FURE INTEGRALLARINI YAQINLASHTIRISH

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Annotatsiya: *Ushbu maqolada ko‘plab masalalari integral va differensial tenglamalar yoki ularning sistemalariga keltiriladi. Bunday tenglamalarni yechish uchun aniq integralni hisoblashga to‘g‘ri keladi. Bunday integrallarni katta aniqlikda taqribiy hisoblash usullarini ishlab chiqish hisoblash matematikasining dolzarb masalalaridan biridir. Integrallarni taqribiy hisoblashning universal usuli kvadratur va kubatur algebraik, ehtimollar nazariyasi, nazariy-sonli va funksional formulalardan foydalanishdir.*

Kalit so'zlar: *Furye almashtirish, kvadratur, kubatur, integral, taqribiy hisoblash, optimal*

Kirish

Fan va texnikaning ko‘plab masalalari integral va differensial tenglamalar yoki ularning sistemalariga keltiriladi. Ko‘p hollarda bunday tenglamalarni yechish uchun aniq integralni hisoblashga to‘g‘ri keladi. Lekin integrallarning juda kam ko‘rinishlarinigina aniq hisoblash mumkin. Bunday integrallarni katta aniqlikda taqribiy hisoblash usullarini ishlab chiqish hisoblash matematikasining dolzarb masalalaridan biridir. Integrallarni taqribiy hisoblashning universal usuli kvadratur va kubatur formulalardan foydalanishdir. Bu maqolada Sobolev fazosida eksponentsial vaznli integrallarni taqribiy hisoblash uchun optimal kvadratur formulalar qurishga bag‘ishlangan. Hozirgi kunda kvadratur va kubatur formulalar qurish nazariyasida quyidagi asosiy yondoshuvlar mavjud: *algebraik, ehtimollar nazariyasi, nazariy-sonli va funksional.* Funksional analiz usullariga asoslangan holda kvadratur formulalar qurish dastlab A.Sard va S.M.Nikolskiyning ishlarida bajarilgan. Kvadratur formulalarni optimallashtirish masalasini integral va kvadratur formulalarning ayirmasini eng kichik bo‘ladigan qilib izlash deb talqin qilish mumkin.

FURYE INTEGRALLARINI TAQRIBIY HISOBLASH UCHUN OPTIMAL KVADRATUR FORMULALAR

Furye almashtirishlari fan va texnikada, xususan, kompyuter tomografiyasi (KT) muammolarida keng qo‘llaniladi. Ma’lumki, to‘liq uzluksiz rentgen ma'lumotlari mavjud bo'lganda, KT tasvirlari turli xil analitik formulalar yordamida aniq qayta tiklanishi mumkin, masalan, filtrlangan orqa proyeksiyalash formulasi [1,2,3] .

Furye almashtirishlarini taqribiy hisoblash uchun odatda

$$I(f,\omega)=\int_a^b e^{2\pi i\omega x} f(x)dx, \qquad \omega\in Ў \qquad (1.1)$$

ko‘rinishidagi integrallarini taqribiy qiymatini hisoblashimiz kerak.

ω-parametrning katta qiymatlari uchun bunday integrallar *kuchli tebranuvchi* deb nomlanadi. Bunday integrallarni hisoblash maxsus samarali sonli usullarni ishlab chiqishni talab qiladi. Integralni hisoblashning bunday birinchi usuli Fileon [4] tomonidan taklif qilingan. Bundan tashqari, kuchli tebranuvchi integrallarga ega bo‘lgan integrallarni hisoblashni har xil turlari uchun ko‘plab maxsus samarali usullar ishlab chiqilgan.

$(m-1)$-tartibli absalyut uzluksiz hosilaga ega bo'lgan va m-tartibli hosilasi [a, b] bo‘yicha kvadrati bilan integrallanuvchi kompleks qiymatli funksiyalarni ko‘rib chiqamiz. $L_2^{(m)}[a,b]$ fazosida skalyar ko‘paytma quyidagicha kiritiladi

$$\langle f,g\rangle_m=\int_a^b f^{(m)}(x)\bar{g}^{(m)}(x)dx$$

va unga mos keladigan norma quyidagicha aniqlanadi

$$\| f \|_{L_2^{(m)}[a,b]}=\sqrt{\langle f,f\rangle_m}.$$

Quyidagi maqolalarda [17,18,19] d^{2m}/dx^{2m} differentsial operatorning diskret analogidan foydalangan holda, $L_2^{(m)}$ va $W_2^{(m,m-1)}$ Hilbert fazosilaridagi ω butun sonlar uchun integral (2.1) ni taqribiy hisoblash uchun optimal kvadratur formulalar qurilgan.

Shu bilan birga, [4,5] ishlarida $L_2^{(m)}[a,b]$ Sobolev fazosida integral (1.1) integralni sonli hisoblash uchun optimal kvadratur formulalari qurilgan va olingan kvadratur formulalar KT tasvirlarini qayta tiklashda foydalaniladi.

Shuni ta'kidlash kerakki, (1.1) Furye koeffitsientlarini davriy funksiyalarning $\tilde{L}_2^{(m)}$ -fazosida ω ning butun qiymati uchun optimal kvadratura formulalar X.M. Shadimetov [5,6] tomonidan qurilgan.

[5] va [6] ishlarda qurilgan ωning butun qiymatlarda Furye koeffitsientlari uchun optimal kvadratur formulalaridan foydalangan holda, haqiqiy ω uchun (1.1) integralni sonli hisoblash uchun approksimatsion formulalar olingan. Keyin ushbu approksimatsion formulalar $\tilde{L}_2(0,1]$ kompleks qiymatli, davriy funksiyalarning fazosida KT tasvirlarini taqribiy qayta tiklash uchun qo'llaniladi.

Ushbu ishning maqsadi [6] natijalarini davom ettirish va rivojlantirishdir. Bu yerda biz kompleks qiymatli funksiyalarning $L_2^{(2)}$-fazosida ωhaqiqiy qiymatlarida (1.1) integralni taqribiy hisoblash uchun hosilali optimal kvadratur formulasini qurish masalasini qarab chiqamiz. Furye almashtirishini aniq funksiyalarda taqribiy hisoblashda olingan optimal kvadratur formulalarni qo‘llaymiz.

Bundan tashqari, bu bobda Sobolev fazosidagi $L_2^{(2)}$-kompleks qiymatli funksiyasining (1.1) integrallarini taqribiy hisoblash uchun optimal kvadratur formula qurishga bag'ishlangan. Uchinchi bobda, ikkinchi bobda qurilgan optimal formuladan aniq funksiyalarning Furye almashtirishni taqribiy hisoblash uchun foydalanamiz.

φ funksiya $L_2^{(2)}[0,1]$ Sobolev fazosiga tegishli bo'lsin. $L_2^{(2)}[0,1]$ - Hilbert fazosi kompleks qiymatli funksiyalarning ikkinchi tartibli umumlashgan hosilalasi kvadrati bilan integrallanuvchi funksiyalar fazosi.

Bu fazoda φ va ϕ ikki funksiyaning skalyar ko‘paytmasi quyidagicha aniqlanadi

$$\langle \varphi, \phi \rangle_2 = \int_0^1 \varphi''(x)\bar{\phi}''(x)dx.$$

Skalyar ko‘paytma yordamida, $L_2^{(2)}[0,1]$ fazoda norma quyidagicha kiritiladi

$$\| \varphi \|_{L_2^{(2)}[0,1]} = \sqrt{\langle \varphi, \varphi \rangle_2} = (\int_0^1 \varphi''(x)\bar{\varphi}''(x)dx)^{\frac{1}{2}} = (\int_0^1 (\varphi''(x))^2 dx)^{\frac{1}{2}}$$

Biz ushbu kvadratur formulani qaraymiz

$$\int_0^1 e^{2\pi i\omega x}\varphi(x)dx \cong \sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta) + \sum_{\beta=0}^{N} C_1[\beta]\varphi'(h\beta), \qquad (1.2)$$

Bu yerda, $C_0[\beta]$ lar $\int_0^1 e^{2\pi i\omega x}\varphi(x)dx \cong \sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta)$, $\varphi \in L_2^{(1)}$ kvadratur formulaning koeffitsiyentlari bo‘lib, quyidagicha aniqlangan.

$$C_0[0] = h \cdot \frac{1 + 2\pi i\omega h - e^{2\pi i\omega h}}{(2\pi\omega h)^2},$$

$$C_0[\beta] = h \cdot \frac{2 \cdot (1 - \cos 2\pi\omega h)}{(2\pi\omega h)^2} \cdot e^{2\pi i\omega h\beta}, \beta = 1, 2, \ldots, N-1,$$

$$C_0[N] = h \cdot \frac{1 - 2\pi i\omega h - e^{-2\pi i\omega h}}{(2\pi\omega h)^2} \cdot e^{2\pi i\omega}.$$

$\omega \in R$ va $\omega \neq 0$, $i^2 = -1$, $h = 1/N$, $N \in \Gamma$.

Integral va yig‘indini orasidagi ushbu

$$\int_0^1 e^{2\pi i\omega x}\varphi(x)dx - \sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta) - \sum_{\beta=0}^{N} C_1[\beta]\varphi'(h\beta),$$

ayirma (2.2) kvadratur formulaning xatoligi deyiladi va bu xatolikka ushbu chiziqli funksional mos keladi

$$l(x) = e^{2\pi i\omega x}\varepsilon_{[0,1]}(x) - \sum_{\beta=0}^{N} C_0[\beta]\delta(x - h\beta) + \sum_{\beta=0}^{N} C_1[\beta]\delta'(x - h\beta). \qquad (1.3)$$

Bu yerda $\varepsilon_{[0,1]}(x)$ - $[0,1]$ kesmaning xarakteristik funksiyasi, $\delta(x)$ - Dirakning delta funksiyasi, xususan

$$\int_{-\infty}^{\infty} \delta(x)\varphi(x)dx = \varphi(0),$$

$$\int_{-\infty}^{\infty} \delta(x-a)\varphi(x)dx = \varphi(a),$$

$$\int_{-\infty}^{\infty} \delta^{(n)}(x-a)\varphi(x)dx = (-1)^n \varphi^{(n)}(a).$$

l funksionalning φ funksiyaga tasiri quyidagicha aniqlanadi va (1.2) kvadratur formulaning xatoligini beradi.

$$(l,\varphi) = \int_{-\infty}^{\infty} l(x)\varphi(x)dx. \qquad (1.4)$$

Bundan,

$$(l,\varphi) = \int_{-\infty}^{\infty} (e^{2\pi i\omega x}\varepsilon_{[0,1]}(x) - \sum_{\beta=0}^{N} C_0[\beta]\delta(x-h\beta) + \sum_{\beta=0}^{N} C_1[\beta]\delta'(x-h\beta))\varphi(x)dx =$$

$$= \int_{-\infty}^{\infty} e^{2\pi i\omega x}\varepsilon_{[0,1]}(x)\varphi(x)dx - \sum_{\beta=0}^{N} C_0[\beta]\int_{-\infty}^{\infty} \delta(x-h\beta)\varphi(x)dx + \sum_{\beta=0}^{N} C_1[\beta]\int_{-\infty}^{\infty} \delta'(x-h\beta)\varphi(x)dx =$$

$$= \int_0^1 e^{2\pi i\omega x}\varphi(x)dx - \sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta) - \sum_{\beta=0}^{N} C_1[\beta]\varphi'(h\beta)$$

Demak

$$(l,\varphi) = \int_0^1 e^{2\pi i\omega x}\varphi(x)dx - \sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta) - \sum_{\beta=0}^{N} C_1[\beta]\varphi'(h\beta). \qquad (1.5)$$

ifoda (1.2) kvadratur formulaning xatoligini beradi.

(1.3) xatolik funksionali $L_2^{(2)}(0,1)$ da aniqlangan bo'lishi uchun quyidagi shartlarni qanoatlantirishi kerak bo'ladi:

$$(l,1) = \int_0^1 e^{2\pi i\omega x}dx - \sum_{\beta=0}^{N} C_0[\beta] = 0, \qquad (1.6)$$

$$(l,x) = \int_0^1 e^{2\pi i\omega x}xdx - \sum_{\beta=0}^{N} (C_0[\beta]\cdot h\beta + C_1[\beta]) = 0 \qquad (1.7)$$

Bu ikki tengliklar (1.2) kvadratur formulaning $ax+b$ ko'rinishidagi chiziqli funksiyaga aniqligini bildiradi. Bizga ma'lum bo'lgan $C_0[\beta]$ koeffitsiyentlar (1.6) shartni qanoatlantiradi. Demak, $C_1[\beta]$ koeffitsiyentlar uchun (1.7) shart qoladi.

Koshi-Shvarts tengsizligiga ko'ra biz quyidagiga egamiz:

$$|(l,\varphi)| \le \| l \|_{L_2^{(2)*}[0,1]} \cdot \|\varphi\|_{L_2^{(2)}[0,1]}, \qquad (1.8)$$

Shuning uchun, (1.2) kvadratur formulaning (1.5) xatoligi (1.3) funksional normasi yordamida yuqoridan quyidagicha baholanadi:

$$\|1\|_{L_2^{(2)*}[0,1]} = \sup_{\|\varphi\|_{L_2^{(2)}[0,1]}=1} |(1,\varphi)| \qquad (1.9)$$

Bu yerda $L_2^{(2)*}[0,1]$-fazo $L_2^{(2)}[0,1]$ fazosiga qo'shma fazo.

Bu ishdan asosiy maqsad berilgan $C_0[\beta]$ koeffitsiyentlar bo'yicha $L_2^{(2)}$ fazoda (1.5) xatolikga minimum beradigan $C_1[\beta]$ koeffitsiyentlarni topishdan iboratdir. Bunday $C_1[\beta]$ koeffitsiyentlar quyidagi tenglikni qanoatlantiradi:

$$\|\overset{\circ}{1} \mid L_2^{(2)*}\| = \inf_{C_1[\beta]} \|1 \mid L_2^{(2)*}\|. \qquad (1.10)$$

(1.10) tenglikni qanoatlantiruvchi $C_1[\beta]$ koeffetsiyentlar optimal koeffitsiyentlar deb ataladi va $\overset{\circ}{C}_1[\beta]$ kabi belgilanadi. Shunday qilib $L_2^{(2)}[0,1]$ fazosida (1.2) ko'rinishdagi Sard ma'nosidagi kvadratur formulani olish uchun biz quyidagi ikkita masalani yechishimiz kerak bo'ladi:

Masala 1. $L_2^{(2)*}$ fazosida (1.2) kvadratur formulaning (1.3) xatolik funksionali normasini hisoblash.

Masala 2. (1.10) tenglikni qanoatlantiruvchi $\overset{\circ}{C}_1[\beta]$ koeffitsiyentlarni topish.

Sonli eksperimentlar

Ushbu bo'limda biz $L_2^{(2)}[a,b]$-fazosida (2.1) integralni $\omega \in Ў$ bo'lgan holda taqribiy hisoblash uchun formulani beramiz.

Ikkinchi bobda $\tilde{L}_2^{(2)}(0,1]$ fazosidagi $\int_0^1 e^{(2\pi i\omega x} f(x)dx$, $\omega \in ў$ integrali uchun (1.2) kvadratur formulaning optimal koeffitsientlari (1.31) va (1.32) lardan foydalanamiz.

(1.2) optimal kvadratur formulaning (1.32) koeffitsientlari bilan $\omega \in Ў$ hol uchun kengaytmalaridan biri bu (1.32) koeffitsientlarni ω argumentning uzluksiz funksiyalari sifatida kengaytirish natijasida olingan taqribiy formuladir.

Keyin $L_2^{(2)}[a,b]$ fazodan f funksiya uchun 1 va 2-natijalar yordamida quyidagi formulani

$$\int_a^b e^{2\pi i\omega x} f(x)dx \cong \sum_{k=0}^{N} C_{k,\omega}[a,b] f(x_k), \qquad (3.1)$$

olamiz. Bu yerda

$$C_{0,\omega}[a,b] = \frac{h}{2} K_{2,\omega} e^{2\pi i\omega a},$$

$$C_{k,\omega}[a,b] = hK_{2,\omega} e^{2\pi i\omega(hk+a)}, \ k = 1,2,\dots,N-1, \qquad (3.2)$$

$$C_{N,\omega}[a,b] = \frac{h}{2} K_{2,\omega} e^{2\pi i\omega b}$$

koeffitsiyentlar, $x_k = hk + a$, $(k = 0,1,\dots,N)$-(3.1) kvadratur formulaning tugun nuqtalari, $i^2 = -1$, $\omega \in Ў$ $h = (b-a)/N$, $N \in Γ$ va

$$K_{2,\omega}=\begin{cases}\left(\dfrac{\sin\pi\omega h}{\pi\omega h}\right)^4\dfrac{3}{\cos(2\pi\omega h)+2} & \text{da } \omega\in R, \ \{0\},\\ 1 & \text{da } \omega=0.\end{cases}$$

Bundan tashqari, (3.1) formulaning (3.2) koeffitsientlari bilan integrallarni hisoblash uchun foydalanamiz.

$$g_{0,\omega}[-1,1]=\int_{-1}^{1}e^{2\pi i\omega x}dx,$$

$$g_{\sin,\omega}[-1,1]=\int_{-1}^{1}e^{2\pi i\omega x}\sin(\pi x+\pi))dx,$$

$$g_{\exp,\omega}[-1,1]=\int_{-1}^{1}e^{2\pi i\omega x}e^{-x-1}dx,$$

bu yerda

$$g_{0,\omega}[-1,1]=\begin{cases}\dfrac{1}{(pi\omega)}(\sin 2\pi\omega), & \omega\neq 0,\\ 2, & \omega=0,\end{cases} \qquad (3.3)$$

$$g_{\sin,\omega}[-1,1]=\begin{cases}\dfrac{2i}{4\pi\omega^2-\pi}\sin(2\pi\omega), & \omega\neq 0,\\ 0, & \omega=0,\end{cases} \qquad (3.4)$$

$$g_{\exp,\omega}[-1,1]=\begin{cases}\dfrac{1}{2\pi i\omega-1}[e^{-2+2\pi i\omega}-e^{-2\pi i\omega}], & \omega\neq 0,\\ 1-e^{-2}, & \omega=0.\end{cases} \qquad (3.5)$$

Yaqinlashish xatoligi quyidagicha belgilanadi

$$R_{f,\omega}[a,b]=\int_a^b e^{2\pi i\omega x}f(x)dx-\sum_{\beta=0}^{N}C_{\beta,\omega}[a,b]f(h\beta+a).$$

Quyidagi funksiyalarni ko'rib chiqamiz,

$$f_0(x)=\begin{cases}1 & x\in[-1,1],\\ 0 & \text{aks holda},\end{cases} \qquad (3.6)$$

$$f_{\sin}(x)=\begin{cases}\sin(\pi(x+1)) & x\in[-1,1],\\ 0 & \text{aks holda},\end{cases} \qquad (3.7)$$

$$f_{\exp}(x)=\begin{cases}e^{-x-1} & x\in[-1,1],\\ 0 & \text{aks holda}.\end{cases} \qquad (3.8)$$

Birinchidan, f_0 , $f_{\sin}$ va $f_{\exp}$ integrallarni hisoblash uchun (3.1) kvadratur formuladan foydalanamiz.

$$\int_{-1}^{1}e^{2\pi i\omega x}f(x)dx$$

Keyin, (3.1) kvadratur formulaning (3.2) koeffitsiyentlaridan foydalangan holda xatoliklar uchun quyidagilarga ega bo'lamiz

$$R_{f_0,\omega}[-1,1] = g_{0,\omega}[-1,1] - \sum_{k=0}^{N} C_{k,\omega}[-1,1], \quad (3.9)$$

$$R_{f_{\sin},\omega}[-1,1] = g_{\sin,\omega}[-1,1] - \sum_{k=0}^{N} C_{k,\omega}[-1,1]\sin(\pi hk), \quad (3.10)$$

$$R_{f_{\exp},\omega}[-1,1] = g_{\exp,\omega}[-1,1] - \sum_{k=0}^{N} C_{k,\omega}[-1,1]e^{-hk}, \quad (3.11)$$

XULOSA

Ushbu maqolada kompyuter tomografiyasi tasvirlarni qayta qurishda Radon almashtirishi va orqaga akslantirishdan foydalanilganda Furye integraliga kelishi bayon qilingan. Shuning uchun, ushbu ish Furye integralini taqribiy hisoblash uchun optimal kvadratur formula qurishga undan tashqari, ko'pgina boshqa amaliy masalalar ham Furye integrallarini hisoblashga keltiriladi va bu integrallarni etarlicha yuqori aniqlikda hisoblash amaliy matematika va hisoblash matematikasining dolzarb masalasidir.

Ushbu ishida Sobolevning $\tilde{L}_2^{(m)}[a,b]$ davriy, kompleks qiymatli funksiyalar fazosida optimal kvadratur formulani qurildi. Ushbu optimal kvadratur formulalar asosida yaqinlashtirish formulasini oldindi. Bu formula ba'zi funksiyalar Furye integrallarini taqribiy hisoblashda ishlatiladi. Olingan yaqinlashtirish formulasining yaqinlashish tartibi 2 ga tengligini aniq funksiyalarda ko'satilgan.

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