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GEOMETRIYA MASALALARINI YECHISHDA VEKTORLARNING BA'ZI BIR TADBIQLARI

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Anatatsiya: *Ushbu maqolada vektorlarning skalyar, vektor va aralash ko'paytmalari yordamida geometriya masalalarini yechishni ko'rib o'tamiz.*

Kalit so'zlar: *Vektorlar ko'paytma, skalyar ko'paytma, aralash ko'paytma, yo'naltiruvchi vektor, komplanarlik.*

Ta'rif. $\overline{a}, \overline{b}$ vektorlarning uzunliklari bilan ular orasidagi burchak kosinusini ko'paytirishdan hosil qilingan son bu *vektorlarning skalyar ko'paytmasi* deb ataladi.

$\overline{a}, \overline{b}$ vektorlarning skalyar ko'paytmasi $\overline{a}\ \overline{b}$ yoki $(\overline{a}\ \overline{b})$ ko'rinishda belgilanadi.

Demak, $\overline{a}\ \overline{b} = |\overline{a}||\overline{b}|\cos\varphi$, $\varphi = (\overline{a} \wedge \vec{в})$.

$\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorning aralash ko'paytmasi deb, (vektorlarning ko'rsatilgan tartibiga ko'ra) $\vec{a}$ va $\vec{b}$ vektorlarning vektor ko'paytmasidan iborat vektorni $\vec{c}$ vektorga skalyar ko'paytirishdan hosil qilingan songa aytiladi:

$([\vec{a}, \vec{b}], \vec{c})$

Aralash ko'paytma

$(\vec{a}, \vec{b}, \vec{c})=([\vec{a}, \vec{b}], \vec{c})$

ko'rinishda belgilanadi. Aralash ko'paytmaning geometrik ma'nosi quyidagicha: $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar biror O nuqtaga qo'yilgan bo'lib, komplanar bo'lmasin hamda o'ng uchlikni hosil qilsin. Qirralari shu berilgan vektorlardan iborat parallelepipedni yasasak, $|[\vec{a}, \vec{b}]|$ miqdor shu parallelepiped asosining yuzini bildiradi. Aralash ko'paytma ta'rifiga asosan $([\vec{a}, \vec{b}], \vec{c})= |[\vec{a}, \vec{b}]||\vec{c}|cos\varphi$, bu yerda φ, $[\vec{a}, \vec{b}]$ va $\vec{c}$ vektorlar orasidagi burchak bo'lib, $|\vec{c}|cos\varphi$ miqdor $\vec{c}$ vektorning $[\vec{a}, \vec{b}]$ vektor yo'nalishidagi to'g'ri chiziqdagi proyeksiyasiga teng bo'lib, parallelepipedning balandligidir. ($|\vec{c}|\ cos\varphi\ = h$).

Demak, agar $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar o'ng uchlik hosil qilsa, bu vektorlarning aralash ko'paytmasi bu vektorlarga yasalgan parallelepiped hajmiga teng bo'ladi. Agar $\vec{a}$, $\vec{b}$, $\vec{c}$ lar chap uchlik tashkil qilsa, $[\vec{a},\vec{b}]$ vektor bilan $\vec{c}$ orasidagi burchak $\varphi \geq \frac{\pi}{2}$ $(cos\varphi \leq 0)$ bo'ladi. U holda $([\vec{a},\vec{b}],\vec{c}) = -V$. Agar $\vec{a},\vec{b}$ va $\vec{c}$ vektorlar o'zlarining koordinatalari

$\vec{a}$=$\{X_1; Y_1; Z_1\}$, $\vec{b}$=$\{X_2; Y_2; Z_2\}$, $\vec{c}$=$\{X_3; Y_3; Z_3\}$ bilan berilgan bo'lsa, u holda aralash ko'paytma.

$$([\vec{a},\vec{b}],\vec{c}) = (\vec{a},\vec{b},\vec{c}) = \begin{vmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{vmatrix}$$

1⁰. $(\vec{a}\,\vec{b}\,\vec{c})=(\vec{b}\,\vec{c}\,\vec{a})$. Haqiqatan ham, bu uch vektorga qurilgan parallelepiped hajmlarining absolyut qiymatlari teng, undan tashqari $\vec{a}, \vec{b}, \vec{c}$ uchlik bilan $\vec{b}, \vec{c}, \vec{a}$ uchlikning orientatsiyalari bir xil. Shuning singari $(\vec{a}\,\vec{b}\,\vec{c})=(\vec{b}\,\vec{c}\,\vec{a})=(\vec{c}\,\vec{a}\,\vec{b})$.

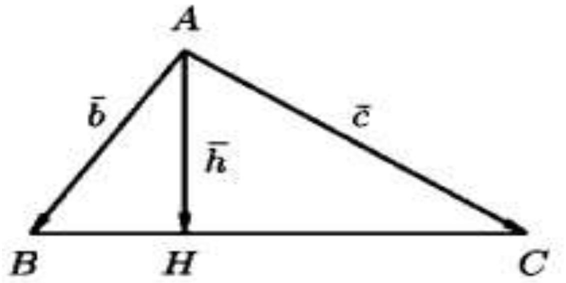
2⁰. $(\vec{a}\,\vec{b}\,\vec{c})=-(\vec{b}\,\vec{a}\,\vec{c})$, chunki $(\vec{a}\,\vec{b}\,\vec{c})=[\vec{a}\,\vec{b}]\vec{c}=-[\vec{b}\,\vec{a}]\vec{c}=-(\vec{b}\,\vec{a}\,\vec{c})$. demak, $(\vec{a}\,\vec{b}\,\vec{c})=-(\vec{b}\,\vec{a}\,\vec{c})$, $(\vec{b}\,\vec{c}\,\vec{a})=-(\vec{c}\,\vec{b}\,\vec{a})$, $(\vec{c}\,\vec{a}\,\vec{b})=(\vec{a}\,\vec{c}\,\vec{b})$.

3⁰. $((\vec{a}+\vec{b})\vec{c}\,\vec{d})=(\vec{a}\,\vec{c}\,\vec{d})+(\vec{b}\,\vec{c}\,\vec{d})$ chunki $((\vec{a}+\vec{b})\vec{c}\,\vec{d})=[\vec{a}+\vec{b}\quad\vec{c}]\vec{d}=$ $=([\vec{a}\;\vec{c}]+[\vec{b}\;\vec{c}]\vec{d})=[\vec{a}\;\vec{c}]\vec{d}+[\vec{b}\;\vec{c}]\vec{d}=(\vec{a}\,\vec{c}\,\vec{d})+(\vec{b}\,\vec{c}\,\vec{d})$.

4⁰. $\forall\lambda\in R$ uchun $(\lambda\,\vec{a}\,\vec{b}\,\vec{c})=\lambda(\vec{a}\,\vec{b}\,\vec{c})$, chunki $(\lambda\,\vec{a}\,\vec{b}\,\vec{c})=[\lambda\,\vec{a}\,\vec{b}\,]\vec{c}=$
$=\lambda[\vec{a}\;\vec{b}]\vec{c}=\lambda(\vec{a}\,\vec{b}\,\vec{c})$.

5⁰. $\vec{a}, \vec{b}, \vec{c}$ komplanar bo'lsa, ularning aralash ko'paytmasi nolga teng, chunki ularga qurilgan parallelepiped tekislikda joylashib qoladi, bunday parallelepipedning balandligi nolga tengligidan hajmi ham nolga teng; aksincha $(\vec{a}\,\vec{b}\,\vec{c})=0\Rightarrow\vec{a}, \vec{b}, \vec{c}$ vektorlar komplanar. Haqiqatan ham $(\vec{a}\,\vec{b}\,\vec{c})=0\Rightarrow[\vec{a}\;\vec{b}]\vec{c}=0$ yoki $[\vec{a}\;\vec{b}]\perp\vec{c}$. Lekin vektor ko'paytmaning ta'rifiga asosan $[\vec{a}\;\vec{b}]\perp\vec{a}$, $[\vec{a}\;\vec{b}]\perp\vec{b}$, bundan $[\vec{a}\;\vec{b}]$ vektorning $\vec{a}, \vec{b}, \vec{c}$ ning har biriga perpendikularligi kelib chiqadi, demak, $\vec{a}, \vec{b}, \vec{c}$ komplanar.

1-misol: ABC uchburchakda $\overrightarrow{AB} = \vec{b}$, $\overrightarrow{AC} = \vec{c}$. $\overrightarrow{AH}$ balandlik bo'yicha yo'nalgan $\vec{h}$ vektorni $\vec{a}$ va $\vec{c}$ lar orqali ifodalang.



$\vec{a}$

$\bar{h} = \bar{c} - \overline{Hc}$ ga egamiz. Bu yerda $\overline{Bc} = \overline{Hc}$ bo'ladi. $\overline{Hc} = \alpha \cdot \overline{Bc}$ deb belgilab olamiz.

$\overline{Bc}$ esa $\bar{a}$ vektorga teng deb olsak, $\overline{Hc} = \alpha \cdot \bar{c}$ bo'ladi. $\overline{hc} = \alpha \cdot \overline{Bc}$ bo'ladi.

$\overline{AH} \perp \overline{BC}$ bundan esa $\overline{AH} \cdot \overline{BC} = 0$ ga teng bo'ladi.

Bizda ma'lumki $\overline{Hc} = \alpha \cdot \bar{a}$ ga $\overline{Bc}$ esa $\bar{a}$ ga teng . Bular orqali $\overline{AH}$ ni topib olamiz. $\overline{AH} = \bar{c} - \alpha \cdot \bar{a}$ endi $(\bar{c} - \alpha \cdot \bar{a}) \cdot a = 0$ $\bar{c} \cdot \bar{a} - \alpha \cdot \overline{a^2} = 0$ α ni olib borib topamiz $\bar{h} = \bar{c} - \frac{\bar{c} \cdot \bar{a}}{|a|^2} \cdot \bar{a}$

2-misol $\vec{a}, \vec{b}, \vec{c}$ - ixtiyoriy vektorlar va α, β, γ - ixtiyoriy haqiqiy sonlar bo'lsa, $\alpha\vec{a} - \beta\vec{b},\ \gamma\vec{b} - \alpha\vec{c},\ \beta\vec{c} - \gamma\vec{a}$ vektorlarning komplanar ekanligi isbotlansin.

Isbot. Shu uchta vektorning $(\alpha\vec{a} - \beta\vec{b}\ \ \gamma\vec{b} - \alpha\vec{c}\ \ \beta\vec{c} - \gamma\vec{a})$ aralash ko'paytmasini hisoblaylik. Buning uchun yuqorida keltirilgan $1^0 - 5^0$-xossalarni nazarda tutsak,

$$(\alpha\vec{a} - \beta\vec{b}\ \gamma\vec{b} - \alpha\vec{c}\ \ \beta\vec{c} - \gamma\vec{a}) = (\alpha\vec{a}(\gamma\vec{b} - \alpha\vec{c})(\beta\vec{c} - \gamma\vec{a}) -$$
$$-(\beta\vec{b}(\gamma\vec{b} - \alpha\vec{c})(\beta\vec{c} - \gamma\vec{a})) = (\alpha\vec{a}\gamma\vec{b}\ \beta\vec{c} - \gamma\vec{a}) - (\alpha\vec{a}\ \beta\vec{c}\alpha\vec{c} - \gamma\vec{a}) -$$
$$-(\beta\vec{b}\ \gamma\vec{b}\ \beta\vec{c} - \gamma\vec{a}) + (\vec{\beta}\vec{b}\alpha\vec{c}\beta\vec{c} - \gamma\vec{a}) = \alpha\gamma\beta(\vec{a}\vec{b}\vec{c}) - \alpha\gamma\gamma(\vec{a}\vec{b}\vec{a}) - \alpha\alpha\beta(\vec{a}\vec{c}$$
$$+\alpha\alpha\gamma(\vec{a}\vec{c}\vec{a}) - \beta\gamma\beta(\vec{b}\vec{b}\vec{c}) + \beta\gamma\beta(\vec{b}\vec{b}\vec{c}) + \beta\gamma\gamma(\vec{b}\vec{b}\vec{a}) + \beta\alpha\beta(\vec{b}\vec{c}\vec{c}) -$$
$$-\beta\alpha\gamma(\vec{b}\vec{c}\vec{a}) = \alpha\beta\gamma(\vec{a}\vec{b}\vec{c}) - \alpha\beta\gamma(\vec{a}\vec{b}\vec{c}) = 0.$$

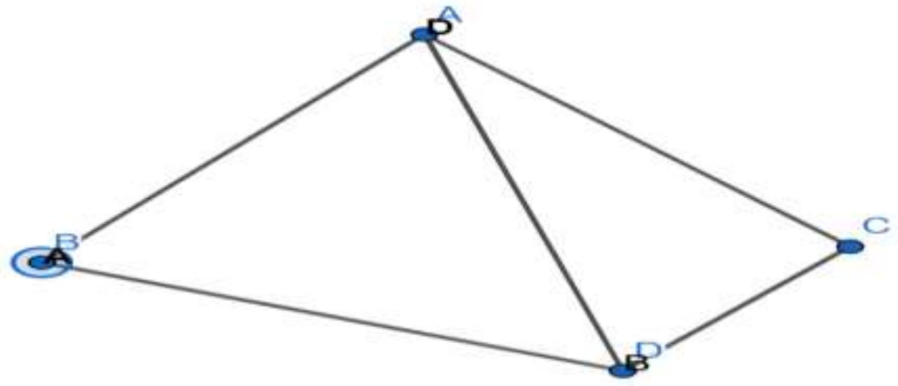
Aralash ko'paytmasi nolga teng bo'lgani uchun 5^0 ga asosan ular komplanardir.

3-misol . $\overrightarrow{AB}$ (2,0.0), $\overrightarrow{AC}$ (3,4,0), $\overrightarrow{AD}$ (3,4,2) vektorlarga qurilgan tetraedr

berilgan . Quyidagilar topilsin : a) tetraedrning hajmi , b) ABC yoqning yuzi , v) D uchdan

tushurilgan bandlik , g) AB va BC qirralar orasidagi φ_1 burchak kosinusi , d) ABC va ADC

yoqlar orasidagi φ_2 burchak kosinusi.



$\overrightarrow{AB}$ (2,0.0), $\overrightarrow{AC}$ (3,4,0), $\overrightarrow{AD}$ (3,4,2)

a) Tetraedr hajmi .

b) ABC yoqning yuzi .

d) D uchidan tushirilgan balandlik .

a) Formulaga kora $V=\pm\frac{1}{6}\overrightarrow{AB}*\overrightarrow{AC}*\overrightarrow{AD} = \pm\frac{1}{6}\begin{vmatrix}2 & 0 & 0\\3 & 4 & 0\\3 & 4 & 2\end{vmatrix}=$

$=\pm\frac{1}{6}(16+0+0-0-0)=\pm\frac{1}{6}*16=\frac{8}{3}$ kub birlik

Maktab giometryasidan ma'lumki

$V=\frac{1}{3}S_a*\text{h} => h=\frac{3*V}{S_{ACB}}$

b) $S_{ACB}=\frac{1}{2}\left|\overrightarrow{AC}\text{ x }\overrightarrow{AB}\right|$

$\overrightarrow{AC}\text{ x }\overrightarrow{AB}=\begin{vmatrix}i & j & k\\3 & 4 & 0\\2 & 0 & 0\end{vmatrix}=$ (0+0+0-8k-0-0)=8k

$S_{ACB}=\frac{1}{2}*8=4$

v) $h=\frac{3*V}{S_{ACB}}=\frac{\frac{8}{3}*3}{4}=\frac{8}{4}=2$ h=2

2) $\overrightarrow{AB}=\bar{B}\bar{A}$ $\overrightarrow{AC}-\overrightarrow{AB}=\bar{C}-\bar{B}=\overline{BC}$

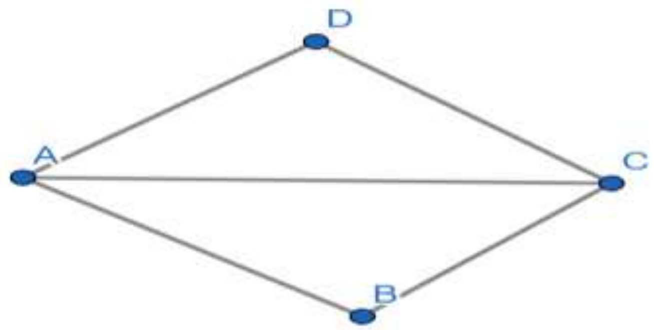
$\overrightarrow{AC}=\bar{C}-\bar{A}$

$\overrightarrow{AB}$ va $\overline{BC}$ qirralar orasidagi burchak

$\cos\varphi=\frac{\bar{a}\bar{b}}{|a||b|}$ $\overrightarrow{AB}$ (2,0.0) BC(1,4,0)

$\cos\varphi=\frac{\overline{AB}*\overline{BC}}{|AB||BC|}=\frac{2+0+0}{\sqrt{4}*\sqrt{1+16}}=\frac{2}{2*\sqrt{17}}=\frac{1}{\sqrt{17}}$

ABC va ADC yoqlar orasidagi φ burchak kosinusi



$\varphi_2 - ?$

1^0 Buning uchun ΔABC dan $\overrightarrow{AB}$ va $\overrightarrow{AC}$ yo'naltiruvchisini va ΔADC dan esa $\overrightarrow{AD}$ va $\overrightarrow{AC}$

ni yo'naltiruvchisini topamiz…

$\overrightarrow{AB}$ va $\overrightarrow{AC}$ to'g'ri chiziq tenglamasiga ko'ra

$\frac{x-2}{3-2} = \frac{y-4}{4-0} = \frac{z-0}{0-0}$ $\frac{x-2}{1} = \frac{y}{4} = \frac{z}{0}$ yo'naltiruvchisi $\overrightarrow{n}(1;4;0)$

2^0 $\overrightarrow{AD}$ va $\overrightarrow{AC}$ dan esa:

$\frac{x-3}{0} = \frac{y-4}{0} = \frac{z-2}{2-0}$ $\frac{x-3}{0} = \frac{y-4}{0} = \frac{z-2}{2}$ yo'naltiruvchisi esa $\overrightarrow{l}(0;0;2)$

$\cos(n \wedge l) = \frac{0}{\sqrt{17}} * \frac{1}{\sqrt{4}} = 0$ $\varphi = 90^0$

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