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ANIQ INTEGRALLARNI TAQRIBIY HISOBLASH .

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Annotatsiya. *Integralning aniq qiymatini topish masalasi juda murakkab bo'lishi mumkin. Bunday hollarda aniq integral qiymatini taqribiy hisoblash usullariga murojaat qilinadi. Ularga to'g'ri to'rtburchaklar va trapetsiyalar formulalarini misol qilib ko'rsatib bo'ladi.*

Kalit so'zlar: *Yuqori chegarasi o'zgaruvchan integral, kvadratur formulalar to'g'ri to'rtburchaklar formulasi, trapetsiyalar formulasi*

Annotation. *The problem of finding the exact value of the integral can be very complicated. In such cases, methods of approximate calculation of the exact integral value are used. They can be shown as examples of formulas of rectangles and trapezoids.*

Kirish

$I = \int_a^b f(x)dx$ aniq integral qiymatini hisoblash masalasi integral ostidagi $f(x)$ funksiyaning biror $F(x)$ boshlang'ich funksiyani topish va uning qiymatlarini hisoblash masalasiga keltiriladi. Ammo ayrim aniq integrallar uchun bu usullarni qo'llashda quyidagi muammolar paydo bo'lishi mumkin:

1) $F(x)$ boshlang'ich funksiyani topish murakkab ;

2) $F(x)$ boshlang'ich funksiya murakkab ko'rinishda bo'lib, uning $F(a)$ va $F(a)$ qiymatlarini hisoblash qiyinchilik tug'diradi ;

3) $F(x)$ boshlang'ich funksiya elementar funksiyalarda ifodalanmaydi;

4) integral ostidagi $f(x)$ funksiya jadval ko'rinishida berilgan .

Bunday hollarda aniq integralning qiymatini taqribiy hisoblash masalasi paydo bo'ladi. Bu masalani yechish uchun matematikada turli formulalar topilgan bo'lib, ular umumiy holda kvadratur formulalar deb ataladi. Shu formulalardan eng soddalaridan ikkitasini qisqacha ko'rib o'tamiz.

I. *To'g'ri to'rtburchaklar formulasi.* Bu formulani keltirib chiqarish uchun dastlab $[a,b]$ kesmani uzunligi bir xil va $\Delta x=(b-a)/n$ bo'lgan n ta $[x_{i-1}, x_i]$ kesmachalarga (i=1, 2, ⋯, n) ajratamiz. Bunda x_i bo'linish nuqtalari

$$x_i = a + i\Delta x = a + \frac{b-a}{n}i, \quad i = 0, 1, 2, \cdots, n \qquad (1)$$

formula bilan topiladi.

So‘ngra integral ostidagi $f(x)$ funksiyaning x_i bo‘linish nuqtalaridagi $f(x_i)$ (i=1, 2, ⋯, n) qiymatlarini hisoblaymiz. Bu qiymatlar va $[x_{i-1}, x_i]$ kesmachalar uzunligi Δx bo‘yicha

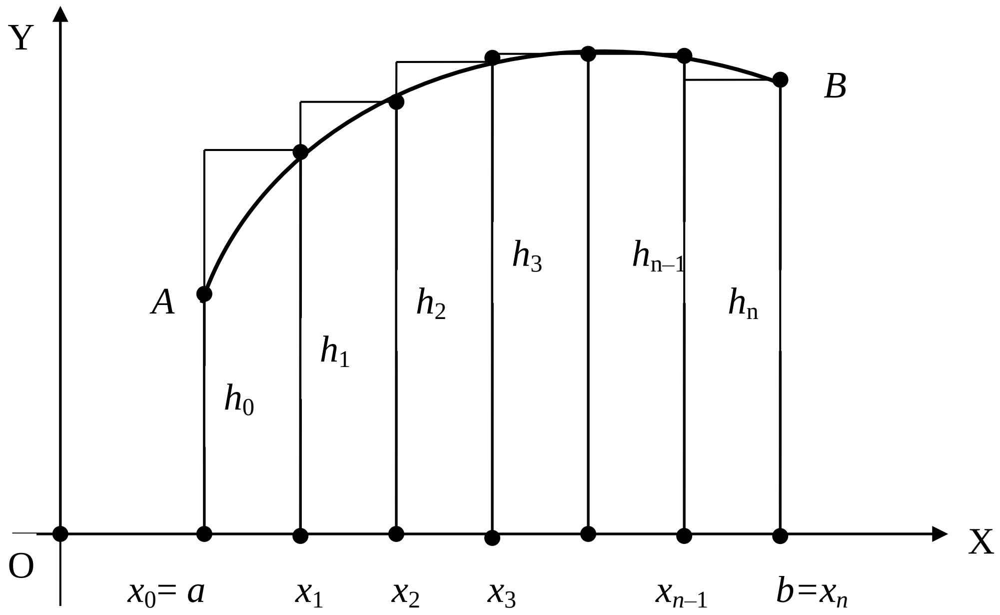
$S_n(f) = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \cdots + f(x_n)\Delta x$

integral yig‘indini hosil qilamiz. Ta’rifga asosan I aniq integral $S_n(f)$ integral yig‘indilar ketma – ketligining $n\to\infty$ bo‘lgandagi limitiga teng. Shu sababli, n katta son bo‘lganda, $I \approx S_n(f)$ deb olish mumkin. Natijada ushbu taqribiy formulaga ega bo‘lamiz:

$$\int_a^b f(x)dx \approx \Delta x \cdot [f(x_1) + f(x_2) + f(x_3) + \cdots + f(x_n)] = \frac{b-a}{n}\sum_{i=1}^{n} f(x_i). \qquad (2)$$

Agar $[a,b]$ kesmada $f(x)>0$ deb olsak, unda (9) taqribiy tenglikning o‘ng

tomonidagi yig‘indi asoslari bir xil Δx uzunlikli $[x_{i-1}, x_i]$ kesmachalardan, balandliklari esa $h_i = f(x_i)$ (i=1, 2, ⋯, n) bo‘lgan to‘g‘ri to‘rtburchaklardan tuzilgan pog‘onasimon geometrik shaklning (74-rasmga qarang) yuzini ifodalaydi. Chap tomondagi aniq integral qiymati esa ***aABb*** egri chiziqli trapetsiya yuziga teng.



1-rasm

Aniq integral uchun (2) taqribiy tenglik ***to‘g‘ri to‘rtburchaklar formulasi*** deyiladi.

To‘g‘ri to‘rtburchaklar formulasining xatoligi

$$\Delta \le M_1 \frac{(b-a)^2}{4n}, \quad M_1 = \max_{x\in[a,b]} |f'(x)| \qquad (3)$$

formula bilan baholanadi.

Misol sifatida to‘g‘ri to‘rtburchaklar formulasi yordamida

$$I = \int_0^1 \frac{dx}{1+x^2} = \mathrm{arctg}x\Big|_0^1 = \mathrm{arctg}1 - \mathrm{arctg}0 = \frac{\pi}{4} \qquad (4)$$

aniq integralning taqribiy qiymatini topamiz. Buning uchun [0,1] integrallash kesmasini n=10 teng bo‘lakka ajratamiz va hisoblashlar natijalarini quyidagi jadval ko‘rinishida ifodalaymiz.

i	$x_i=0.1i$	$1+x_i^2$	$f(x_i)=\dfrac{1}{1+x_i^2}$	$\sum_i f(x_i)$
1	0.1	1.01	0.9901	0.9901
2	0.2	1.02	0.9615	1.9516
3	0.3	1.09	0.9174	2.8690
4	0.4	1.16	0.8621	3.7311
5	0.5	1.25	0.8000	4.5311
6	0.6	1.36	0.7353	5.2664
7	0.7	1.49	0.6711	5.9375
8	0.8	1.64	0.6098	6.5473
9	0.9	1.81	0.5525	7.0998
10	1.0	2.0	0.5000	7.5998

Bizning misolda $\Delta x=(1–0)/10=0.1$ bo‘lgani uchun, (9) formulaga asosan, ushbu natijani olamiz:

$$\int_0^1 \frac{dx}{1+x^2} \approx 0.1\cdot 7.5998 = 0.75998 .$$

Bu taqribiy natijani xatoligini (10) formula bo‘yicha baholaymiz. Bizning misolda

$$f(x)=\frac{1}{1+x^2} \Rightarrow f'(x)=-\frac{2x}{(1+x^2)^2} \Rightarrow |f'(x)|=\left|-\frac{2x}{(1+x^2)^2}\right| < \frac{2\cdot 1}{(1+0^2)^2}=2$$

va shu sababli (10) formulada $M_1=2$ deb olish mumkin. Bu holda

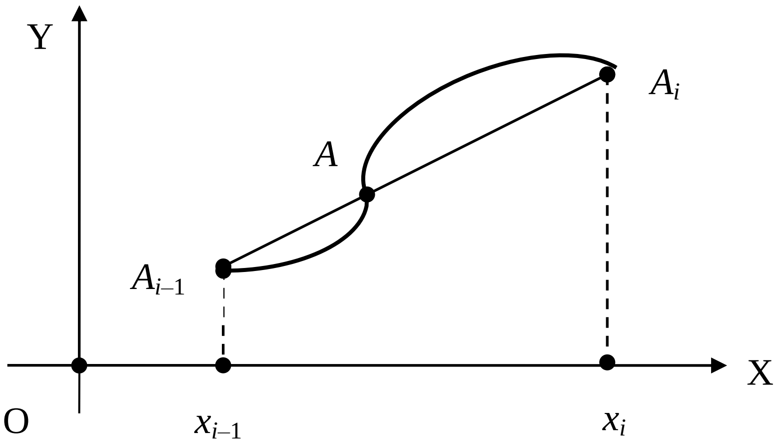
$\Delta\le 2\cdot(1–0)^2/(4\cdot 10)=1/20=0.05$

bo‘lgani uchun (11) aniq integralning qiymati

$0.75998–0.05 < I < 0.75998+0.05 \Rightarrow 0.70998 < I < 0.80998$

oraliqda yotadi. Bu natijani (4) integralning aniq qiymati $\pi/4\approx 0.7854$ bilan taqqoslab, yo‘l qo‘yilgan absolut xatolik $\Delta=0.0255$ ekanligini ko‘rishimiz mumkin. Shunday qilib, hatto unchalik katta bo‘lmagan n=10 holda ham (2) to‘g‘ri to‘rtburchaklar formulasi ancha yaxshi natija berdi.

II. *Trapetsiyalar formulasi.* Soddalik uchun bu formulani I integral ostidagi funksiya $f(x)>0$ bo‘lgan holda qaraymiz.Bu yerda ham $[a,b]$ integrallash kesmasini (8) nuqtalar bilan bir xil Δx uzunlikli n ta $[x_{i-1}, x_i]$ (i=1, 2, ⋯, n) kesmachalarga bo‘laklaymiz. So‘ngra $y=f(x)$ funksiya grafigidagi $A_{i-1}(x_{i-1}, f(x_{i-1}))$ va $A_i(x_i, f(x_i))$ nuqtalarni to‘g‘ri chiziq kesmasi (vatar) bilan tutashtirib, egri chiziqli $x_{i-1}A_{i-1}\,AA_ix_i$ trapetsiyani to‘g‘ri chiziqli $x_{i-1}A_{i-1}A_ix_i$ trapetsiya bilan (2-rasmga qarang) almashtiramiz.



2-rasm

Bu holda to‘g‘ri chiziqli $x_{i-1}A_{i-1}A_ix_i$ trapetsiyaning yuzi

$$S_i = \frac{f(x_{i-1}) + f(x_i)}{2}(x_i - x_{i-1}) = \frac{f(x_{i-1}) + f(x_i)}{2}\Delta x \quad (i = 1,2,3,\cdots,n)$$

egri chiziqli $x_{i-1}A_{i-1}AA_ix_i$ trapetsiyaning yuziga taqriban teng deb olish mumkin. Unda bu yuzalarning yig‘indisi aniq integralning taqribiy qiymatiga teng bo‘ladi, ya’ni

$$\int_a^b f(x)dx \approx \frac{b-a}{n}[\frac{f(a)+f(b)}{2} + f(x_1) + f(x_2) + \cdots + f(x_{n-1})] \qquad (5)$$

taqribiy formula o‘rinli bo‘ladi.

Aniq integral uchun (5) taqribiy tenglik ***trapetsiyalar formulasi*** deyiladi.

Trapetsiyalar formulasining absolut xatoligi

$$\Delta \le \frac{(b-a)^3}{12n^2}M_2 \quad , \quad M_2 = \max_{x\in[a,b]}|f''(x)| \qquad (6)$$

formula bilan baholanadi.

Misol sifatida (4) aniq integralning taqribiy qiymatini n=10 bo‘lgan holda trapetsiyalar formulasi orqali hisoblaymiz. Oldingi hisoblash natijalaridan foydalanib,

$$\int_0^1 \frac{dx}{1+x^2} \approx 0.1\cdot[\frac{1+0.5}{2} + 7.0998] = 0.78498$$

taqribiy tenglikni hosil etamiz. Bunda hosil qilingan taqribiy natijaning absolut xatoligi

Δ=π/4–0.78498=0.7854–0.78498=0.0004

bo‘lib, to‘g‘ri to‘rtburchaklar formulasi absolut xatoligiga (unda Δ=0.0255 ekanligini eslatib o‘tamiz) qaraganda ancha kichikdir. Demak, trapetsiyalar formulasi to‘g‘ri to‘rtburchaklar formulasiga nisbatan aniqroq natija beradi. Buni ularning xatoliklarini ifodalovchi (3) va (6)formulalar orqali ham ko‘rish mumkin.

Ko‘rib o‘tilgan to‘g‘ri to‘rtburchaklar va trapetsiyalar formulalariga nisbatan aniq integralning taqribiy qiymatini aniqroq hisoblashga imkon beradigan boshqa kvadratur formulalar ham mavjudligini ta’kidlab o‘tamiz. Masalan, ingliz matematigi Simpson

(1710 – 1761) tomonidan topilgan parabolalar formulasi, Chebishevning kvadratur formulasi shular jumlasidandir.

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