PARALLEL TIP O‘ZGARISH CHIZIG‘IGA EGA PARABOLIK-GIPERBOLIK TIPDAGI TENGLAMA UCHUN INTEGRAL ULASH SHARTLI CHEGARAVIY MASALA

F.R.Boymirzayev
Farg‘ona davlat universiteti
e-mail farhodjonboymirzayev@gmail.com

Annotatsiya: Ushbu ishda Riman-Liuvill kasr tartibli hosila ishtrok etgan aralash tenglama uchun aralash sohada umumiy integral shartli chegaraviy masalaning bir qiymatli yechilishi tadqiq qilinadi.

Kalit so‘zlar: Aralash tenglama, kasr tartibli hosila, integral ulash sharti.

BOUNDARY PROBLEM WITH INTEGRAL GLUING CONDITION FOR PARABOLIC-HYPERBOLIC TYPE EQUATION WITH PARALLEL LINE OF TYPE-CHANGING

F.Boymirzaev

Abstract: In this work a unique solvability of a boundary problem with general integral gluing condition for mixed equation involving the Riemann-Liouville fractional derivative has been proved.

Keywords: Mixed equation, fractional derivative, integral gluing condition.

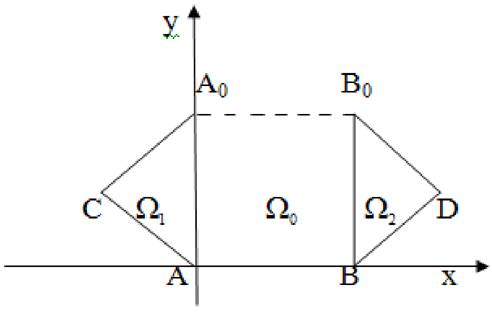
$$f(x,y)=\begin{cases} U_{xx}(x,y)-D_{0y}^{\alpha}U(x,y), & (x,y)\in\Omega_0, \\ U_{xx}(x,y)-U_{yy}(x,y), & (x,y)\in\Omega_i \quad (i=1,2) \end{cases} \qquad (1)$$

tenglamani $\Omega=\Omega_0\cup\Omega_1\cup\Omega_2\cup AA_0\cup BB_0$ aralash sohada tadqiq qilamiz.

Bu yerda $f(x,y)$-berilgan funksiya, $D_{0y}^{\alpha}U$ esa α kasr tartibli Riman-Liuvill integro-differensial operatori bo‘lib, u $0<\alpha<1$ uchun quydagicha aniqlangan $[1]$:

$$D_{0y}^{\alpha}g(t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-z)^{-\alpha}g(z)dz.$$

(1) tenglama uchun Ω sohada



1-rasm

quydagi masalani tadqiq etamiz:

1-Masala. (1) tenglamaning Ω sohada

$U(x,y)\in C(\bar{\Omega})\cap AC^1(\Omega_0)\cap C^2(\Omega_i),\quad U_{xx}\in C(\Omega_0)$

funksiyalar sinfiga tegishli quyidagi shartlarni qanoatlantiradigan regulyar yechimi topilsin:

$$U(x,0)=0,\qquad 0\le x\le 1, \tag{2}$$

$$U|_{A_0C}=\varphi(y),\qquad \frac{1}{2}\le y\le 1, \tag{3}$$

$$U|_{B_0D}=\psi(y),\qquad \frac{1}{2}\le y\le 1, \tag{4}$$

$$U_x(0+,y)=I_1(U(x,y)|_{x=0-}),\quad U_y(0+,y)=U_y(0-,y),\qquad 0<y<1, \tag{5}$$

$$U_x(1-0,y)=I_2(U_x(x,y)|_{x=1+0}),\quad U_y(1-0,y)=U_y(1+0,y),\qquad 0<y<1. \tag{6}$$

Bu yerda $\varphi(y)$, $\psi(y)$-berilgan funksiyalar, I_1, I_2 lar esa hozircha ixtiyoriy integral operatorlar.

Bunday tipdagi masalalar I_1 *va* I_2 integral operatorlarning maxsus ko‘rinishida $[2]$ da ($\alpha=1$ holda) hamda $0<\alpha<1$ uchun $[3]$ tadqiq etilgan.

(1) tenglamaning Ω_0 sohada (2) va

$$U_x(0+,y)=\nu_0^+(y),\qquad U_x(1-0,y)=\nu_1^-(y),\qquad 0<y<1 \tag{7}$$

shartlarni qanoatlantiruvchi yechimi quydagicha yoziladi $[4]$:

$$U(x,y)=\int_0^y \nu_1^-(\eta)G(x,y;1,\eta)d\eta-\int_0^y \nu_0^+(\eta)G(x,y;0,\eta)d\eta-$$
$$-\int_0^y\int_0^1 f(\xi,\eta)G(x,y;\xi,\eta)d\xi d\eta, \tag{8}$$

bu yerda

$$G(x,y;\xi,\eta)=\frac{(y-\eta)^{\beta-1}}{2}\sum_{n=-\infty}^{+\infty}\left[e_{1,\beta}^{1,\beta}\left(-\frac{|x-\xi+2n|}{(y-\eta)^{\beta}}\right)+e_{1,\beta}^{1,\beta}\left(-\frac{|x+\xi+2n|}{(y-\eta)^{\beta}}\right)\right],$$

(9)

$e_{1,\beta}^{1,\beta}(z)=\sum_{n=0}^{+\infty}\frac{z^n}{n!\Gamma(\beta-\beta n)}$ $\rightarrow$ Rayt tipidagi funksiya $[4]$, $\beta=\frac{\alpha}{2}$

(1) tenglama uchun Ω_1 va Ω_2 sohalardagi Koshi masalasi yechimini Dalamber formulasi orqali yozib olamiz $[5]$:

$$U(x,y)=\frac{1}{2}\left\{\tau_0^-(y+x)+\tau_0^-(y-x)+\int_{y-x}^{y+x}\nu_0^-(t)dt+\int_0^y\int_{x-y+\eta}^{x+y-\eta}f(\xi,\eta)d\xi d\eta\right\},$$

$(x,y)\in\Omega_1$ (10)

$$U(x,y)=\frac{1}{2}\left\{\tau_1^+(y-x+1)+\tau_1^+(y+x-1)+\int_{y+x-1}^{y-x+1}\nu_1^+(t)dt+\int_0^y\int_{1-x-y+\eta}^{1-x+y-\eta}f(\xi,\eta)d\xi d\eta\right\},$$

$(x,y)\in\Omega_2$ (11)

Bu yerda $U(-0,y)=\tau_0^-(y)$, $U(1+0,y)=\tau_1^+(y)$.

(10) ni (3) ga qo‘yamiz:

$$U(y-1,y)=\varphi(y)=\frac{1}{2}\left\{\tau_0^-(y+y-1)+\tau_0^-(y-y+1)+\int_{y-y+1}^{y+y-1}\nu_0^-(t)dt+\int_0^y\int_{y-1-y+\eta}^{y-1+y-\eta}f(\xi,\eta)d\xi d\eta\right\},$$

$\frac{1}{2}\le y\le 1.$

$$\varphi(y)=\frac{1}{2}\left\{\tau_0^-(2y-1)+\tau_0^-(1)+\int_1^{2y-1}\nu_0^-(t)dt+\int_0^y\int_{\eta-1}^{2y-\eta-1}f(\xi,\eta)d\xi d\eta\right\},$$

yoki (2) ni hisobga olsak $\tau_0^-(1)=0$ va y bo‘yicha bir marta differensiallab y ni $\frac{y+1}{2}$ ga almashtirib

$$\varphi'(\frac{y+1}{2})=\tau_0^{-\prime}(y)+\nu_0^-(y)+\int_0^{\frac{y+1}{2}}f(y-\eta,\eta)d\eta\,,\qquad 0<y<1 \qquad (12)$$

ni olamiz.

Xuddi shuningdek, (11) ni (4) ga qo‘yib, yuqoridagidek amallarni bajarganda quyidagini olamiz:

$$\psi'(\frac{y+1}{2})=\tau_1^{+\prime}(y)+\nu_1^+(y)+\int_0^{\frac{y+1}{2}}f(y-\eta,\eta)d\eta\,,\qquad 0<y<1. \qquad (13)$$

(5) va (6) ulash shartlarini quydagicha yozib olish mumkin:

$$\nu_0^+(y)=I_1(\nu_0^-(y)),\qquad \nu_1^-(y)=I_2(\nu_1^+(y)),\qquad 0<y<1, \qquad (14)$$

(12), (13) ni hisobga olsak, (14) dan

$$\nu_0^+(y)=I_1\left[\varphi'(\frac{y+1}{2})-\tau_0^{-\prime}(y)-\int_0^{\frac{y+1}{2}}f(y-\eta,\eta)d\eta\right],$$

$$\nu_1^-(y)=I_2\left[\psi'(\frac{y+1}{2})-\tau_1^{+\prime}(y)-\int_0^{\frac{y+1}{2}}f(y-\eta,\eta)d\eta\right] \qquad (15)$$

ifodalarni hosil qilamiz. (15) ni (8) ga qo‘yamiz:

$$U(x,y)=\int_0^y I_2\left[\psi'\left(\frac{y+1}{2}\right)-\tau_1^{+}{}'(y)-\int_0^{\frac{y+1}{2}} f(y-\eta,\eta)d\eta\right]G(x,y;1,\eta)d\eta-$$

$$-\int_0^y I_1\left[\varphi'\left(\frac{y+1}{2}\right)-\tau_0^{-}{}'(y)-\int_0^{\frac{y+1}{2}} f(y-\eta,\eta)d\eta\right]G(x,y;0,\eta)d\eta- \qquad (16)$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(x,y;\xi,\eta)d\xi d\eta$$

(16) da $x\to 0+$ va $x\to 1-0$ holatlarda limitlarga o‘tamiz:

$$\tau_0^{-}(y)=\int_0^y G(0,y;1,\eta)I_2\left[\psi'\left(\frac{\eta+1}{2}\right)-\tau_1^{+}{}'(\eta)-\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)d\xi\right]d\eta-$$

$$-\int_0^y G(0,y;0,\eta)I_1\left[\varphi'\left(\frac{\eta+1}{2}\right)-\tau_0^{-}{}'(\eta)-\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)d\xi\right]d\eta- \qquad (17)$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(0,y;\xi,\eta)d\xi d\eta\,,$$

$$\tau_1^{+}(y)=\int_0^y G(1,y;1,\eta)I_2\left[\psi'\left(\frac{\eta+1}{2}\right)-\tau_1^{+}{}'(\eta)-\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)d\xi\right]d\eta-$$

$$-\int_0^y G(1,y;0,\eta)I_1\left[\varphi'\left(\frac{\eta+1}{2}\right)-\tau_0^{-}{}'(\eta)-\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)d\xi\right]d\eta- \qquad (18)$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(1,y;\xi,\eta)d\xi d\eta\,.$$

Agar I_1 va I_2 integral operatorlarni quyidagi ko‘rinishda olsak:

$$I_1(g)=\alpha_1 g(\eta)+\int_0^\eta g(z)K_1(\eta,z)dz,$$

$$(19)$$

$$I_2(g)=\alpha_2 g(\eta)+\int_0^\eta g(z)K_2(\eta,z)dz,$$

(17) va (18) larni Volterra integral tenglamalar sistemasiga keltirsak bo‘ladi:

$$\tau_0^-(y)=\int_0^y G(0,y;1,\eta)\left(\begin{array}{l}\alpha_2\left(\left(\psi'\dfrac{\eta+1}{2}\right)-\tau_1^{+\prime}(\eta)-\displaystyle\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)\right)+\\ +\displaystyle\int_0^{\eta}K_2(\eta,z)dz\left[\psi'(\dfrac{z+1}{2})-\tau_1^{+\prime}(z)-\displaystyle\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)d\xi\right]\end{array}\right)d\eta-$$

$$-\int_0^y G(0,y;0,\eta)\left(\begin{array}{l}\left(\alpha_1(\left(\varphi'\dfrac{\eta+1}{2}\right)-\tau_0^{-\prime}(\eta)-\displaystyle\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)\right)+\\ +\displaystyle\int_0^{\eta}K_1(\eta,z)dz\left[\varphi'(\dfrac{z+1}{2})-\tau_0^{-\prime}(z)-\displaystyle\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)d\xi\right]\end{array}\right)d\eta-$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(0,y;\xi,\eta)d\xi d\eta\,, \tag{20}$$

$$\tau_1^+(y)=\int_0^y G(1,y;1,\eta)\left(\begin{array}{l}\alpha_2\left(\psi'\left(\dfrac{\eta+1}{2}\right)-\tau_1^{+\prime}(\eta)-\displaystyle\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)d\xi\right)+\\ +\displaystyle\int_0^{\eta}K_2(\eta,z)dz\left[\psi'(\dfrac{z+1}{2})-\tau_1^{+\prime}(z)-\displaystyle\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)d\xi\right]\end{array}\right)d\eta-$$

$$-\int_0^y G(1,y;0,\eta)\left(\begin{array}{l}\alpha_1\left(\varphi'\left(\dfrac{\eta+1}{2}\right)-\tau_0^{-\prime}(\eta)-\displaystyle\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)d\xi\right)+\\ +\displaystyle\int_0^{\eta}K_1(\eta,z)dz\left[\varphi'(\dfrac{z+1}{2})-\tau_0^{-\prime}(z)-\displaystyle\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)d\xi\right]\end{array}\right)d\eta-$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(1,y;\xi,\eta)d\xi d\eta\,. \tag{21}$$

Endi (20) ni soddalashtiramiz:

$$\tau_0^-(y)=\alpha_2\int_0^y G(0,y;1,\eta)\psi'\left(\frac{\eta+1}{2}\right)d\eta-\alpha_2\int_0^y G(0,y;1,\eta)\tau_1^{+\prime}(\eta)d\eta-$$

$$-\alpha_2\int_0^y G(0,y;1,\eta)\left(\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)\,d\xi\right)d\eta+\int_0^y G(0,y;1,\eta)\left(\int_0^{\eta} K_2(\eta,z)\psi'\left(\frac{z+1}{2}\right)dz\right)d\eta-$$

$$-\int_0^y G(0,y;1,\eta)(\int_0^{\eta} K_2(\eta,z)\tau_1^{+\prime}(z)dz)d\eta-\int_0^y G(0,y;1,\eta)\left(\int_0^{\eta} K_2(\eta,z)\left(\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)d\xi\right)dz\right)d\eta-$$

$$-\int_0^y G(0,y;0,\eta)\alpha_1\varphi'\left(\frac{\eta+1}{2}\right)d\eta+\int_0^y G(0,y;0,\eta)\alpha_1\tau_0^{-\prime}(\eta)d\eta+$$

$$+\int_0^y G(0,y;0,\eta)\alpha_1\left(\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)\,d\xi\right)d\eta-\int_0^y G(0,y;0,\eta)(\int_0^{\eta} K_1(\eta,z)\varphi'\left(\frac{z+1}{2}\right)dz)d\eta+$$

$$+\int_0^y G(0,y;0,\eta)(\int_0^{\eta} K_1(\eta,z)\tau_0^{-\prime}(z)dz)d\eta+\int_0^y G(0,y;0,\eta)(\int_0^{\eta} K_1(\eta,z)(\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)d\xi)dz)d\eta-$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(0,y;\xi,\eta)\,d\xi\,d\eta. \qquad (22)$$

(22) da $\tilde{K}_1(y,z)$ va $\tilde{F}(y)$ belgilashlar kiritsak u quyidagi ko‘rinishga keladi:

$$\tau_0^-(y)-\int_0^y G(0,y;0,\eta)\alpha_1\tau_0^{-\prime}(\eta)d\eta-\int_0^y G(0,y;0,\eta)(\int_0^{\eta} K_1(\eta,z)\tau_0^{-\prime}(z)dz)d\eta=\tilde{F}(y)$$

$$\tau_0^-(y)-\int_0^y G(0,y;0,\eta)\alpha_1\tau_0^{-\prime}(\eta)d\eta-\int_0^y \tau_0^{-\prime}(z)K_1(\eta,z)dz=\tilde{F}(y)$$

yoki

$$\tau_0^-(y)-\int_0^y \tau_0^{-\prime}(\eta)\left[\alpha_1 G(0,y;0,\eta)+\tilde{K}_1(y,\eta)\right]d\eta=\tilde{F}(y), \qquad (23)$$

bu yerda

$$\tilde{K}_1(y,z)=\int_z^y G(0,y;0,\eta)K_1(\eta,z)d\eta,$$

$$\tilde{F}(y)=\int_0^y G(0,y;1,\eta)\alpha_2\psi'\left(\frac{\eta+1}{2}\right)d\eta-\int_0^y G(0,y;1,\eta)\alpha_2\tau_1^{+\prime}(\eta)d\eta-$$

$$-\int_0^y G(0,y;1,\eta)\alpha_2\left(\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)\,d\xi\right)d\eta+\int_0^y G(0,y;1,\eta)\left(\int_0^{\eta} K_2(\eta,z)\psi'\left(\frac{z+1}{2}\right)dz\right)d\eta-$$

$$-\int_0^y G(0,y;1,\eta)\left(\int_0^\eta K_2(\eta,z)\tau_1^{+\prime}(z)dz\right)d\eta-\int_0^y G(0,y;1,\eta)\left(\int_0^\eta K_2(\eta,z)\left(\int_0^{\frac{z+1}{2}} f\left(z-\xi,\xi\right)d\xi\right)dz\right)d\eta-$$

$$-\int_0^y G\left(0,y;0,\eta\right)\alpha_1\varphi'\left(\frac{\eta+1}{2}\right)d\eta+\int_0^y G\left(0,y;0,\eta\right)\alpha_1\left(\int_0^{\frac{\eta+1}{2}} f\left(\eta-\xi,\xi\right)d\xi\right)d\eta-$$

$$-\int_0^y G(0,y;0,\eta)\left(\int_0^\eta K_1(\eta,z)\varphi'\left(\frac{z+1}{2}\right)dz\right)d\eta+\int_0^y G(0,y;0,\eta)\left(\int_0^\eta K_1(\eta,z)\left(\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)d\xi\right)dz\right)d\eta-$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(0,y;\xi,\eta)d\xi d\eta.$$

(23) ni (9) dan foydalanib quyidagicha yozib olamiz:

$$\tau_0^-\left(y\right)-\frac{\alpha_1}{\Gamma\left(\beta\right)}\int_0^y\frac{\tau_0^{-\prime}\left(z\right)}{\left(y-z\right)^{1-\beta}}dz-\alpha_1\int_0^y\tau_0^{-\prime}\left(z\right)\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}\left(y-z\right)^{\beta-1}e_{1,\beta}^{1,\beta}\left(\frac{-2n}{\left(y-z\right)^{\beta}}\right)dz-$$

$$-\int_0^y\tau_0^{-\prime}\left(z\right)\tilde{K}_1\left(y,z\right)dz=\tilde{F}\left(y\right) \qquad (24)$$

yoki

$$\frac{1}{\Gamma\left(\beta\right)}\int_0^y\frac{\tau_0^{-\prime}\left(z\right)}{\left(y-z\right)^{1-\beta}}dz=\frac{1}{\alpha_1}\left[\begin{array}{l}\tau_0^-\left(y\right)-\alpha_1\displaystyle\int_0^y\tau_0^{-\prime}\left(z\right)\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{\left(y-z\right)^{\beta}}\right)}{2\left(y-z\right)^{1-\beta}}dz-\\ -\displaystyle\int_0^y\tau_0^{-\prime}\left(z\right)\tilde{K}_1\left(y,z\right)dz-\tilde{F}\left(y\right)\end{array}\right]$$

(25)

Riman-Liuvill integrali ko'rinishdan foydalansak,

$$I_{0y}^{\beta}\left(\tau_0^{-\prime}\left(y\right)=\tilde{F}\left(y\right)\right) \qquad (26)$$

hosil bo'ladi. (26) ning har ikki tomoniga ${}_cD_{0y}^{\beta}$ operatorni ta'sir ettiramiz:

$${}_cD_{0y}^{\beta}\left(I_{0y}^{\beta}\tau_0^{-\prime}\left(y\right)\right)={}_cD_{0y}^{\beta}\left(\tilde{F}\left(y\right)\right). \qquad (27)$$

(28) ning o'ng tomonini hisoblab olamiz:

$${}_cD_{0y}^{\beta}\left(\tau_0^-\left(y\right)\right)=\frac{1}{\Gamma\left(\beta\right)}\int_0^y\left(y-z\right)^{-\beta}\tau_0^{-\prime}\left(z\right)dz, \qquad (28)$$

$$ {}_cD_{0y}^{\beta}\left[\int_0^y \tau_0^{-\prime}(z)\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(y-z)^{\beta}}\right)}{(y-z)^{1-\beta}}dz\right]=\frac{1}{\Gamma(1-\beta)}\int_0^y (y-s)^{-\beta}\frac{d}{ds}\left(\int_0^s \tau_0^{-\prime}(z)\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(s-z)^{\beta}}\right)}{(s-z)^{1-\beta}}dz\right)ds= $$

$$ =\frac{1}{\Gamma(1-\beta)}\int_0^y (y-s)^{-\beta}ds\left[\lim_{s\to z}\left(\tau_0^{-\prime}(z)\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(s-z)^{\beta}}\right)}{(s-z)^{1-\beta}}dz\right)+\int_0^s \tau_0^{-\prime}(z)\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}\frac{\partial}{\partial s}\left(\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(s-z)^{\beta}}\right)}{(s-z)^{1-\beta}}\right)dz\right], $$

(29) bu yerda

$$ \lim_{s\to z}\left(\tau_0^{-\prime}(z)\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(s-z)^{\beta}}\right)}{(s-z)^{1-\beta}}\right)=0 $$

Bu tasdiqning isbotini 1-lemmada keltirib o'tamiz. Endi

$$ \frac{\partial}{\partial s}\left(\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(s-z)^{\beta}}\right)}{(s-z)^{1-\beta}}\right) $$ ni hisoblab olamiz:

$$ \frac{\partial}{\partial s}\left((s-z)^{\beta-1}e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(s-z)^{\beta}}\right)\right)=(s-z)^{\beta-2}e_{1,\beta}^{1,\beta-1}\left(\frac{-2n}{(s-z)^{\beta}}\right) $$

Olingan natijalarni (29) ga qo'ysak,

$$ \frac{1}{\Gamma(1-\beta)}\int_0^y (y-s)^{-\beta}\left(\int_0^s \tau_0^{-\prime}(z)(s-z)^{\beta-2}\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}e_{1,\beta}^{1,\beta-1}\left(\frac{-2n}{(s-z)^{\beta}}\right)dz\right)ds= $$

$$ =\frac{1}{\Gamma(1-\beta)}\int_0^y \tau_0^{-\prime}(z)dz\int_0^y (y-s)^{-\beta}(s-z)^{\beta-2}\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}e_{1,\beta}^{1,\beta-1}\left(\frac{-2n}{(s-z)^{\beta}}\right)ds \qquad (30) $$

ko'rinishga keladi.(30) da $M(y,z)$ belgilash kiritib olamiz:

$$ M(y,z)=\int_z^y (y-s)^{-\beta}(s-z)^{\beta-2}\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}e_{1,\beta}^{1,\beta-1}\left(\frac{-2n}{(s-z)^{\beta}}\right)ds $$

$M(y,z)$ ni soddalashtirishni 2-lemmada ko'ramiz:

(27) ning o‘ng tomonidagi oxirgi ifodani hisoblaymiz:

$$ {}_cD_{0y}^{\beta}\left[\int_0^y \tau_0^{-\prime}(z)\tilde{K}_1(y,z)dz\right]= $$

$$ =\frac{1}{\Gamma(1-\beta)}\int_0^y (y-t)^{-\beta}\frac{\partial}{\partial t}\left[\int_0^y \tau_0^{-\prime}(z)\tilde{K}_1(t,z)dz\right]dt= $$

$$ =\frac{1}{\Gamma(1-\beta)}\int_0^y (y-t)^{-\beta}\left[\int_0^y \tau_0^{-\prime}(z)\tilde{K}_1(t,t)+\int_0^t \tau_0^{-\prime}(z)\frac{\partial \tilde{K}_1(t,z)}{\partial t}dz\right]dt= $$

$$ =\frac{1}{\Gamma(1-\beta)}\int_0^y \tau_0^{-\prime}(t)\frac{\tilde{K}_1(t,t)}{(y-t)^\beta}dt+\frac{1}{\Gamma(1-\beta)}\int_0^y\frac{dt}{(y-t)^\beta}\cdot $$

$$ \cdot\int_0^t \tau_0^{-\prime}(z)\frac{\partial\tilde{K}_1(t,z)}{\partial t}dz=\frac{1}{\Gamma(1-\beta)}\int_0^y\tau_0^{-\prime}(t)\frac{\tilde{K}_1(t,t)}{(y-t)^\beta}dt+ $$

$$ +\frac{1}{\Gamma(1-\beta)}\int_0^y\tau_0^{-\prime}(z)dz\int_0^y\frac{\frac{\partial\tilde{K}_1(t,z)}{\partial t}}{(y-t)^\beta}dt= $$

$$ =\frac{1}{\Gamma(1-\beta)}\int_0^y\tau_0^{-\prime}(s)\left[\frac{\tilde{K}_1(s,s)}{(y-s)^\beta}+\int_0^y\frac{\frac{\partial \tilde{K}_1(t,s)}{\partial t}}{(y-t)^\beta}dt\right]ds. $$

Demak,

$$ \tau_0^{-\prime}(y)-\int_0^y\tau_0^{-\prime}(z)\left[\begin{array}{l}\dfrac{(y-z)^{-\beta}}{\alpha_1\Gamma(1-\beta)}+\dfrac{M(y,z)}{\Gamma(1-\beta)}+\\ +\dfrac{1}{\Gamma(1-\beta)}\left(\dfrac{\tilde{K}_1(z,z)}{(y-z)^\beta}+\displaystyle\int_0^y\frac{\frac{\partial\tilde{K}_1(t,s)}{\partial t}}{(y-t)^\beta}dt\right)\end{array}\right]dz. \qquad (31) $$

(31) da $\bar{F}(y)$ va $\bar{K}(y,z)$ belgilashlar kiritib olsak,

$$ \tau_0^{-\prime}(y)-\frac{1}{\Gamma(1-\beta)}\int_0^y\tau_0^{-\prime}(z)\bar{K}(y,z)=\bar{F}(y) \qquad (32) $$

ni olamiz, bu yerda

$$ \bar{K}(y,z)=\frac{1}{\alpha_1}\frac{\Gamma(1-\beta)}{\Gamma(\beta)}\frac{1}{(y-z)^\beta}+M(y,z)+\frac{\tilde{K}_1(z,z)}{(y-z)^\beta}+\int_0^y\frac{\frac{\partial\tilde{K}_1(t,z)}{\partial t}}{(y-t)^\beta}dt $$

$$\bar{F}(y) = -\frac{1}{\alpha_1}\,{}_cD_{0y}^{\beta}\tilde{F}(y)$$

$\bar{F}(y)$ - uzuluksiz, $\bar{K}(y,z)$ - kichik maxsuslikka ega.

1-lemma.

Agar $\frac{1}{3} \le \beta \le \frac{1}{2}$ va $n \ne 0$ bo'lsa, u holda

$$\lim_{s\to z}(z-s)^{\beta-1} e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right) = 0$$ bo'ladi.

Isbot:

Avvalo,

$$e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right) = \underbrace{\frac{-2n}{(z-s)^{\beta}} e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right)}\frac{(z-s)^{\beta}}{-2n}$$

tarizda shakl almashtiramiz va

$$ze_{\alpha,\beta}^{\mu,\delta}(z) = e_{\alpha,\beta}^{\mu-\alpha,\delta+\beta}(z) - \frac{1}{\Gamma(\mu-\alpha)\Gamma(\delta+\beta)}$$

formulada $\mu = 1, \alpha = 1, \quad \delta = \beta,\ \beta = \beta$ holda

$$e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right) = \left(\frac{(z-s)^{\beta}}{-2n}\right)^2\left(\frac{-2n}{(z-s)^{\beta}} e_{1,\beta}^{0,2\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right)\right)$$ ni olamiz

Demak,

$$\lim_{s\to z}\frac{(z-s)^{3\beta-1}}{(-2n)^2}\left(\frac{-2n}{(z-s)^{\beta}} e_{1,\beta}^{0,2\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right)\right)$$

Agar $3\beta - 1 \ge 0$ *yoki* $\beta \ge \frac{1}{3}$ *desak*

$$\lim_{s\to\infty} ze_{\alpha,\beta}^{\mu,\delta}(z) = -\frac{1}{\Gamma(\mu-\alpha)\Gamma(\delta+\beta)}$$ *ni hisobga olib*

$$\lim_{s\to z}\frac{(z-s)^{3\beta-1}}{(-2n)^2}\left(\frac{-2n}{(z-s)^{\beta}} e_{1,\beta}^{0,2\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right)\right) = \frac{0}{(-2n)^2}\frac{-1}{\Gamma(0-1)\Gamma(3\beta)} = 0$$

ni holis qilamiz. $\beta \in \left[0, \frac{1}{2}\right]$ ekanligini hisobga olsak,demak $\beta \in \left[\frac{1}{3}, \frac{1}{2}\right]$ bo'lishi kerak.

2-lemma. $n \ne 0$ uchun

$$\int_z^y (y-s)^{-\beta}(s-z)^{\beta-2} e_{1,\beta}^{1,\beta-1}\left(\frac{-2n}{(z-s)^{\beta}}\right) ds = 0$$

tenglik o'rinli.

Isbot:

$$e_{\alpha,\beta}^{\mu,\delta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+\mu)\Gamma(\delta-\beta k)} \quad \textit{formulaga ko'ra}$$

$$\mu=1,\ \alpha=1,\quad \delta=\beta-1,\ \beta=\beta$$

holda quyidagini olamiz:

$$e_{1,\beta}^{1,\beta-1}\left(\frac{-2n}{(s-z)^{\beta}}\right) = \sum_{k=0}^{+\infty} \frac{(-1)^k (2n)^k (s-z)^{-\beta k}}{\Gamma(k+1)\Gamma(\beta-1-\beta k)}.$$

Demak,

$$M(y,z) = \int_z^y (y-s)^{-\beta}(s-z)^{\beta-2} \sum_{k=0}^{+\infty} \frac{(-1)^k (2n)^k (s-z)^{-\beta k}}{\Gamma(k+1)\Gamma(\beta-1-\beta k)} ds =$$

$$= \sum_{k=0}^{+\infty} \frac{(-1)^k (2n)^k}{\Gamma(k+1)\Gamma(\beta-1-\beta k)} \int_z^y (y-s)^{-\beta}(s-z)^{\beta-2-\beta k} ds.$$

Bu yerda

$$(y-z)\xi + z = s, \qquad s-z = (y-z)\xi$$

$$y-s = y-(y-z)\xi+z = (1-\xi)(y-z)$$

$$(y-z)d\xi = ds$$

almashtirish bajarsak,

$$M(y,z) = \sum_{k=0}^{+\infty} \frac{(-1)^k (2n)^k}{\Gamma(k+1)\Gamma(\beta-1-\beta k)} \int_0^1 \left((1-\xi)(y-z)\right)^{-\beta} \left((y-z)\xi\right)^{\beta-2-\beta k} (y-z) d\xi =$$

$$= \sum_{k=0}^{+\infty} \frac{(-1)^k (2n)^k}{\Gamma(k+1)\Gamma(\beta-1-\beta k)} \int_0^1 (1-\xi)^{-\beta} \xi^{\beta-2-\beta k} (y-z)^{-1-\beta k} d\xi =$$

$$= \sum_{k=0}^{+\infty} \frac{(-1)^k (2n)^k}{\Gamma(k+1)\Gamma(\beta-1-\beta k)} \frac{1}{(y-z)^{1+\beta k}} \int_0^1 (1-\xi)^{-\beta} \xi^{\beta-2-\beta k} d\xi$$

hosil bo'ladi.

Endi $B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1} dx = \dfrac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ *ni hisobga olib,*

$$a = \beta-\beta k-1, \quad b = 1-\beta \ da$$

$$M(y,z)=\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k}{\Gamma(k+1)\Gamma(\beta-1-\beta k)}\frac{1}{(y-z)^{1+\beta k}}\frac{\Gamma(\beta-\beta k-1)\Gamma(1-\beta)}{\Gamma(\beta-\beta k-1+1-\beta)}=$$

$$=\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k}{\Gamma(k+1)\Gamma(\beta-1-\beta k)}\frac{1}{(y-z)^{1+\beta k}}\frac{\Gamma(\beta-1-\beta k)\Gamma(1-\beta)}{\Gamma(-\beta k)}=$$

$$=\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k\Gamma(1-\beta)}{\Gamma(k+1)\Gamma(-\beta k)(y-z)^{1+\beta k}}$$

$$\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k\Gamma(1-\beta)}{\Gamma(k+1)\Gamma(-\beta k)(y-z)^{1+\beta k}}=\frac{1}{y-z}e_{1,\beta}^{1,0}\left(\frac{-2n}{(y-z)^{\beta}}\right)$$

$M(y,z)=\frac{1}{y-z}e_{1,\beta}^{1,0}\left(\frac{-2n}{(y-z)^{\beta}}\right)$ ni hosil qilamiz.

3-lemma. Agar $\beta\in\left[\frac{1}{3},\frac{1}{2}\right]$ va $n\neq 0$ bo‘lsa, u holda $\lim_{z\to y}\frac{1}{y-z}e_{1,\beta}^{1,0}\left(\frac{-2n}{(y-z)^{\beta}}\right)=0$.

Bu lemmaning isboti 1-lemmaning isbotiga o‘xshash bo‘lib, $\lim_{a\to -2}\frac{1}{\Gamma(a)}=0$ tasdiqdan foydalaniladi.

Bunda ham $\beta\in\left[\frac{1}{3},\frac{1}{2}\right]$ bo‘lishi talab etiladi.

Demak, $\tau_1^{+}(y)$ funksiyani vaqtincha ma’lum desak, (32) ni 2-tur Volterra integral tenglamasi sifatida qarab, uning yechimini rezolventa orqali yozib olish mumkin:

Bu yechimni (21) ga olib borib qo‘yilganda $\tau_1^{+}(y)$ ga nisbatan yana bir 2-tur Volterra integral tenglamasini olamiz.

Berilganlarga ma’lum shartlar asosida bu integral tenglamaning yechimi ham rezolventa orqali yoziladi. (15) munosabatlardan va (5), (6) ulash shartlari asosida $v_0^{\pm}(y)$, $v_1^{\pm}(y)$ funksiyalar topiladi. Undan so‘ng Ω_0 sohada yechim (8) formula, Ω_1 sohada (10), Ω_2 sohada esa (11) formulalar bilan tiklanadi.

Masala ekvivalaent tarzda 2-tur Volterra integral tenglamalar sistemasiga keltirilganligi tufayli, masala yechimi yagona bo‘ladi.

Demak, quyidagi tasdiq o‘rinli:

Teorema. Agar $\alpha\in\left[\frac{2}{3},1\right]$, $\varphi,\psi\in C^1[0,1]\,\mathrm{I}\ C^2(0,1)$, $f(x,y)\in(\overline{\Omega})\,\mathrm{I}\ C^1(\Omega)$,

$I_1\ va\ I_2$ integral operatorlar (19) ko‘rinishda berilgan bo‘lsa va $\alpha_1\neq 0, \alpha_2\neq 0$ bo‘lganda 1-masalaning yechimi mavjud va yagona bo‘ladi.

FOYDALANILGAN ADABIYOTLAR:

1. Нахушев А.М. Элементы дробного исчислетления и их применение. Нальчик, 2000.

2. Каримов Э.Т. Краевые задачи для уравнений параболо-гиперболического типа со спектральным параметром. Автореферат кандидатской диссертации. Ташкент , 2006 г.

3. Berdyshev A. S. , Cabada A. ,Karimov E.T. On a non-local boundary problem for a parabolic-hyperbolic equation involving Riemann-Liouville factional differential operator. Nonlinear Analysis, 2002, 75, pp.3268-3273.

4. Псху А.В. Решение краевых задач для уравнения диффузии дробного порядка методом функции Грина. Дифференциальные уравнения, 2003. 39(10),pp.1430-1433.

5. M. Salohiddinov. Matematik fizika tenglamalari. Toshkent - “O‘zbekiston” 2002.