GARMONIK FUNKSIYALAR VA ULARNING XOSSALARI

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Annotasiya: *Laplas tenglamasining fundamental yechimlari keltirilgan. Elliptik tipdagi tenglamalardan eng soddasi va muhimi Laplas Karleman matrisasini tuzish esa masalani yechishning dolzarbligini ifodalaydi.*

Kalit so'zlar: *Elliptik tipli tenglamalar uchun asosiy chegaraviy masalalar keltirilgan. Grin formulasi, Gauss formulasi.*

Garmonik funksiyalarning xossalarini o'rganish uchun avvalo ularning integral munosabatlarini qarab chiqamiz. Bu integral munosabatlarini keltirish uchun esa. Grin formulalaridan foydalanishga to'g'ri keladi. Grin formulalarining o'zlari esa Ostogradskiy formulasining xususiy hollaridan iboratdir. Ostogradskiy formulasining ma'nosi esa quyidagichadir.

Faraz qilaylik P, Q, R funksiyalar $S+T$ yopiq sohada o'zining bir tartibli hosilalari bilan uzluksiz funksiyalardan iborat bo'lsin. Agarda S sirtda o'tkazilgan tashqi normal n ni Ox, Oy, Oz o'qlar bilan tashkil etgan burchaklari mos holda

$$\alpha = (n, x); \;\; \beta = (n, y); \;\; \gamma = (n, z)$$

lardan iborat bo'lsin.Bu holda Ostogradskiy formulasi quyidagicha edi.

$$\iiint_T \left[\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right] d\tau = \iint_S (P\,cos\alpha + Q\,cos\beta + R\,cos\gamma)ds \qquad (1)$$

Bu tenglikning chap tomonidagi qavs ichidagi ifoda:

$$\bar{a}(x, y, z) = iP + jQ + kR$$

ko'rinishdagi vektorlar maydonining yo'nalishi yoki divergensiyasini bildiradi.

$$div\bar{a} = div\left(iP + iQ + \bar{k}R\right) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

skalyardan iborat.

$$\bar{n} = i\,cos\alpha + j\,cos\beta + k\,cos\gamma$$

Bularni hisobga olganda (1) tenglikning o'ng tomonidagi qavs ichidagi ifoda

$$(\bar{a}, \bar{n}) = (iP + jQ + kR;\; i\,cos\alpha + j\,cos\beta + k\,cos\gamma) =$$
$$= P\,cos\alpha + Q\,cos\beta + R\,cos\gamma = a_n$$

Shu ko'rinishda (1.1.1) ning ko'rinishi quyidagicha bo'ladi.

$$\iiint_T \; diva\,d\tau = \iint_S \; a_n d\tau, \qquad (2)$$

Endi keltirilgan formulalardan foydalanib Grin formulasini hosil qilamiz. Buning uchun faraz qilaylik: S sirt bilan chegaralangan T sohada

$$\left.\begin{matrix} u = u(x, y, z) \\ v = v(x, y, z) \end{matrix}\right\}$$

Funksiyalar o'zlarining argumentlari bo'yicha ikkinchi tartibgacha differensiallanuvchi funksiyalardan iborat bo'lsin. Quyidagicha belgilashlar kiritamiz:

$$\left.\begin{aligned} P &= u\frac{\partial v}{\partial x} \\ Q &= u\frac{\partial v}{\partial y} \\ R &= u\frac{\partial v}{\partial z} \end{aligned}\right\} \quad (3)$$

(3) ni (1) ga qo'yamiz.

$$\iiint_T \left[\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + u\frac{\partial^2 v}{\partial x^2} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y} + u\frac{\partial^2 v}{\partial y^2} + \frac{\partial u}{\partial z}\frac{\partial v}{\partial z} + u\frac{\partial^2 y}{\partial z^2}\right] d\tau =$$

$$\iint_S u\left[\frac{\partial v}{\partial x}cos\alpha + \frac{\partial v}{\partial y}cos\beta + \frac{\partial v}{\partial z}cos\gamma\right] ds = d\iint_S u\frac{\partial v}{\partial \bar{n}} ds.$$

$$\frac{\partial u}{\partial \bar{n}}\Big|_s = \left[\frac{\partial u}{\partial x}cos\alpha + \frac{\partial u}{\partial y}cos\beta + \frac{\partial u}{\partial z}cos\gamma\right] / S$$

$$\iiint_T n\Delta V d\tau = \iint_S u\frac{\partial v}{\partial \bar{n}} d\tau$$

$$\left.\begin{aligned} \nabla u &= gradu = i\frac{\partial u}{\partial x} + j\frac{\partial u}{\partial y} + k\frac{\partial u}{\partial z} \\ \nabla v &= gradv = i\frac{\partial v}{\partial x} + j\frac{\partial v}{\partial y} + k\frac{\partial v}{\partial z} \end{aligned}\right\}$$

$$(\nabla u.\nabla v) = \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial v}{\partial z}$$

Demak quyidagi masalaga kelamiz.

$$\iiint_T u\Delta v d\tau = \iint_S u\frac{\partial v}{\partial \bar{n}} d\sigma - \iiint_T (\nabla u.\nabla v) d\tau$$

(3) dagi u va v larning o'rinlarini almashtirib, quyidagilarga kelamiz.

$$\iiint \quad v\Delta u d\tau = \iint_S v\frac{\partial u}{\partial \bar{n}} d\sigma - \iiint_T (\nabla u.\nabla v) d\tau \quad (4)$$

(4) va (5) ga Grinning birinchi formulasi deb yuritiladi. Endi (1.1.4) va (1.115) larni mos ravishda ayirib quyidagiga kelamiz:

$$\iiint_T (u\Delta v - v\Delta u) d\curlyvee = \iint_S \left[u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial n}\right] d\tau \quad (5)$$

Bunga Grinning ikkinchi formulasi deyiladi. Agarda bu keltirilgan formulalarda qatnashayotgan funksiyalar

$$\left.\begin{aligned} u &= u(x,y) \\ v &= v(x,y) \end{aligned}\right\}$$

ko'rinishda bo'lsa, u holda (4) va (5) hamda (6) ning ko'rinishi quyidagicha bo'ladi.

$$\iint_S u\Delta v ds_1 = \int_{c_1} u\frac{\partial v}{\partial \bar{n}} d\, c_1 - \iint_S (\nabla u.\nabla v) dS_1 \quad (6)$$

$$\iint_S v\Delta I ds_1 = \int_{c_1} v\frac{\partial u}{\partial \bar{n}} d\, c_1 - \iint_S (\nabla v.\nabla n) dS_1 \quad (7)$$

$$\iint_S [u\Delta v - v\Delta u]\, dS_1 = \int_c \left[u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial \bar{n}}\right] dS_1 \quad (8)$$

$$\begin{cases} \Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 n}{\partial y^2}; \ \dfrac{\partial u}{\partial \bar{n}} = \dfrac{\partial u}{\partial x} \cos\alpha + \dfrac{\partial u}{\partial y} \cos\beta \\ \nabla u = gradu = i \dfrac{\partial u}{\partial x} + j \dfrac{\partial u}{\partial y} \end{cases}$$

Ya'ni keltirilgan (1.3.4), (1.3.5) va (1.3.6) formulalar tekislik uchun Grin formulalaridan iboratdir. Endi shu funksiyaning ba'zi integral munosabatlarini hosil qilish masalasini qaraymiz.

Endi yuqoridagi garmonik funksiyalarga tegishli bo'lgan formulalardan foydalangan holda shu funksiyalarning ba'zi soda xossalarini qarab chqamiz. Buning uchun esa quyidagi xossalarni alohida–alohida qaraymiz.

Xossa–1: Agar $u = u(M)$ va $v = v(M)$ o'zining funksiyalar S+T yopiq sohada o'zining 1–tartibli xosilasi bilan birga uzluksiz, hamda T sohaning ichida garmonik funksiyadan iborat bo'lsa, u holda quyidagi tengliklar o'rinlidirlar.

$$\iint_S \frac{\partial u}{\partial n} d\tau = 0; \quad \iint_S \frac{\partial v}{\partial n} d\tau = 0$$

Isbot:bunga ishonch hosil qilish Grinning (1.4) formulasidan foydalanamiz.

$$\iiint_T u \Delta v d\tau = \iint_S u \frac{\partial v}{\partial n} d\tau - \iint_T (\Delta u, \Delta v) ds$$

$$\iiint_T v \Delta u d\tau = \iint_S v \frac{\partial u}{\partial n} d\tau - \iint_T (\Delta u, \Delta v) ds$$

Agar bu formulalarning birisida

$v = c = const$ (hamma vaqt garmonik funksiyadan iborat) deb olsak,

$$\Delta(c) = i \frac{\partial c^{=0}}{\partial x} + j \frac{\partial c^{=0}}{\partial y} + k \frac{\partial c^{=0}}{\partial z} = 0, \quad \Rightarrow \Delta(c) = 0$$

bo'ladi.

Bu holda formulalarning ko'rinishlari quyidagicha bo'ladi.

$$\left. \begin{aligned} 0 &= \iint c \frac{\partial u}{\partial \bar{n}} \partial\tau - 0 \\ 0 &= \iint c \frac{\partial v}{\partial \bar{n}} \partial\tau - 0 \end{aligned} \right\} \Rightarrow \begin{cases} \iint_S \dfrac{\partial u}{\partial \bar{n}} \partial\tau = 0 \\ \iint_S \dfrac{\partial v}{\partial \bar{n}} \partial\tau = 0 \end{cases}$$

Sizga ma'lumki, elliptic tipdagi tenglamalar uchun 2 chegaraviy masalani qo'yilishi quyidagidan iborat edi.

$$\left. \begin{aligned} &\Delta u = 0 \\ &\frac{\partial u}{\partial \bar{n}} / S = f \end{aligned} \right\} \qquad \text{(A)}$$

Bu masalaning yechimini topishda xossa 1-ni tadbiq etamiz. Xossa 1-ni asosan (A) masalaning chegaraviy sharti quyidagidan iborat bo'lidi

$$\iint_S \frac{\partial u}{\partial \bar{n}} d\tau = \iint_S f d\tau = 0 \qquad \text{(B)}$$

Demak (A) ko'rinishidagi Neyman masalasi yechimiga ega bo'lishi uchun (B) ko'rinishidagi shart bajarilishi kerak. (B) shart qaralayotgan prosessdagi manbani yo'qotish sharti deyiladi.

Bu shartga garmonik funksiyaning xossasini chegaraviy masalani yechishdagi tadbiqi deb yuritiladi.

Xossa–2: (garmonik funksiya uchun o'rta qiymat haqidagi Gauss tebranmasi).

Teorema:agar $u = u(M)$ funksiya $S+T$ yopiq sohada o'zining birinchi tartibli hosilasi bilan uzluksiz hamda T sohada garmonik funksiyadan iborat bo'lsa va $M_0(x_0, y_0, z_0)T$ sohadagi ixtiyoriy nuqtadan iborat bo'lsa, g holda garmonik funksiya U ning M_0 nuqtadagi qiymati quyidagi formula yordamida aniqlanadi.

$$u(M_0) = \frac{\iint_{SR} u(p) d\tau}{4TR^2}$$

Bunga garmonik funksiya uchun o'rta qiymat haqidagi Gauss formulasi deb yuritiladi.

Isbot: Bunga ishonch hosil qilish uchun T sohaning ichida M_0 nuqtadagi olib, shu nuqtani markaz hisoblab, K radiusli T_R sferani hosil qilamiz.

Sferaning sirti S_R dan iborat bo'lsin. Bizga ma'lumki, garmonik funksiyaning M_0 nuqtadagi qiymati Grinning integral formulasi yordamida quyidagicha aniqlanar edi.

$$u(\mu_0) = \frac{1}{4\pi} \iint_{S_R} \left[\frac{1}{r} \frac{\partial u}{\partial \bar{n}} - u(p) \frac{\partial}{\partial \bar{n}} \left(\frac{1}{r} \right) \right] d\sigma \qquad \text{(A)}$$

Bundagi integrallarning alohida–alohida hisoblaymiz.

a) S_R sirtga o'tkazilgan tashqi normal $\bar{n}$ ning yo'nalishini sfera radiusining yo'nalishi bilan bir xil qilib yo'naltiramiz. Ya'ni

$$\frac{\partial}{\partial \bar{n}} \left(\frac{1}{r} \right) \bigg| S_R = \frac{\partial}{\partial r} \bigg| S_R = -\frac{1}{r^2} | S_R = -\frac{1}{R^2}$$

b) $\frac{1}{R} \frac{\partial u}{\partial \bar{n}} S_R = \frac{1}{R} \frac{\partial u}{\partial \bar{n}} | S_R$

$$\iint_{S_R} \frac{1}{R} \frac{\partial u}{\partial \bar{n}} d\sigma = \frac{1}{R} \iint_{S_R} \frac{\partial u}{\partial \bar{n}} \partial\sigma = 0$$

Xossa 1 ga asosan.

Bularga asosan (A) ning ko'rinishi quyidagicha bo'ladi:

$$u(\mu_0) = -\frac{1}{4\pi} \iint_{S_R} M(p) \frac{\partial}{\partial \bar{n}} \left(\frac{1}{r} \right) d\tau = \frac{\iint_{S_R} u(p) d\tau}{4RT^2}$$

Isbot bo'ldi.

Xossa–3:(garmonik funksiyalar uchun maksimal qiymat prinsipi haqidagi teorema).

Agar $u=u(M)$ funksiya $T+S$ yopiq sohada o'zining 1 tartibli hosilasi bilan uzluksiz, hamda T sohada garmonik funksiyadan iborat bo'lsa, u holda u funksiya o'zining eng katta yoki eng kichik qiymatlariga shu T sohaning S sirtida erishadi.

Bunga ishonch hosil qilish uchun T sohaning ichidagi M_0 nuqtani markaz qilib, K radiusli T_R sferani chizamiz. Bu sferaning sirti S_R bo'lsin. Yuqoridan ma'lumki, T sohanng ichida yotuvchi M_0 nuqtadagi garmonik funksiyaning qiymati Gauss formulasi yordamida aniqlanadi.

Isbot: Teskarisini faraz qilaylik:

Faraz qilaylik: Teorema shartini qanoatlantiruvchi $u=u(M)$ funksiya o'zining eng katta qiymatini sferani S_R sirtida emas, balki uning ichki biror M_0 nuqtasida erishsin. Ma'lumki garmonik funksiyaning M_0 nuqtadagi erishgan qiymati quyidagicha bo'ladi:

$$U_0=u(M_0)\geq u(M) \qquad \text{(k)}$$

Ikkinchi tomonidan ma'lumki, bu funksiyaning M_0 dagi qiymati quyidagi formula yordamida aniqlanar edi.

$$u(M_0)=\frac{\iint_{S_R} u(p)d\tau}{4\pi R^2}$$

Agar bu formulaga (k) tengsizlikni tadbiq etsak, quyidagiga kelamiz.

$$u(M_0)=\frac{\iint_{S_R} u(p)d\tau}{4\pi R^2}=\frac{u(M_0)}{4\pi R^2}\iint_{S_R} u4\pi R^2 d\tau=\frac{4\pi R^2 u(M)}{4\pi R^2}=u(M_0)$$

Demak,

$$u(M_0)\geq u(M_0) \qquad (k_1)$$

Agar qaralayotgan T_R sferaning birorta nuqtasida (k) munosabatdagi katta ishora o'rniga kichik ishora o'rinli bo'lsa, u holda (k_1) dagi kichik ishoraga kelamiz. Ya'ni $u(M_0)\prec u(M_0)$

Buning bo'lishi mumkin emas. Bu natijaga esa garmonik funksiya o'zining eng katta qiymatiga sohaning ichki nuqtasiga erishadi, degan farazimiz tefayli kelinadi.

Demak, bu funksiya o'zining eng katta qiymatini shu sohaning sirtida qabul qiladi, degan xulosaga kelamiz. Xuddi yuqoridagilar kabi garmonik funksiyalarning xossasidan foydalanib, eliptik tipdagi tenglamalar uchun qo'yilgan chegaraviy masalalar yechimining yagonaligini, hamda chegaraviy shartlardan uzluksiz bog'langan ko'rsatish mumkin.

Agar Dirixle masalasining yechimi mavjud bo'lsa, u holda u yagonadir.

Ya'ni

$$\left.\begin{array}{l}\Delta u_1=0\\ u_1/s=f\end{array}\right\}(a) \qquad \left.\begin{array}{l}\Delta u_2=0\\ u_2/s=f\end{array}\right\}(b)$$

Teskarisini faraz qilamiz: $u_1 \neq u_2$ bo'lsin. Mos ravishda yuqoridagilarni ayiramiz.

$$\left.\begin{array}{l}\Delta(u_1-u_2)=0\\(u_1-u_2)/s=0\end{array}\right\}\Rightarrow u_1-u_2\equiv 0\Rightarrow u_1\equiv u_2$$

Xuddi shuningdek masala yechimining chegaraviy shartlardan uzluksiz bog'liqligini ham ko'rsatish mumkin.

Laplas tenglamasining fundamental yechimlari keltirilgan.

Elliptik tipdagi tenglamalardan eng soddasi va muhimi Laplas

$\Delta u = 0,\ \Delta = \sum_{i=1}^{n}\frac{\partial^2}{\partial x_i^2}$ va Puassson $\Delta u = f(x)$ tenglamalaridir.

E^n fazodagi ikki $x = (x_1, \dots x_n), (\xi_1, \dots \xi_n)$ nuqta orasidagi masofani r orqali belgilab olamiz, yani $r = |x-\xi| = \sqrt{\sum_{i=1}^{n}(x_i-\xi_i)^2}$. Bevosita tekshirish bilan ishonch hosil qilish mumkinki, ushbu

$$E(x,\xi)=\begin{cases} r^{2-n}, & n>2 \\ ln\frac{1}{r}, & n=2 \end{cases}$$

funksiya $x \neq \xi$ bo'lganda x bo'yicha ham, ξ bo'yicha ham Laplas tenglamasini qanoatlantiradi.(0.1.1) formula bilan aniqlangan $E(x,\xi)$ funksiyani Laplas tenglamasining elementar yoki fundamental yechimi deyiladi.

birinchi tartibli elliptik tipli tenglamalar sistemasi uchun integral formula keltirilgan.

Xususiy hosilalardagi birinchi tartibli

$$D\left(\frac{\partial}{\partial x}\right)f(x) = 0 \qquad (0.2.1)$$

chiziqli differensial tenglamalar sistemasini qaraymiz, uning xarakteristik matrisasi

$$D(x) \in A_{k\times l}(p(x), x) \qquad (0.2.2)$$

$$f(x) = \begin{Vmatrix} f_1(x) \\ \dots \\ f_l(x) \end{Vmatrix} \qquad (0.2.3)$$

esa kompleks qiymatli umumiy holda vektor funksiya .

0.2.1-teorema. $G \in R^n$ - silliq bo'lakli chegarali chegaralangan soha va (2.2.5) $f(x) \in C^1(G) \cap C(\bar{G})$ vektor funksiya (2.2.2) shartda (2.2.1) sistemaning yechimi bo'lsin , u holda $\int M(y-x)f(y) = \begin{cases} f(x), & x \in G, \\ 0, & x \notin \bar{G}. \end{cases}$

(1) formuladan natijadan xususan, sistemaning $C^1(G)$sinfdagi yechimi (2) shart bajarilganda haqiqatdan, G sohada analitik funksiyalar bo'ladi. Bundan va (3) misoldan ko'rinadiki, (4) sistemaga (5) shartning qo'yilishi bu sistema elliptik bo'lishiga teng kuchli.

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