LAPLAS TEOREMASI YORDAMIDA 4-TARTIBLI DETERMINANTNI HISOBLASH

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Anatatsiya: *Laplas teoremasida tarkibida nol ishtirok etgan satr yoki ustunlarni tanlab olish, hisob kitoblarni ancha yengillashtiradi. Ushbu maqolada yuqori tartibli determinantlarni Laplas teoremasi yordamida hisoblashni ko'rib o'tamiz.*

Kalit so'zlar: *To'ldiruvchi minor,* k —tartibli minor, algebraik to'ldiruvchi, *Laplas teoremasi*

Biz avval $A_{i,j}$ algebraik to'ldiruvchi va (n-1)- tartibli $\Delta_{i,j}$ minor tushunchalarini kiritgan edik . Ushbu mavzuda ixtiyoriy k- tartibli minor tushunchasini kiritamiz.

Berilgan $n-$ tartibli determinantning ixtiyoriy k ta satr va k ta ustunining kesishgan joylaridagi elementlardan hosil qilingan $k-$ determinantga k —tartibli minor deyiladi. Determinantning $i_1, i_2, \dots, i_k$ satrlari va $j_1, j_2, \dots, j_k$ ustunlari kesishmasidan tuzilgan minor . $M^{j_1,j_2 \dots\dots j_k}_{i_1,i_2,\dots\dots i_k}$kabi belgilanadi. Xususan , determinantning elemntini ham birinchi tartibli minor deb qarash mumkin.

Tanlab olingan k ta ustun va k ta satrlarni o'chirib tashlash natijasidan hosil bolgan (n-k)- tartibli determinantga , berilgan minorning to'ldiruvchi minori deyiladi. $M^{j_1,j_2 \dots\dots j_k}_{i_1,i_2,\dots\dots i_k}$ minorning to'ldiruvchi minori $\overline{M}^{j_1,j_2 \dots\dots j_k}_{i_1,i_2,\dots\dots i_k}$ kabi belgilanadi.

$k-$ tartibli . $M^{j_1,j_2 \dots\dots j_k}_{i_1,i_2,\dots\dots i_k}$ minorning algebraik to'ldiruvchisi deb

$$\bar{A}^{j_1,j_2 \dots\dots j_k}_{i_1 i_2 \dots\dots i_k} = (-1)^{S_M} \, \overline{M}^{j_1,j_2 \dots\dots j_k}_{i_1,i_2,\dots\dots i_k} \qquad (1)$$

ifodaga aytiladi, bu yerda

$S_M = (i_1 + i_2 + \dots + i_k) + (j_1 + j_2 + \dots + j_k)$

Ta'kidlash joizki $A_{i,j} = \overline{A_i^j}$. $(n-1)$ —tartibli $\Delta_{i,j}$ minor esa birinchi tartibli $a_{i,j}$ minorning to'ldiruvchi minori deyiladi.

M: Ushbu $\begin{vmatrix} -1 & 6 & 2 & 4 \\ 4 & 5 & -2 & 3 \\ 1 & -3 & 3 & 7 \\ -4 & 2 & 0 & -5 \end{vmatrix}$ determinant uchun

$M_{1,3} = 2,$ 1- tartibli minor

$M_{1,3}^{2,3} = \begin{vmatrix} 6 & 2 \\ -3 & 3 \end{vmatrix},$ 2- tartibli minor

$M_{1,2,3}^{1,3,4} = \begin{vmatrix} -1 & 2 & 4 \\ 4 & -2 & 3 \\ -4 & 0 & 5 \end{vmatrix}$, 3- tartibli minor

Berilgan matritsaning bosh dioganalida joylashgan

$a_{1,1}$, $\begin{vmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{vmatrix}$, $\begin{vmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{vmatrix}$,..., $\begin{bmatrix} a_{1,1} & \cdots & a_{1,k} \\ \vdots & \ddots & \vdots \\ a_{k,1} & \cdots & a_{k,k} \end{bmatrix}$ minorlar matritsaning *bosh minorlari* deb ataladi.

Minor, hamda unga mos keluvchi to'ldiruvchi minor va algebraik to'ldiruvchilarni qulaylik uchun M, $\overline{M}$ va $\bar{A}$ lar bilan belgilab olamiz .

1.1-lemma. $M \cdot \bar{A}$ ko'paytmaning hadlari $|A|$ determinantning hadlari bo'lib ular har xil ishorali bo'ladi.

Isbot . Lemma isbotini dastlab , berilgan M minor k −tartibli bosh minor bo'lgan hol uchun ko'rsatamiz:

$M \cdot \bar{A} = M \cdot (-1)^{S_M} \cdot \overline{M} = (-1)^{S_M} \cdot \overline{M} \cdot M$.

U holda

$$S_M = (1 + 2 + \cdots + k) + (1 + 2 + \cdots + k) = 2(1 + 2 + \cdots + k)$$

juft son bo'ladi . Demak ,

$M \cdot \bar{A} = \overline{M} \cdot M$

1.1-teorema .(Laplas teoremasi). Determinantning tanlab olingan k ta $(1 \le k \le n-1)$ satri bo'yicha barcha k −tartibli minorlarning o'z algebraik to'ldiruvchilariga ko'paytmalari yig'indisi determinantning qiymatiga teng.

Isbot. Teoremaning shartig asosan biz

$|A| = M_1A_1 + M_2A_2 + \cdots + M_zA_z$ (2)

Yoyilmaning to'g'ri ekanligini ko'rsatishimiz kerak . Bu yerda M_i lar tanlab olingan $i_1, i_2, \dots, i_k$ satrlar bo'yicha olingan barcha minorlar va A_i lar minorlarga mos keluvchi algebraik to'ldiruvchilardir. Yuqoridagi lemmaga asosan

$M_iA_i, i = \overline{1,z}$ ko'paytmalarning har bir hadi determinantning hadi bo'lib , ular bir xil ishorali bo'ladi.

Aytaylik,

$$a_{1,\alpha_1} \cdot a_{2,\alpha_2} \cdot \dots \cdot a_{n,\alpha_n}$$

determinantning ixtiyoriy hadi bo'lsin. Bu ko'paytmadan tanlab olingan $i_1, i_2, \dots, i_k$ satrlariga tegishli bo'lgan elementlarning ko'paytmasini olamiz.

$$a_{i_1,\alpha_{i_1}} \cdot a_{i_2,\alpha_{i_2}} \cdot \dots \cdot a_{i_k,\alpha_{i_k}}$$

Bu ko'paytma $i_1, i_2, \dots, i_k$ satrlar va $\alpha_{i_1}, \alpha_{i_2}, \dots, \alpha_{i_k}$ ustunlarining kesishmasida turuvchi k −tartibli M minorning umumiy hadi bo'lib olinmay qolgan ko'paytuvchilar $(n-k)$ −tartibli $\overline{M}$ to'ldiruvchi minorning umumiy hadi bo'ladi.

Shunday qilib , determinantning har qanday hadi tanlab olingan satrlar bo'yicha M minor bilan to'ldiruvchi $\overline{M}$ minorning tarkibiga kiradi . Determinantda bo'lgan hadni hosil qilish uchun esa, to'ldiruvchi minorni algebraik to'ldiruvchi bilan almashtirish kifoya.

Endi biz **(2)** tenglikning o'ng tomonidagi hadlar soni chap tomonidagi hadlar soniga teng ekanligini ko'rsatamiz. Bizga ma'lumki , M_i minorda k! ta hadi bo'lib , A_i algebraik to'ldiruvchida esa $(n-k)!$ ta had mavjud . Demak, M_iA_i ko'paytmada $k!\,(n-k)!$ ta had ishtirok etadi. Ma'lumki tanlab olingan k ta satrda hosil qilinadigan barcha k –tartibli minorlar soni n ta sondan k ta sonni tanlab olishlar soniga, ya'ni C_n^k ga teng. Demak, o'ng tomondagi barcha hadlar soni

$$C_n^k \cdot k! \cdot (n-k) = \frac{n!}{k!\,(n-k)!} \cdot k! \cdot (n-k)! = n!$$

ga teng bu esa chap tomondagi hadlar soni bilan o'ng tomondagi hadlar soni teng ekanligini bildiradi. Chunki, n –tartibli determinantning $n!$ ta hadi mavjud. Demak, biz determinantning barcha hadi o'ng tomonda ham aynan bir marotaba ishtirok etishini ko'rsatdik.

Misol.1. Ushbu 4-tartibli tartibli determinantni 2ta satrini tanlab olib Laplas teoremasidan foydalanib hisoblaymiz.

$$\Delta=\begin{vmatrix} -1 & 0 & 2 & 0 \\ 4 & 5 & -2 & 3 \\ 1 & 0 & 3 & 0 \\ -4 & 2 & 1 & -5 \end{vmatrix} \quad i_1 = 1 \;\; i_2 = 3\,, k = 2,\; C_4^2 = \frac{4!}{2!(4-2)!} = 6.$$

$$d = M_{1,3}^{1,2}A_{1,3}^{1,2} + M_{1,3}^{1,3}A_{1,3}^{1,3} + M_{1,3}^{1,4}A_{1,3}^{1,4} + M_{1,3}^{2,3}A_{1,3}^{2,3} + M_{1,3}^{2,4}A_{1,3}^{2,4} + M_{1,3}^{3,4}A_{1,3}^{3,4} =$$

$$= (-1)^{1+3+1+2}\begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix}\begin{vmatrix} -2 & 3 \\ 1 & -5 \end{vmatrix} + (-1)^{1+3+1+3}\begin{vmatrix} -1 & 2 \\ 1 & 3 \end{vmatrix}\begin{vmatrix} 5 & 3 \\ 2 & -5 \end{vmatrix} +$$
$$+(-1)^{1+3+1+4}\begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix}\begin{vmatrix} 5 & -2 \\ 2 & 1 \end{vmatrix} + (-1)^{1+3+2+3}\begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix}\begin{vmatrix} 4 & 3 \\ -4 & -5 \end{vmatrix} + (-1)^{1+3+2+4}$$
$$\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}\begin{vmatrix} 4 & -2 \\ -4 & 1 \end{vmatrix} + (-1)^{1+3+2+4}\begin{vmatrix} 2 & 0 \\ 3 & 0 \end{vmatrix}\begin{vmatrix} 4 & 5 \\ -4 & 2 \end{vmatrix} = (-1)^8\begin{vmatrix} -1 & 2 \\ 1 & 3 \end{vmatrix}\begin{vmatrix} 5 & 3 \\ 2 & -5 \end{vmatrix} = (-3-2)\,\cdot$$
$$(-25-6) = (-5)\,\cdot\,(-31) = 155$$

Misol.2. Ushbu 4-tartibli tartibli determinantni 3 ta satrini tanlab olib Laplas teoremasidan foydalanib hisoblaymiz.

$$\Delta=\begin{vmatrix} 1 & 2 & -2 & 0 \\ 2 & 0 & 3 & 4 \\ -1 & 6 & 7 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} \quad i_1 = 1, i_2 = 2\,, i_3 = 3\;k = 3, C_4^3 = \frac{4!}{3!(4-3)!} = 4$$

$$d = M_{1,2,,3}^{1,2,3} \cdot A_{1,2,3}^{1,2,,3} + M_{12,,3}^{12,4} \cdot A_{1,2,3}^{1,24,} + M_{1,2,3}^{1,3,,4} \cdot A_{12,,3}^{1,3,4} + M_{12,,3}^{2,3,4} \cdot A_{1,2,4}^{2,3,4} =$$

$$(-1)^{12}\begin{vmatrix} 1 & 2 & -2 \\ 2 & 0 & 3 \\ -1 & 6 & 7 \end{vmatrix} \cdot 1 + (-1)^{13}\begin{vmatrix} 1 & 2 & 0 \\ 2 & 0 & 4 \\ -1 & 6 & 0 \end{vmatrix} \cdot 0 + (-1)^{14}\begin{vmatrix} 1 & -2 & 0 \\ 2 & 3 & 4 \\ -1 & 7 & 0 \end{vmatrix} \cdot$$
$$1 + (-1)^{15}\begin{vmatrix} 2 & -2 & 0 \\ 0 & 3 & 4 \\ 6 & 7 & 0 \end{vmatrix} \cdot 0 = (-76) \cdot 1 + (-20) \cdot 0 + (-20) \cdot 1 - 104 \cdot 0 = (-76) + 0 -$$
$$20 = (-96)$$

Yuqoridagi misoldan ko'rinib turibdiki, Laplas teoremasini qo'llashda tarkibida nol ishtirok etgan satr yoki ustunlarni tanlab olish, hisob kitoblarni ancha yengillashtiradi. Demak, determinantda yetarlicha nollar ishtirok etgan holda, aynan noli ko'p satrlar uchun Laplas teoremasini qo'llash orqali determinantlarni tez va oson hisoblash mumkin.

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