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HOSILA VA UNING TADBIQLARI

To'ychi Bobojonov
TIQXMMI, Milliy tadqiqot universiteti
"International Hause Tashkent" akademik litseyi matematika fani o'qituvchisi

Annotatsiya: *Hosila - differensial hisobning asosiy tushunchasi bo'lib, u funksiya o'zgarishi tezligini ifodalaydi. Ushbu maqolada hosila, uning ma'nolari, elementar funksiyalarning hosilalari haqida malumot berilgan, hosilaga doir misollar ishlab ko'rsatilgan.*

Kalit so'zlar: *hosila, differensial, limit, orttirma, argument, funksiya, nuqta, urinma, qiymat, algoritm, tenglama, ta'rif, teorema.*

ПРОИЗВОДНАЯ И ЕЕ ПРИМЕНЕНИЕ

Тойчи Бобожонов
TIQXMMI, Национальный исследовательский университет
Учитель математики академического лицея "International Hause Tashkent"

Аннотация: *производная - это основная концепция дифференциального исчисления, которая представляет скорость изменения функции. В этой статье представлена информация о производной, ее значениях, производных элементарных функций, разработаны примеры производной.*

Ключевые слова: *производная, дифференциал, предел, вычитание, аргумент, функция, точка, попытка, значение, алгоритм, уравнение, определение, теорема.*

DERIVATIVE AND ITS APPLICATIONS

Toychi Bobojonov
TIQXMMI, National Research University
Mathematics teacher of "International Hause Tashkent" academic lyceum

Annotation: *the derivative is a fundamental concept of differential calculus that represents the rate at which a function changes. This article provides information about the derivative, its meanings, derivatives of elementary functions, examples of the derivative are shown by producing.*

Key words: *derivative, differential, limit, gain, argument, function, point, urination, value, algorithm, equation, definition, theorem.*

Hosila matematikaning asosiy tushunchalaridan biri hisoblanadi. Hosila matematika, fizika va boshqa fanlarning bir qancha masalalarini yechishda,

xususan har xil jarayonlarning tezliklarini oʻrganishda keng qoʻllaniladi.

$y = f(x)$ funksiyaning x_0 nuqtadagi hosilasi deb, $f(x)$ funksiyaning x_0 nuqtadagi Δy orttirmasini argument orttirmasi Δx ga nisbatining Δx nolga intilgandagi limitiga aytiladi va u, $y', y'(x_0),\ f'(x_0), \frac{dy}{dx}$ lardan biri bilan belgilanadi.

Hosilaning ta'rifiga ko'ra, funksiyaning ixtiyoriy x nuqtadagi hosilasini topish uchun quyidagi algoritmni ko'rsatish mumkin.

1) x ga Δx orttirma beriladi, u holda $y = f(x)$ funksiya ham Δy orttirma oladi va $y + \Delta y = f(x + \Delta x)$ bo'ladi;

2) Funksiyaning Δy orttirmasi topiladi;

$$\Delta y = f(x + \Delta x) - f(x);$$

3) Funksiya orttirmasining argument orttirmasiga nisbati topiladi;

$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x};$

4) Bu nisbatning Δx nolga intilgandagi limiti topiladi;

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x)$$

Berilgan $f(x)$ funksiyaning $f'(x)$ hosilasini topish amaliga funksiyani differensiallash deyiladi.

Agar $y = f(x)$ funksiya biror oraliqda aniqlangan boʻlsa va $f'(x)$ hosila bu oraliqning har bir nuqtasida mavjud boʻlsa, u holda

$f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{\Delta x}$ formula $f'(x)$ hosilani x ning funksiyasi sifatida aniqlaydi. Bundan keyin, agar $y = f(x)$ funksiyani differensiallashda nuqta koʻrsatilmagan boʻlsa, hosilani x ning mumkin boʻlgan barcha qiymatlarida topamiz va $y'(x)$ deb yozamiz.

$f'(x_0)$ ga funksiya hosilasining x_0 nuqtadagi qiymati deyiladi.

$y = f(x)$ egri chiziqning $M_0(x_0; y_0)$ nuqtasiga o'tkazilgan urinmaning k burchak koeffisienti $y = f(x)$ funksiya hosilasining $x = x_0$ nuqtadagi qiymatiga teng. Ya'ni, $k = f'(x_0)$.

$y = f(x)$ egri chiziqning $M_0(x_0; y_0)$ nuqtasiga o'tkazilgan urinmaning tenglamasi

$$y - y_0 = f'(x_0)(x - x_0)$$

formula yordamida tuziladi. Bu yerda $y_0 = f(x_0)$.

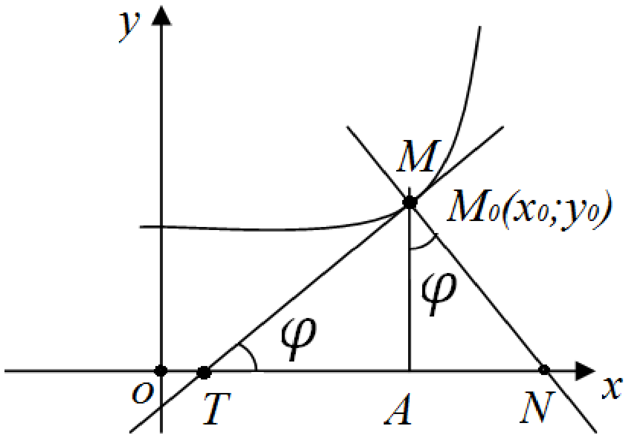
Nuqta Ox o'qi bo'yicha harakat qilib, vaqtning t paytida $x = f(t)$ koordinataga ega bo'lsin, u holda vaqtning t paytida

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}, \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{d^2x}{dt^2}$$ bo'ladi.

Har qanday funksiyaning hosilasini hosilani hisoblash algoritmi bo'yicha aniqlash har doim ham oson emas va ancha murakkab hisoblashlarni talab etadi. Shu sababli amalda $y = f(x)$ funksiyaning hosilasi quyidagi qoidalarni qo'llash yordamida topiladi.

1. $(c)' = 0$ (c-o'zgarmas son).
2. $(cf)' = c \cdot f'$ (c-o'zgarmas son).
3. $(f \pm g)' = f' \pm g'$,
4. $(f \cdot g)' = f' \cdot g + g' \cdot f$.
5. $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - g' \cdot f}{g^2}$ $(g(x) \neq 0)$.

Bu yerda f va g lar x nuqtada hosilaga ega bo'lgan funksiyalardir. Egri chiziqning $M_0(x_0; y_0)$ nuqtasiga o'tkazilgan normal tenglamasi $y - y_0 = -\frac{1}{k}(x - x_0)$ dan iborat bo'ladi (1-chizma).



1-chizma

$TA = y_0 \cdot ctg\varphi$, $AN = y_0 \cdot tg\varphi$ kesmalar mos ravishda urinma osti va normal osti deyiladi. Ularning uzunliklari urinma va normal uzunliklari deyiladi.

Ta'rif. $y = f(x)$ *funksiyaning* x_0 *nuqtadagi o'ng* (*chap*) *hosilasi* deb

$$f'_+(x_0) = \lim_{\Delta x \to 0+} \frac{\Delta y}{\Delta x} \quad \left(f'_-(x_0) = \lim_{\Delta x \to 0-} \frac{\Delta y}{\Delta x} \right)$$ limitga aytiladi.

Misol. $f(x) = |x - 3|$ funksiyaning $x_0 = 3$ nuqtadagi o'ng va chap hosilalarini topamiz. Berilgan funksiyaning $x_0 = 3$ nuqtadagi orttirmasini topamiz:

$$\Delta y = f(3 + \Delta x) - f(3) = |3 + \Delta x - 3| - |3 - 3| = |\Delta x|.$$

U holda

$$\lim_{\Delta x \to 0+} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0+} \frac{|\Delta x|}{\Delta x} = \lim_{\Delta x \to 0+} \frac{\Delta x}{\Delta x} = 1, \quad \lim_{\Delta x \to 0-} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0-} \frac{|\Delta x|}{\Delta x} = \lim_{\Delta x \to 0-} \frac{-\Delta x}{\Delta x} = -1.$$

Bu misolda $f'_{+}(0) \neq f'_{-}(0)$. Shu sababli $f(x)=|x-3|$ funksiya uchun $\Delta x \to 0$ da $\frac{\Delta y}{\Delta x} = \frac{|\Delta x|}{\Delta x}$ nisbatning limiti mavjud emas va $f(x)=|x-3|$ funksiya $x_0 = 3$ nuqtada hosilaga ega bo‘lmaydi.

Funksiya hosilasining yuqorida keltirilgan ta’riflaridan ushbu tasdiqlar kelib chiqadi: agar funksiya x_0 nuqtada hosilaga ega bo‘lsa, funksiya shu nuqtada bir-biriga teng bo‘lgan o‘ng va chap hosilalarga ega bo‘lib, $f'_{+}(x_0) = f'_{-}(x_0) = f'(x_0)$ bo‘ladi; agar funksiya x_0 nuqtada o‘ng va chap hosilalarga ega bo‘lib, $f'_{+}(x_0) = f'_{-}(x_0)$ bo‘lsa, funksiya shu nuqtada hosilaga ega va $f'_{+}(x_0) = f'_{-}(x_0) = f'(x_0)$ bo‘ladi.

Agar $y = f(x)$ funksiyaning hosilasi $f'(x)$ o’z navbatida hosilaga ega bo’lgan funksiya bo’lsa, u holda uning hosilasi ikkinchi tartibli hosila deyiladi va $f''(x)$ deb belgilanadi.

Agar $f''(x)$ ikkinchi tartibli hosila yana hosilaga ega bo’lgan funksiya bo’lsa, u holda uning hosilasi uchunchi tartibli hosila deyiladi va $f'''(x)$ kabi yoziladi.

Xuddi shunday to’rtinchi, beshinchi va xakazo $n-$tartibli hosilalarga ta’rif berish mumkin.

Hosilaning fizik ma’nosi. Faraz qilaylik, harakat qilayotgan moddiy nuqtaning harakat qonuni *s(t)=f(t)*, ya’ni vaqtning uzluksiz funksiyasi ko’rinishida berilgan bo’lsin.

Argument *t* ga Δt orttirma berib s(t) funksiyaning *Δt* orttirmasini topamiz. Ma’lumki, ***Δs(t)=s(t+Δt)-s(t)* bu tenglikdan,**

$\frac{\Delta s(t)}{\Delta t} = \frac{s(t+\Delta t)-s(t)}{\Delta t} = v_{o'rt}$ ni olamiz, ya’ni, $\frac{\Delta s(t)}{\Delta t}$ nisbat harakatdagi moddiy nuqtaning $[t, t+\Delta t]$ vaqt orasidagi o’rtacha tezligini beradi. Hosila ta’rifiga ko’ra :

s`(t)= $\lim_{\Delta x \to 0} \frac{\Delta s}{\Delta t} = v_{oniy}$ ya’ni harakatdagi moddiy nuqtaning yo’l tenglamasidan vaqt bo’yicha olingan hosila moddiy nuqtaning *t* vaqt momentidagi oniy tezligini beradi.

Umulashtirgan holda, agar $y = f(x)$ funksiya biror fizik jarayonni ifodalasa, u holda y' hosila bu jarayonnig ro‘y berish tezligini ifodalaydi deyish mumkin. Bu jumla ***hosilaning fizik ma’nosini*** anglatadi.

Hosilaning geometrik ma’nosi. Hosilaning geometrik ma’nosi: *y=f(x)* funksiya grafigiga biror $M_0(x_0;f(x_0))$ nuqtada urinma o’tkazish bilan bog’liqdir. *y=f(x)* funksiya grafigiga biror $M_0(x_0; f(x_0))$ nuqtada o’tkazilgan urinma deb, $M_0 M$ kesishuvchini *M* nuqta grafik bo’ylab M_0 nuqtaga intilgandagi limit holatiga aytiladi.

Namuna. $y=x^3$ funksiyaning argument qiymati x dan *x+ Δx* ga orttirmasini toping. Yechish. $f(x)= x^3$, $f(x+\Delta x)=(x+\Delta x)^3$ *Demak*, $\Delta f= f(x+\Delta x)- f(x)= (x+\Delta x)^3-x^3= x^3+3x^2 \Delta x+3x(\Delta x)^2+(\Delta x)^3$.

Shunday qilib, $\Delta f=(3x^2 +3x\Delta x + \Delta x)^2 \Delta x$

f(x)=C funksiyaning hosilasini toping, bunda C – berilgan son.

$$\frac{f(x+h)-f(x)}{h}=\frac{C-C}{h}=0$$

Ayirmali nisbat istalgan h≠0 da nolga teng bo'lgani, ya'ni uning qiymati h→0 da o'zgarmagani uchun bu nisbatning limiti ham nolga teng bo'ladi. Shunday qilib, o'zgarmas sonning hosilasi nolga teng: **(C)`=0**

f(x)=kx+b chiziqli funksiyaning hosilasini toping.

$$\frac{f(x+h)-f(x)}{h}=\frac{k(x+h)+b-(kx+b)}{h}=\frac{kh}{h}=k$$

Ayirmali nisbat istalgan h≠0 da k ga teng bo'lgani uchun bu nisbatning limiti ham h→0 da k ga teng bo'ladi. Demak, **(kx+b)`=k.**

Kimyoviy reaksiyaga kirishish tezligi. $m=m(t)$ funksiya bilan vaqtning t onida reaksiyaga kirishuvchi kimyoviy modda miqdori aniqlanayotgan bo'lsin. Bunda t vaqtning Δt orttirmasiga m kattalikning Δm orttirmasi mos keladi va $\frac{\Delta m}{\Delta t}$ nisbat Δt vaqt oralig'ida kimyoviy reaksiyaning o'rtacha tezligini ifodalaydi. Bu nisbatning Δt nolga intilganidagi limiti, ya'ni

$$v(t)=\lim_{\Delta t\to 0}\frac{\Delta m}{\Delta t} \quad \text{yoki} \quad v(t)=\lim_{\Delta t\to 0}\frac{m(t+\Delta t)-m(t)}{\Delta t}$$

*kimyoviy moddaning t ondagi reaksiyaga kirishish tezligi*ni aniqlaydi.

Tabiatning turli sohalariga tegishli ko'plab masalalarini limitlar orqali topish mumkin. Masalan, agar $p=p(t)$ vaqtning t onida tabletkadagi dori moddasining miqdori bo'lsa, u holda *dori moddasining t ondagi erishi tezligi*

$$v(t)=\lim_{\Delta t\to 0}\frac{\Delta p}{\Delta t}=\lim_{\Delta t\to 0}\frac{p(t+\Delta t)-p(t)}{\Delta t}$$ tenglik bilan aniqlanadi.

Asosiy elementar funksiyalarning hosilalari. Asosiy elementar funksiyalarning hosilalarini topishda cheksiz kichik funksiyalardan, teskari va murakkab funksiyalarni differensiallash formulalaridan hamda yig'indi, ayirma, ko'paytma va bo'linmani differensiallash qoidalaridan foydalanamiz.

1. ***O'zgarmas funksiya***: $y=C\,(C\in R)$. O'garmas funksiya butun sonlar o'qida o'zgarmas qiymatini saqlagani uchun ixtiyoriy nuqtada uning orttirmasi nolga teng bo'ladi. Shu sababli $(C)'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{0}{\Delta x}=0.$

2. ***Darajali funksiya***: $y=x^{\alpha}$, bunda $\alpha\in R, \alpha\neq 0$. Bu funksiya uchun $x>0$da

$$\Delta y=(x+\Delta x)^{\alpha}-x^{\alpha}=x^{\alpha}\left(\left(1+\frac{\Delta x}{x}\right)^{\alpha}-1\right)$$ bo'ladi.

Bundan $$\frac{\Delta y}{\Delta x}=x^{\alpha}\frac{\left(1+\frac{\Delta x}{x}\right)^{\alpha}-1}{\Delta x}.$$

$\Delta x \to 0$ da $\left(1+\frac{\Delta x}{x}\right)^{\alpha} - 1 \sim \alpha\frac{\Delta x}{x}$ ni hisobga olib, topamiz:

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = x^{\alpha}\lim_{\Delta x\to 0}\frac{\left(1+\frac{\Delta x}{x}\right)^{\alpha}-1}{\Delta x} = x^{\alpha}\lim_{\Delta x\to 0}\frac{\alpha\Delta x}{\Delta x\cdot x} = x^{\alpha}\lim_{\Delta x\to 0}\frac{\alpha}{x} = \alpha x^{\alpha-1}.$$

Demak, $(x^{\alpha})' = \alpha x^{\alpha-1}$.

Xususan, $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$, $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$.

3. *Korsatkichli funksiya*: $y = a^x$, bunda $a \in R, a > 0, a \neq 1$. Bu funksiyaning orttirmasi $\Delta y = a^{x+\Delta x} - a^x = a^x(a^{\Delta x} - 1)$ ga teng bo‘lib, $\frac{\Delta y}{\Delta x} = a^x \cdot \frac{a^{\Delta x}-1}{\Delta x}$ bo‘ladi. Bundan $\Delta x \to 0$ da $a^{\Delta x} - 1 \sim \Delta x \ln a$ ni hisobga olib, topamiz:

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0} a^x \cdot \frac{a^{\Delta x}-1}{\Delta x} = a^x \lim_{\Delta x\to 0}\frac{\Delta x \ln a}{\Delta x} = a^x \ln a.$$

Demak, $(a^x)' = a^x \ln a$.

Xususan, $(e^x)' = e^x$.

4. *Logorifmik funksiya*: $y = \log_a x$, bunda $a \in R,\ a > 0,\ a \neq 1$. $y = \log_a x$ funksiya $x = a^y$ funksiyaga teskari funksiya. Bunda $x'(y) = a^y \ln a$.

U holda $y'(x) == \frac{1}{x'(y)} = \frac{1}{a^y \ln a} = \frac{1}{x \ln a}$.

Demak, $(\log_a x)' = \frac{1}{x\ln a}$.

Xususan, $(\ln x)' = \frac{1}{x}$.

5. *Trigonometrik funksiyalar*. $y = \sin x$ funksiyaning orttirmasi

$$\Delta y = \sin(x+\Delta x) - \sin x = 2\sin\frac{\Delta x}{2}\cos\left(x+\frac{\Delta x}{2}\right)$$

bo‘lib, $\frac{\Delta y}{\Delta x} = \frac{2\sin\frac{\Delta x}{2}\cos\left(x+\frac{\Delta x}{2}\right)}{\Delta x}$.

Bu tenglikdan $\Delta x \to 0$ da $\sin\frac{\Delta x}{2} \sim \frac{\Delta x}{2}$ ni hisobga olib, topamiz:

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{2\frac{\Delta x}{2}\cos\left(x+\frac{\Delta x}{2}\right)}{\Delta x} = \lim_{\Delta x\to 0}\cos\left(x+\frac{\Delta x}{2}\right) = \cos(x+0) = \cos x.$$

Demak, $(\sin x)' = \cos x.$

$y = \cos x$ funksiyaning hosilasini murakkab funksiyaning hosilasi formulasidan foydalanib topamiz:

$$(\cos x)' = \left(\sin\left(\frac{\pi}{2} - x\right)\right)' = \cos\left(\frac{\pi}{2} - x\right) \cdot \left(\frac{\pi}{2} - x\right)' = \cos\left(\frac{\pi}{2} - x\right) \cdot (-1) = -\sin x.$$

Demak, $(\cos x)' = -\sin x.$

$y = tgx$ funksiyaning hosilasini bo‘linmaning hosilasi formulasidan foydalanib topamiz: $(tgx)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)' \cos x - (\cos x)' \sin x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}.$

Demak, $(tgx)' = \frac{1}{\cos^2 x}.$

$y = ctgx$ funksiyaning hosilasini topishda murakkab funksiyaning hosilasi formulasidan foydalanamiz:

$$(ctgx)' = \left(tg\left(\frac{\pi}{2} - x\right)\right)' = \frac{1}{\cos^2\left(\frac{\pi}{2} - x\right)} \cdot (-1) = -\frac{1}{\sin^2 x}.$$

Demak, $(ctgx)' = -\frac{1}{\sin^2 x}.$

6. ***Teskari trigonometrik funksiyalar***. $y = \arcsin x$ funksiya $x = \sin y$ funksiyaga teskari. Bunda $x'(y) = \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$.

U holda $y'(x) == \frac{1}{x'(y)} = \frac{1}{\sqrt{1 - x^2}}$.

Demak, $(\arcsin x)' = \frac{1}{\sqrt{1 - x^2}}.$

$y = \arccos x$ funksiyaning hosilasini $\arcsin x + \arccos x = \frac{\pi}{2}$ formuladan foydalanib topamiz: $(\arccos x)' = \left(\frac{\pi}{2} - \arcsin x\right)' = -(\arcsin x)' = -\frac{1}{\sqrt{1 - x^2}}.$

Demak, $(\arccos x)' = -\frac{1}{\sqrt{1 - x^2}}.$

$y = arctgx$ funksiyaning hosilasini teskari funksiyaning hosilasi formulasidan foydalanib topamiz:

$$(arctgx)' = \frac{1}{(tgy)'} = \cos^2 y = \frac{1}{1 + tg^2 y} = \frac{1}{1 + x^2}.$$

Demak, $(arctgx)' = \frac{1}{1+x^2}$.

$arctgx$ va $arcctgx$ funksiyalar $arctgx + arcctgx = \frac{\pi}{2}$ bog‘lanishga ega.

Bundan $(arcctgx)' = \left(\frac{\pi}{2} - arcctgx\right)' = -(arctgx)' = -\frac{1}{1+x^2}$.

Demak, $(arcctgx)' = -\frac{1}{1+x^2}$.

Hosilaga doir topshiriqlardan namunalar keltiramiz.

1. $y = x^3$ funksiyaning $x = 1$ nuqtadagi hosilasi topilsin.

Yechish: 1) x argumentga Δx orttirma beramiz. U holda y funksiya y Δy orttirma oladi. $y + \Delta y = (x + \Delta x)^3 = x^3 + 3x^2 \cdot \Delta x + 3x \cdot (\Delta x)^2 + (\Delta x)^3$;

2) Δy ni topamiz:

$$\Delta y = (x + \Delta x)^3 - x^3 = x^3 + 3x^2 \cdot \Delta x + 3x \cdot (\Delta x)^2 + (\Delta x)^3 - x^3 =$$
$$= [3x^2 + 3x\Delta x + (\Delta x)^2] \cdot \Delta x;$$

3) $\frac{\Delta y}{\Delta x}$ ni topamiz:

$$\frac{\Delta y}{\Delta x} = \frac{[3x^2+3x\cdot\Delta x+(\Delta x)^2]\cdot\Delta x}{\Delta x} = 3x^2 + 3x \cdot \Delta x + (\Delta x)^2;$$

4) $\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$ ni topamiz: Agar bu limit mavjud bo’lsa, u holda y berilgan funksiyaning hosilasidan iborat bo’ladi.

$$y' = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}[3x^2 + 3x \cdot \Delta x + (\Delta x)^2] = 3x^2.$$

$y'(1) = 3 \cdot 1^2 = 3.$

2. $y = 2x^2 - 2$ parabolaning absissasi $x_0 = -2$ bo’lgan nuqtasiga o’tkazilgan urinmaning tenglamasi tuzilsin.

Yechish: Parabolaga tegishli bo’lgan va absissasi $x_0 = -2$ bo’lgan nuqtaning ordinatasini topamiz:

$$y_0 = y(x_0) = y(-2) = 2 \cdot (-2)^2 - 2 = 2 \cdot 4 - 2 = 6.$$

$y = f(x)$ egri chiziqning $M_0(x_0; y_0)$ nuqtasiga o’tkazilgan urinma tenglamasi $y - y_0 = y'(x_0)(x - x_0)$ dan iborat bo’lgani uchun dastlab y' ni so’ngra $y'(x_0) = y'(2)$ ni topamiz.

$$y' = (2x^2 - 2)' = (2x^2)' - (2)' = 4x - 0 = 4x;$$

$y'(-2) = 4 \cdot (-2) = -8.$

Demak urinma tenglamasi

$y - y_0 = y'(x_0)(x - x_0)$, $y - 6 = -8(x + 2)$, $y = -8x - 10$ dan iborat.

3. $y = x^2 - cosx + 2$ funksiyaning hosilasi topilsin.

Yechish: Funksiyalar yig’indisining hosilasini topish formulasidan foydalanamiz:

$y' = (x^2 - cosx + 2)' = (x^2)' - (cosx)' + (2)' = 2x + sinx + 0 = = 2x + sinx.$

4. Quyidagi funksiyalarning hosilalari topilsin:

1) $y = (2x + 1)(3x - 1)$; 2) $y = x \cdot cosx$; 3) $y = \frac{3+2x}{1+x}$.

Yechish: 1) Ko'paytmani hosilasini toppish qoidasidan foydalanamiz:

$y' = [(2x + 1)(3x\text{-}1)]' = (2x + 1)' \cdot (3x\text{-}1) + +(3x\text{-}1)' \cdot (2x + 1) = 2 \cdot (3x\text{-}1) + 3 \cdot (2x + 1) = 6x\text{-}2 + 6x +$

$$y' = [(2x + 1)(3x\text{-}1)]' = (2x + 1)' \cdot (3x\text{-}1) + +(3x\text{-}1)' \cdot (2x + 1)$$
$$= 2 \cdot (3x\text{-}1) + 3 \cdot (2x + 1) = 6x\text{-}2 + 6x + \ y' = [(2x + 1)(3x\text{-}1)]'$$
$$= (2x + 1)' \cdot (3x\text{-}1) + +(3x\text{-}1)' \cdot (2x + 1) = 2 \cdot (3x\text{-}1) + 3 \cdot (2x + 1)$$
$$= 6x\text{-}2 + 6x +$$

$+3 = 12x + 1$;

2) $y' = (xcosx)' = (x)' \cdot cosx + (cosx)' \cdot x = cosx - x \cdot sinx$;

3) Bu funksiyani hosilasini topish uchun bo'linmani hosilasini topish qoidasidan foydalanamiz: $y' = \left(\frac{3+2x}{1+x}\right)' = \frac{(3+2x)' \cdot (1+x) - (1+x)' \cdot (3+2x)}{(1+x)^2} = \frac{2 \cdot (1+x) - 1 \cdot (3+2x)}{(1+x)^2} = \quad = \frac{2+2x-3-2x}{(1+x)^2} = -\frac{1}{(1+x)^2}$.

5. To'g'ri chiziqli harakat qonuni

$$S = 4t^3 - t^2 + 1\ (m)$$

formula bilan berilgan. Bu harakatning $t = 4c$ bo'lgan paytdagi tezlanishi topilsin.

Yechish: Harakatning t paytdagi tezligi:

$v(t) = s'(t) = (4t^3 - t^2 + 1\ )' = (12t^2 - 2t)\frac{m}{c}$,

t paytdagi tezlanishi esa

$a(t) = v'(t) = (12t^2 - 2t)' = (24t - 2)\frac{m}{c^2}$;

ga teng bo'lib undan $a(4) = 24 \cdot 4 - 2 = 96 - 2 = 94\frac{m}{c^2}$,

6. $y = \frac{1}{3}x^6 + 2x^5 - 4x + 10$ funksiyaning beshinchi tartibli hosilasi topilsin.

Yechish: $y^I = \left(\frac{1}{3}x^6 + 2x^5 - 4x + 10\right)' = 2x^5 + 10x^4 - 4$;

$$y^{II} = (2x^5 + 10x^4 - 4)' = 10x^4 + 40x^3;$$
$$y^{III} = (10x^4 + 40x^3)' = 40x^3 + 120x^2;$$
$$y^{IV} = (40x^3 + 120x^2)' = 120x^2 + 240x;$$

$y^V = (120x^2 + 240x)' = 240x + 240 = 240(x + 1)$.

7. $y = cosx$ funksiyaning to'rtinchi tartibli hosilasi topilsin.

Yechish: $y' = (cosx)' = -sinx$; $y'' = (-sinx)' = -cosx$; $y''' = (-cosx)' = sinx$; $y^{IV} = (sinx)' = cosx$.

8. $f(x) = x^3 + 5x^2 - 1$ funksiyaning ikkinchi tartibli hosilasi topilsin va $f''(0.5)$ hisoblansin.

Yechish: $f'(x) = (x^3 + 5x^2 - 1)' = 3x^2 + 10x$;

$$f''(x) = (3x^2 + 10x)' = 6x + 10;$$
$$f''(0.5) = 6 \cdot 0.5 + 10 = 3 + 10 = 13.$$

Hosila tushunchasiga olib keladigan masalalar jumlasiga qattiq jismni to`g`ri chiziqli harakatini, yuqoriga vertikal holda otilgan jismning harakatini yoki dvigatel silindridagi porshen harakatini tekshirish kabi masalalarni kiritish mumkin. Bunday harakatlarni tekshirganda jismning konkret o`lchamlarini va shaklini e‘tiborga olmay, uni harakat qiluvchi moddiy nuqta shaklida tasavvur qilamiz.

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