AYLANMA JISM HARAKATI TURG'UNLIGINI CHETAYEV VA RAUS TEOREMALARI YORDAMIDA ANIQLASH

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Annotatsiya: *Ushbu ishda konik mayatnikning harakati turg'unligini o'rganishning ikkita usuli keltirilgan bo'lib, ularning birinchisi integral bog'lovchilarini qurishning Chetaev usuli ikkinchisi esa Raus teoremasini qo'llashdir.*

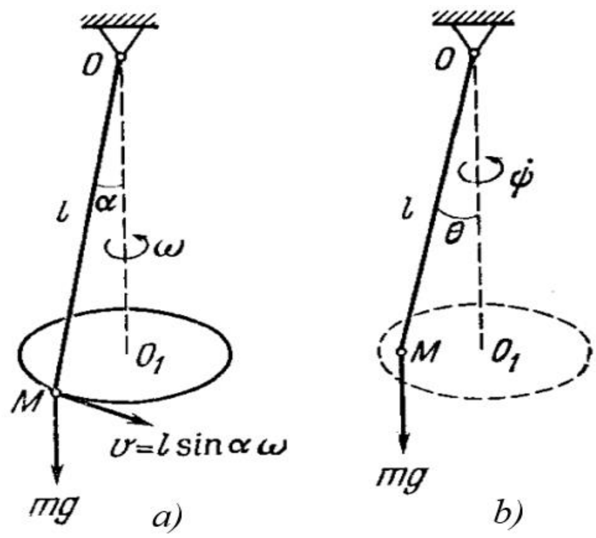
Kalit so'zlar: *mayatnik, statsionar harakat, harakat turg'unligi, integral bog'lovchilar, kinetik energiya, potensial energiya, siklik koordinata, pozitsion koordinata, giroskopik sistema.*

Barchaga ma'lumki hayot harakatdan iborat. Harakat bor joyda ish bajariladi, kuch sarflanadi, energiya paydo bo'ladi. Bu esa o'z navbatida insoniyat ehtiyojini qondiradigan jarayonni yuzaga keltiradi. Har qanday jism yoki tizim harakatini uzluksiz, turg'un bo'lishi muhim hisoblanadi. Bu masala hozirda taraqqiy etayotgan jamiyatimizda dolzarb masalalardan biri hisoblanadi. Inson ongi taraqqiy etgani sayin, u tomonidan harakatga keltirilayotgan jarayonlar ham murakkablashib boradi. Bunday murakkab jarayonlar turg'unligini o'rganishda olimlar tomonidan bir nechta usullar taklif etilgan.

Harakat turg'unligini o'rganish davomida eng effektiv usullardan biri integral bog'lovchilarini qurishning Chetaev usuli hisoblanadi. Bu maqolada Konik mayatnikning statsionar harakati turg'unligini integral bog'lovchilarini qurishning Chetaev usulidan foydalanib va Raus teoremasini qo'llab aniqlaymiz.

1. Konik mayatnikning statsionar harakati turg'unligini integral bog'lovchilarini qurishning Chetaev usulidan foydalanib tekshiramiz.

m massali M moddiy nuqtaning statsionar harakatini qaraylik. Bu nuqta vaznsiz va l uzunlikka ega bo'lgan, gorizontal aylana bo'ylab og'irlik kuchi ta'sirida doimiy tezlik bilan harakatlanuvchi ipga osilgan bo'lsin (1 a-rasm).



1-rasm

O nuqtaga mahkamlangan mayatnik ipi aylana konusni o’z statsionar harakati davomida tasvirlaydi; ip va OO_1 vertikal orasidagi burchakni α orqali belgilaymiz, OO_1 vertikal bo’ylab ip aylanishi burchak tezligini ω orqali belgilaymiz. α burchak, ω burchak tezlik va l mayatnik uzunligi orasida statsionar harakatdan yaxshi ma’lum bo’lgan munosabat bor:

$$\omega^2 cos\alpha = g/l, \qquad (1)$$

va u Dalamber prinsipi orqali hosil qilinishi mumkin.

Mayatnikning aylana bo’ylab statsionar harakatini qo’zg’almagan harakat deb olamiz. Faraz qilaylik, bu harakatga kichik qo’zg’alishlar qo’yilgan bo’lsin. Qo’zg’algan harakatda ip va OO_1 vertikal orasidagi burchakni θ orqali belgilaymiz (1 b-rasm). OO_1M tekislikning OO_1 vertikal atrofida aylanishi burchak tezligini ψ orqali belgilaymiz.

Quyidagi belgilashlarni kiritaylik:

$$\theta = \alpha + x_1, \ \ \dot{\theta} = x_2, \ \ \psi = \dot{\omega} + x_3. \qquad (2)$$

Qo’zg’almagan harakat turg’unligini $\theta, \dot{\theta}\ va\ \dot{\psi}$ kattaliklarga nisbatan o’rganamiz. Mayatnikning T −kinetik va Π – potensial energiyalari

$$T = \frac{ml^2}{2}\left(\dot{\theta}^2 + sin^2\theta\dot{\psi}^2\right), \qquad \Pi = -mglcos\theta.$$

tengliklar orqali hisoblanadi.

Mayatnikka ta’sir qiluvchi og’irlik kuchi potensial bo’lgani sabab, ψ koordinata esa siklik ekanidan (kinetik energiya T umumiy tezlik $\dot{\psi}$ ga bog’liq, lekin ψ koordinataga bog’liq emas va bu koordinataga mos keladigan kuch nolga teng) harakatning ikkita integrali mavjud (h va n – o’zgarmaslar):

$$T + \Pi = \frac{ml^2}{2}\left(\dot{\theta}^2 + sin^2\theta\dot{\psi}^2\right) - mglcos\theta = \frac{ml^2}{2}h,$$

$$\frac{\partial T}{\partial \dot{\psi}} = ml^2 sin^2\theta\dot{\psi} = ml^2 n$$

($ml^2/2$ va ml^2 xadlar qulaylik uchun kiritilgan).

Ikkinchi tenglik mayatnikning OO_1 vertikaliga nisbatan harakat miqdori oni integralini tasvirlaydi va uni elementar tasavvur orqali olish mumkin.

(2.35) tengliklardan foydalanib, bu integrallarni quyidagi ko'rinishda yozamiz:

$$F_1(x_1,x_2,x_3)=[x_2^2+sin^2(\alpha+x_1)(\omega+x_3)^2]-\frac{2g}{l}cos(\alpha+x_1)=h,$$

$$F_2(x_1,x_2,x_3)=sin^2(\alpha+x_1)(\omega+x_3)=n. \qquad (3)$$

(3) integrallar dinamikaning umumiy teoremalaridan olingan.

Mayatnikning statsionar harakatini turg'un harakatini $\theta,\dot{\theta}$ va $\dot{\psi}$ kattaliklarga nisbatan o'rganishga o'tamiz. Topilgan integrallarning hech biri x_1,x_2 va x_3 ga nisbatan aniq ishorali emas. Shuning uchun (3) integrallar bog'lovchisini tuzib olamiz, $\lambda_1=1,\lambda_2=\lambda$:

$$V=F_1-F_1(0)+\lambda[F_2-F_2(0)]=[x_2^2+sin^2(\alpha+x_1)(\omega+x_3)^2]-$$
$$-\frac{2g}{l}cos(\alpha+x_1)-\left(\omega^2sin^2\alpha-\frac{2g}{l}cos\alpha\right)+$$
$$+\lambda sin^2(\alpha+x_1)(\omega+x_3)-\lambda sin^2\alpha\cdot\omega.$$

$\left(\omega^2sin^2\alpha-\frac{2g}{l}cos\alpha\right)$ va $\lambda sin^2\alpha\cdot\omega$ xadlar, V funksiya $x_1=x_2=x_3=0$ bo'lganda nolga aylanishi uchun kiritilgan. g/l nisbatni (1) tenglikdagi qiymati bilan almashtiramiz va V funksiyani x_1,x_2,x_3 darajalar bo'yicha qatorlarga yoyamiz.

$$sin^2(\alpha+x_1)=sin^2\alpha+sin2\alpha\cdot x_1+cos2\alpha\cdot x_1^2+\cdots,$$
$$\cos(\alpha+x_1)=cos\alpha-sin\alpha\cdot x_1-\frac{1}{2}cos\alpha\cdot x_1^2+\cdots,$$

ga egamiz, bu yerda nuqtalar orqali yuqori darajali xadlar belgilangan.

Bu qiymatlarni V funksiyaning oxirgi ifodasiga $sin^2(\alpha+x_1)$ va $\cos(\alpha+x_1)$ uchun kiritamiz va xadlarni gruppalaymiz

$$V=\omega[(\lambda+\omega)cos2\alpha+\omega cos^2\alpha]x_1^2+x_2^2+sin^2\alpha\cdot x_3^2+$$
$$+\omega sin2\alpha\cdot(\lambda+2\omega)x_1+sin^2\alpha\cdot(\lambda+2\omega)x_3+$$
$$+sin2\alpha\cdot(\lambda+2\omega)x_1x_3+\cdots$$

V funksiya aniq – musbat bo'lishi uchun birinchi darajadagi x_1,x_2,x_3 variatsiyalarni o'z ichiga olgan xadlardan qutulish zarur.

Bu holatda,

$$\lambda=-\alpha\cdot\omega$$

deb olish yetarli. λ ning bu qiymatida V funksiya quyidagi ko'rinishga ega:

$$V=\omega^2sin^2\alpha\cdot x_1^2+x_2^2+sin^2\alpha\cdot x_3^2+\cdots$$

V funksiyaning kvadratik qismi x_1,x_2 va x_3 ga nisbatan aniq – musbat bo'lsa, u holda x_1,x_2 va x_3 ning yetarlicha kichik qiymatlarida butun V funksiya aniq – musbat bo'ladi. V funksiyaning vaqt bo'yicha hosilasi (3) integrallar asosida ayniy nolga teng va haqiqatdan konik mayatnikning statsionar harakati $\theta,\dot{\theta}\ va\ \dot{\psi}$ ga nisbatan turg'un bo'ladi.

2. Konik mayatnik statsionar harakati turg'unligini Raus teoremasi yordamida keltirib chiqaramiz.

Oldin mayatnikning T kinetik va Π potensial energiyasi uchun quyidagi ifodalar olingan edi:

$$T=\frac{1}{2}ml^2\left(\dot{\theta}^2+\dot{\psi}^2 sin^2\theta\right),\quad \Pi=-mglcos\theta.$$

Bu ifodalardan ko’rinib turibdiki, ψ koordinata siklik, θ koordinata esa pozitsion. Endi siklik integralni tuzib olamiz:

$$p=\frac{\partial T}{\partial\dot{\psi}}=ml^2 sin^2\theta\dot{\psi}=c. \qquad (4)$$

Bundan $\dot{\psi}$ ni topamiz va uni kinetik energiya ifodasiga kiritamiz:

$$\dot{\psi}=\frac{c}{ml^2 sin^2\theta},\quad T^*=\frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}\frac{c^2}{ml^2 sin^2\theta}.$$

Raus funksiyasini tuzamiz:

$$R=T^*-c\dot{\psi}=\frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}\frac{c^2}{ml^2 sin^2\theta}-c\cdot\frac{c}{ml^2 sin^2\theta}$$

yoki

$$R=\frac{1}{2}ml^2\dot{\theta}^2-\frac{1}{2}\frac{c^2}{ml^2 sin^2\theta}.$$

Bundan ko’rinib turibdiki:

$$R_2=\frac{1}{2}ml^2\dot{\theta}^2,\quad R_1=0,\quad R_0=-\frac{1}{2}\frac{c^2}{ml^2 sin^2\theta}$$

$R_1=0$ ekanidan, sistema giroskopik bog’langan emas. Umumiy nazariyaga asosan keltirilgan sistemani $W=\Pi-R_0$ potensial energiyasini tuzib olamiz:

$$W=-mglcos\theta+\frac{1}{2}\frac{c^2}{ml^2 sin^2\theta}. \qquad (5)$$

θ burchak qiymatini harakatda α orqali belgilaylik, $\dot{\psi}$ siklik tezlik qiymatini ω orqali belgilaymiz. $\left(\frac{\partial W}{\partial q_j}\right)_{q_0}=0 \quad (j=1,\dots,s)$ shart quyidagi ko’rinishga keladi:

$$\left(\frac{\partial W}{\partial\theta}\right)_{\theta=\alpha}=mglsin\alpha-\frac{c^2 cos\alpha}{ml^2 sin^3\alpha}=0 \qquad (6)$$

yoki soddalashtirgandan so’ng:

$$\frac{sin^4\alpha}{cos\alpha}=\frac{c^2}{m^2gl^3}. \qquad (7)$$

Bu tenglik, (6) tenglamaning bir parametrlik yechimlar oilasini aniqlaydi, konik mayatnikning burchak tezligi mos qiymati (4) tenglik orqali topiladi:

$$\dot{\psi}=\dot{\psi}_0=\omega=\frac{c}{ml^2 sin^2\alpha}. \qquad (8)$$

(7) va (8) tengliklardan c parametrni chiqarib yuborish orqali quyidagini topamiz:

$$\omega^2 cos\alpha=\frac{g}{l}.$$

Konik mayatnikni statsionar harakatining bu sharti elementar farazlar orqali topiladi.

Mayatnikning statsionar harakatini qo’zg’almagan deb olamiz va uning turg’unligini Raus teoremasi va Lyapunov to’ldirishi yordamida o’rganamiz. $\partial=\alpha+x$

deb olamiz va (5) ifodaga W funksiya uchun qo'llab, $W - W_0$ ayirmani x darajalar bo'yicha qatorga yoyamiz:

$$W - W_0 = \left(\frac{\partial W}{\partial \theta}\right)_{\theta=\alpha} \cdot x + \frac{1}{2}\left(\frac{\partial^2 W}{\partial \theta^2}\right)_{\theta=\alpha} \cdot x^2 + \cdots,$$

yoki (6) tenglikni qo'llab:

$$W - W_0 = \frac{1}{2}\left(\frac{\partial^2 W}{\partial \theta^2}\right)_{\theta=\alpha} \cdot x^2 + \cdots,$$

bu yerda nuqtalar orqali x ning darajasi 2 dan yuqori xadlar belgilangan.

Hosilani hisoblab quyidagini topamiz:

$$W - W_0 = \frac{1}{2}\left[mglcos\alpha + \frac{c^2}{ml^2}\frac{sin^2\alpha + 3cos^2\alpha}{sin^4\alpha}\right]x^2 + \cdots$$

x^2 dagi ko'paytiruvchi musbat ekanidan W funksiya statsionar harakatda minimumga ega. Bundan tashqari $\forall\, \theta = \alpha \neq \frac{\pi}{2}$ yechim (7), (4) integralning c o'zgarmasiga uzluksiz bog'liq, $\dot{\psi} = \omega$ siklik tezlik esa $\theta = \alpha \neq 0$ bo'lganda shu o'zgarmasga uzluksiz bog'liq. Shuning uchun Raus teoremasi va Lyapunov qo'shimchasi asosida konik mayatnikning θ, $\dot{\theta}$ va $\dot{\psi}$ ga nisbatan statsionar harakati turg'un.

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