E_{\alpha,\alpha}\left[-\lambda(z-s)^{\alpha}\right]e^{\gamma s}f(s)ds$$

tenglikni yozib olamiz. Integral tartibini almashtirib,

$$I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_2(t)\right)=\frac{1}{\Gamma\left((1-\beta)(1-\alpha)\right)}\int_0^t e^{\gamma s}f(s)ds\int_s^t(t-z)^{(1-\beta)(1-\alpha)-1}E_{\alpha,\alpha}\left[-\lambda(z-s)^{\alpha}\right]dz$$

tenglikni hosil qilamiz. $z=(t-s)\xi+s$ almashtirish bajaramiz. Ba'zi soddalashtirishlar amalga oshirib,

$$I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_2(t)\right)=\int_0^t(t-s)^{-\beta(1-\alpha)}E_{\alpha,1-\beta(1-\alpha)}\left[-\lambda(t-s)^{\alpha}\right]e^{\gamma s}f(s)ds$$

(22)

tenglikni hosil qilamiz. Ko'rsatish qiyin emaski uni

$$I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_2(t)\right)=\frac{d}{dt}\int_0^t(t-s)^{1-\beta(1-\alpha)}E_{\alpha,2-\beta(1-\alpha)}\left[-\lambda(t-s)^{\alpha}\right]e^{\gamma s}f(s)ds$$

ko'rinishda yozib olish mumkin. Bu integralni bo'laklab integrallaymiz,

$$I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_2(t)\right)=$$

$$=\frac{d}{dt}\left[f(0)t^{2-\beta(1-\alpha)}E_{\alpha,3-\beta(1-\alpha)}\left[-\lambda t^{\alpha}\right]+\int_0^t\left[e^{\gamma s}f(s)\right]'(t-s)^{2-\beta(1-\alpha)}E_{\alpha,3-\beta(1-\alpha)}\left[-\lambda(t-s)^{\alpha}\right]ds\right]$$

tenglikni hosil qilamiz.

$$\frac{d}{dt}\left[t^{2-\beta(1-\alpha)}E_{\alpha,3-\beta(1-\alpha)}\left(-\lambda t^{\alpha}\right)\right]=t^{1-\beta(1-\alpha)}E_{\alpha,2-\beta(1-\alpha)}\left(-\lambda t^{\alpha}\right) \qquad (23)$$

formuladan foydalanib,

$$I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_2(t)\right)=$$

$$=f(0)t^{1-\beta(1-\alpha)}E_{\alpha,2-\beta(1-\alpha)}\left[-\lambda t^{\alpha}\right]+\int_0^t(t-s)^{1-\beta(1-\alpha)}E_{\alpha,2-\beta(1-\alpha)}\left[-\lambda(t-s)^{\alpha}\right]\left[e^{\gamma s}f(s)\right]'ds$$

tenglikni hosil qilamiz. (23) formulaga o'xshash ravishdagi hosila formulasidan foydalanamiz,

$$\frac{d}{dt}I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_2(t)\right)=f(0)t^{-\beta(1-\alpha)}E_{\alpha,1-\beta(1-\alpha)}\left[-\lambda t^{\alpha}\right]+$$

$$+\int_0^t\left[e^{\gamma s}f(s)\right]'(t-s)^{-\beta(1-\alpha)}E_{\alpha,1-\beta(1-\alpha)}\left[-\lambda(t-s)^{\alpha}\right]ds \qquad (24)$$

tenglikni hosil qilamiz. (24) ga $I_{0t}^{\beta(1-\alpha)}$ ni ta'sir ettirib,

$$I_{0t}^{\beta(1-\alpha)}\frac{d}{dt}I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_2(t)\right)=$$

$$=\frac{1}{\Gamma\left(\beta(1-\alpha)\right)}\int_0^t f(0)z^{-\beta(1-\alpha)}E_{\alpha,1-\beta(1-\alpha)}\left[-\lambda z^{\alpha}\right](t-z)^{\beta(1-\alpha)-1}dz+$$

$$+\frac{1}{\Gamma\left(\beta(1-\alpha)\right)}\int_0^t(t-z)^{\beta(1-\alpha)-1}dz\int_0^z(z-s)^{-\beta(1-\alpha)}E_{\alpha,1-\beta(1-\alpha)}\left[-\lambda(z-s)^{\alpha}\right]\left[e^{\gamma s}f(s)\right]'ds$$

tenglikni hosil qilamiz. Bu integrallarga mos ravishda $z=t\xi$ va $z=(t-s)\xi+s$ almashtirishlarni bajarib, ba'zi hisoblashlarni amalga oshirib,

$$I_{0t}^{\beta(1-\alpha)}\frac{d}{dt}I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_2(t)\right)=\int_0^t\left[e^{\gamma s}f(s)\right]'E_{\alpha,1}\left[-\lambda(t-s)^{\alpha}\right]ds+f(0)E_{\alpha,1}\left[-\lambda t^{\alpha}\right]$$

tenglikni hosil qilamiz. Bu integralni bo'laklab integrallab, $E_{\alpha,1}[0]=1$ dan foydalanib,

$$I_{0t}^{\beta(1-\alpha)}\frac{d}{dt}I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y_2(t)\right)=$$

$$=e^{\gamma t}f(t)-\lambda\int_0^t(t-s)^{\alpha-1}E_{\alpha,\alpha}\left[-\lambda(t-s)^{\alpha}\right]e^{\gamma s}f(s)ds \qquad (25)$$

tenglikni hosil qilamiz. (25) ni $e^{-\gamma t}$ ga ko'paytirib ${}^{TH}D_{0t}^{\alpha,\beta,\gamma}y_2(t)$ ni

$${}^{TH}D_{0t}^{\alpha,\beta,\gamma}y_2(t)=f(t)-\lambda e^{-\gamma t}\int_0^t(t-s)^{\alpha-1}E_{\alpha,\alpha}\left[-\lambda(t-s)^{\alpha}\right]e^{\gamma s}f(s)ds \quad (26)$$

(26) ni topamiz. (26) va (17) da $y_2(t)$ (1) tenglamani qanoatlantirishi kelib kelib chiqadi.

(18) , (22) va $f(x)\in C^1[0,T]$ lardan foydalanib ko'rsatish qiyin emaski (15) formula (2) shartni qanoatlantirishi kelib chiqadi. Teorema isbotlandi.

Bevosita ko'rsatish qiyin emaski (2) shartni $\lim_{t\to 0} t^{(1-\beta)(1-\alpha)} y(t) = B$ shartga almashtirish mumkin.

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