TIP O‘ZGARISH CHIZIG‘I PERPENDIKULYAR PARABOLIK-GIPERBOLIK TENGLAMA UCHUN INTEGRAL ULASH SHARTLI CHEGARAVIY MASALA

A’zamov Valiahror Halimjon o‘g‘li
Farg‘ona davlat universiteti
Matematika (yo‘nalishlar bo‘yicha) yo‘nalishi magistranti
E-mail: valiahror@mail.ru

Annotatsiya: Tip o‘zgarish chizig‘i perpendikulyar parabolik-giperbolik tenglama uchun integral ulash shartli chegaraviy masalaning bir qiymatli yechilishi isbotlangan. Masala yechimining yagonaligi energiya integrallari usulida, mavjudligi esa integral tenglamalar usulida va Abel integral tenglamasi yechimidan foydalanib isbotlangan.

Kalit so‘zlar: parabolik-giperbolik teglama; integral ulash sharti; energiya integrallari; Grin funksiyasi; Dalamber formulasi; integral tenglama;Abel tenglamasi.

BOUNDARY PROBLEM WITH INTEGRAL GLUING CONDITION FOR PARABOLIC-HYPERBOLIC EQUATION WITH PERPENDICULAR LINE OF TYPE-CHANGING

A’zamov Valiahror Halimjon o‘g‘li
Ferghana State University
Master's student of mathematics (by majors)
E-mail: valiahror@mail.ru

Abstract: A unique solvability of a boundary problem with integral gluing condition for parabolic-hyperbolic equation with perpendicular line of type-changing has been proved. The uniqueness of the solution to the problem is proved by energy integral’s method, the existence by the method of integral equations, using solution of Abel’s integral equation.

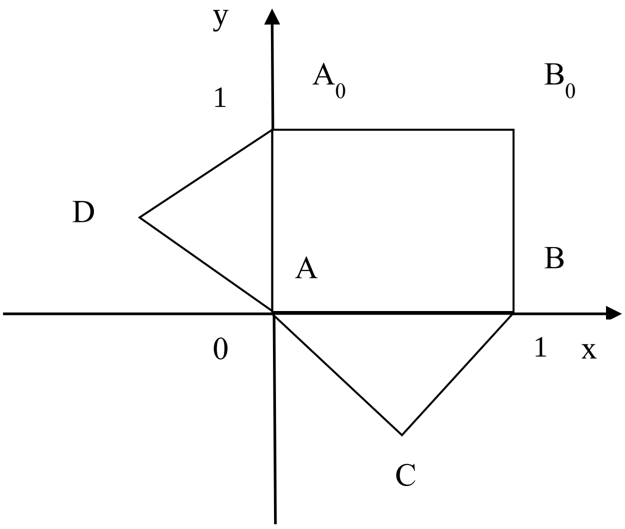
Keywords: Parabolic-hyperbolic equation; integral gluing condition; energy integrals; Green’s function; d’Alambert’s formula; integral equations, Abel’s equetion.

Bizga Ω sohada quyidagi tenglama berilgan bo‘lsin:

$$\begin{cases} U_{xx} - U_y = 0, & (x, y) \in \Omega_0, \\ U_{xx} - U_{yy} = 0, & (x, y) \in \Omega_i, (i = 1, 2), \end{cases} \quad (1)$$

bu yerda $\Omega = \Omega_0 \cup \Omega_1 \cup \Omega_2 \cup AB \cup AA_0$,

$$A(0,0); A_0(0,1); B(1,0); B_0(1,1); C\left(\frac{1}{2}, -\frac{1}{2}\right); D\left(-\frac{1}{2}, \frac{1}{2}\right).$$



1-rasm

Masala. Ω sohada (1) tenglamani qanoatlantiruvchi shunday $U(x,y)$ funksiya topilsinki, u $U(x,y)\in C^{2,1}_{x,y}(\Omega_0)\cap C(\bar{\Omega})\cap C^2(\Omega_i),(i=1,2)$ regulyarlik shartlarini hamda quyidagi shartlarni qanoatlantirsin:

$$U(x,y)\big|_{AC}=\psi_1(x), \qquad 0\le x\le\frac{1}{2}, \qquad (2)$$

$$U(x,y)\big|_{A_0D}=\psi_2(y), \qquad \frac{1}{2}\le y\le 1, \qquad (3)$$

$$U(x,y)\big|_{BB_0}=\varphi(y), \qquad 0\le y\le 1, \qquad (4)$$

$$U_y(x,+0)=I_1(U_y(x,-0)),\ U_x(x,+0)=U_x(x,-0), \qquad 0<x<1, \qquad (5)$$

$$U_x(+0,y)=I_2(U_x(-0,y)),\ U_y(+0,y)=U_y(-0,y), \qquad 0<y<1. \qquad (6)$$

Bu yerda $\psi_1(x),\psi_2(x),\varphi(x)$ - berilgan funksiyalar, I_1va I_2 lar hozircha ixtiyoriy integral operatorlar.

$\Omega_0$sohada $U_{xx}-U_y=0$ tenglama uchun birinchi chegaraviy masala yechimini quyidagicha yozamiz [1]:

$$u(x,y)=\int_0^1\tau_1^{\ +}(\xi)G_1(x,y;\xi,0)d\xi+\int_0^y\tau_2^{\ +}(\eta)G_{1\xi}(x,y;0,\eta)d\eta-\int_0^y\varphi(\eta)G_{1\xi}(x,t;l,\eta)d\eta$$

,

bu yerda, $\tau_1^{\ +}(x)=U(x,0),\ 0\le x\le 1,\ \tau_2^{\ +}(y)=U(0,y),\ \ \varphi(y)=U(1,y),$ $0\le y\le 1.$

Ω_1 sohada $U_{xx}-U_{yy}=0$ tenglamaning $U(x,-0)=\tau_1^-(x)$, $0\le x\le 1$, $U_y(x,-0)=\nu_1^-(x)$, $0<x<1$ boshlang'ich shartlarni qanoatlantiruvchi yechimini Dalamber formulasi orqali yozamiz [2]:

$$U(x,y)=\frac{1}{2}\left[\tau_1^-(x-y)+\tau_1^-(x+y)\right]+\frac{1}{2}\int_{x-y}^{x+y}\nu_1^-(z)dz.$$

Bu yechimni (2) shartga bo'ysundiramiz:

$$2\psi_1(x)=\tau_1^-(0)+\tau_1^-(2x)+\int_{2x}^{0}\nu_1^-(z)dz,\quad 0\le x\le\frac{1}{2}.$$

Endi $2x$ ni x bilan almashtirsak

$$2\psi_1\left(\frac{x}{2}\right)=\tau_1^-(0)+\tau_1^-(x)-\int_0^x\nu_1^-(z)dz,\quad 0\le x\le 1$$

hosil bo'ladi. Keyin bu tenglikni x bo'yicha differensiallab

$$\psi_1'\left(\frac{x}{2}\right)=\tau_1'(x)-\nu_1^-(x),\quad 0<x<1 \qquad (7)$$

munosabatni olamiz.

Ω_2 sohada $U_{xx}-U_{yy}=0$ tenglamaning $U(-0,y)=\tau_2^-(y)$, $0\le y\le 1$, $U_x(-0,y)=\nu_2^-(y)$, $0<y<1$ boshlang'ich shartlarni qanoatlantiruvchi yechimini Dalamber formulasi orqali yozamiz [2]:

$$U(x,y)=\frac{1}{2}\left[\tau_2^-(y-x)+\tau_2^-(y+x)\right]-\frac{1}{2}\int_{y+x}^{y-x}\nu_2^-(z)dz.$$

Bu yechimni (3) shartga bo'ysundiramiz:

$$2\psi_2(y)=\tau_2^-(1)+\tau_2^-(2y-1)-\int_{2y-1}^{1}\nu_2^-(z)dz,\ 0\le y\le\frac{1}{2}.$$

Endi $2y-1$ ni y bilan almashtirsak,

$$2\psi_2\left(\frac{y+1}{2}\right)=\tau_2^-(1)+\tau_2^-(y)+\int_1^y\nu_2^-(z)dz,\ 0\le y\le 1$$

hosil bo'ladi. Keyin bu tenglikni y bo'yicha differensiallasak

$$\psi_2'\left(\frac{y+1}{2}\right)=\tau_2^{-\prime}(y)+\nu_2^-(y),\quad 0<y<1 \qquad (8)$$

tenglikka ega bo'lamiz.

Masala yechimining yagonaligi.

Masala yechimi 2 ta u_1 va u_2 funksiyalar bo‘lsin. U holda $v = u_1 - u_2$ funksiyaga nisbatan quyidagi masalani qarashimiz mumkin bo‘ladi:

$$\begin{cases} v_{xx} - v_y = 0, & (x,y) \in \Omega_0, \\ v_{xx} - v_{yy} = 0, & (x,y) \in \Omega_i, (i = 1,2). \end{cases}$$

$$, v(x,y)\big|_{BB_0} = 0 \qquad , v(x,y)\big|_{A_0D} = 0, v(x,y)\big|_{AC} = 0$$

$$, \quad v_x(+0,y) = I_2(v_x(-0,y)) . v_y(x,+0) = I_1(v_y(x,-0))$$

Bu esa bir jinsli tenglama uchun bir jinsli shartli masaladir. Agar ushbu masala yechimining $v(x,y) \equiv 0$ ekanligini ko‘rsata olsak, asosiy masalaning yechimi yagonaligi kelib chiqadi.

Dastlab, $v_{xx} - v_y = 0$ tenglamada $y \to +0$ holda limitga o‘tamiz va

$$\tau_1^{+\prime\prime}(x) - \nu_1^+(x) = 0$$

tenglamani olamiz, bu yerda $\tau_1^+(x) = v(x,+0)$, $\nu_1^+(x) = v_y(x,+0)$. Ushbu tenglamani $\tau_1^+(x)$ ga ko‘paytirib (0,1) oraliqda integrallab

$$\int_0^1 \tau_1^+(x)\tau_1^{+\prime\prime}(x)dx - \int_0^1 \tau_1^+(x)\nu_1^+(x)dx = 0$$

ni hosil qilamiz.

Ushbu integralni bo‘laklab integrallasak

$$\tau_1^+(x)\tau_1^{+\prime}(x)\Big|_0^1 - \int_0^1 \left[\tau_1^{+\prime}(x)\right]^2 dx - \int_0^1 \tau_1^+(x)\nu_1^+(x)dx = 0$$

va $\tau_1^+(0) = \tau_1^+(1) = 0$ ekanligini inobatga olsak, yuqoridagi ifoda quyidagi ko‘rinishga keladi:

$$. \int_0^1 \left[\tau_1^{+\prime}(x)\right]^2 dx + \int_0^1 \tau_1^+(x)\nu_1^+(x) = 0$$

Bundan $\int_0^1 \left[\tau_1^{+\prime}(x)\right]^2 dx \geq 0$ bo‘lgani uchun $\int_0^1 \tau_1^+(x)\nu_1^+(x) \geq 0$ ekanligini ko‘rsatamiz.

Buning uchun I_1 operatorni quyidagicha tanlaymiz:

$$. I_1\left(f(x)\right) = \int_x^1 f(z)K(x,z)dz$$

Demak,

$$J_1 = \int_0^1 \tau_1^+(x)\nu_1^+(x)dx = \int_0^1 \tau_1^+(x)I_1\left(\tau_1^{+\prime}(x)\right)dx = \int_0^1 \tau_1^+(x)dx\int_x^1 \tau_1^{+\prime}(z)K(x,z)dz =$$

$$=\int_0^1 \tau_1^+(x)dx\left[\tau_1^+(z)K(x,z)\Big|_x^1-\int_x^1 \tau_1^+(z)\frac{\partial}{\partial z}K(x,z)dz\right]=$$

$$=-\int_0^1 \tau_1^2(x)K(x,x)dx-\int_0^1 \tau_1(x)dx\int_x^1 \tau_1^+(z)\frac{\partial}{\partial z}K(x,z)dz.$$

Bundan $K(x,x)\le 0$ deb olib, $K(x,z)$ uchun yana quyidagi shartni qo'yamiz:

$$.\frac{\partial}{\partial z}K(x,z)=-K_1(x)K_1(z)$$

U holda J_1 integralimizni quyidagicha ifodalashimiz mumkin bo'ladi:

$$J_1=-\int_0^1 \tau_1^2(x)K(x,x)dx+\frac{1}{2}\int_0^1\frac{d}{dx}\left[\int_x^1 \tau_1^+(z)K_1(z)dz\right]^2dx=$$

$$=-\int_0^1 \tau_1^2(x)K(x,x)dx+\frac{1}{2}\left(\int_0^1 \tau_1^+(z)K_1(z)dz\right)^2.$$

Shunday qilib yuqoridagi shartlarni qanoatlantiruvchi $K(x,z)$ funksiya uchun $\tau_1^-(x)$ funksiyani nolga tengligi kelib chiqadi. Bundan esa $\nu_1^-(x)$ funksiyani ham nolga tengligini olish mumkin.

Agar $\tau_1^-(x)=0, \nu_1^-(x)=0$ bo'lsa Dalamber formulasidan $v(x,y)$ ni $v(x,y)\equiv 0$ ekanligi kelib chiqadi, bu esa bizga yechim Ω_1 sohada yagonaligini keltirib chiqaradi.

Endi $v_{xx}-v_y=0$ tenglamani olib uni v ga ko'paytirib Ω_0 soha bo'yicha integrallaymiz:

$$\text{yoki}\iint_{\Omega_0} v(v_{xx}-v_y)dxdy=0$$

$$.\iint_{\Omega_0}\left[(vv_x)_x-v_x{}^2-\frac{1}{2}(v^2)_y\right]dxdy=0$$

Grin formulasini qo'llab [2], soha bo'yicha integralni soha chegarasi bo'yicha integralga keltiramiz:

$$.\int_{\partial\Omega_0} vv_x dy+\frac{1}{2}(v^2)dx-\iint_{\Omega_0} v_x{}^2dxdy=0$$

Endi chegara bo'yicha integrallashni amalga oshirib olamiz:

$$\int_0^1\frac{1}{2}\left[\tau_1^+(x)\right]^2dx+\int_0^1 v(1,y)v_x(1,y)dy-\frac{1}{2}\int_0^1 v^2(x,1)dx-\int_0^1 \tau_2^+(y)\nu_2^+(y)dy-$$

$$-\iint_{\Omega_0} v_x^2(x,y)dxdy=0.$$

, $v(1,y)=0$ ekanligidan integral quyidagi ko‘rinishga keladi: $\tau_1^+(x)=0$

$$.\frac{1}{2}\int_0^1 v^2(x,1)dx+\int_0^1 \tau_2^+(y)v_2^+(y)dy+\iint_{\Omega_0} v_x^2(x,y)dxdy=0$$

Agar $\int_0^1 \tau_2^+(y)v_2^+(y)dy \ge 0$ ni ekanligini ko‘rsata olsak yechim Ω_2 sohada yagonaligini ko‘rsatishimiz mumkin.

$$J_2=\int_0^1 \tau_2^+(y)v_2^+(y)dy=\int_0^1 \tau_2^+(y)I_2\left(\tau_2{}'(y)\right)dy.$$

$I_2(f(y))=\int_y^1 f(z)P(y,z)dz$ ga ko’ra

$$J_2=\int_0^1 \tau_2^+(y)\left[\int_y^1 \tau_2{}'(z)P(y,z)dz\right]dy=\int_0^1 \tau_2^+(y)\left[\tau_2^+(z)P(y,z)\Big|_y^1-\int_y^1 \tau_2(z)\frac{\partial}{\partial z}P(y,z)dz\right]dy=$$

$$=-\int_0^1 \left(\tau_2^+(y)\right)^2 P(y,y)dy-\int_0^1 \tau_2^+(y)\left[\int_y^1 \tau_2(z)\frac{\partial}{\partial z}P(y,z)dz\right]dy$$

Agar $\frac{\partial}{\partial z}P(y,z)=-P_1(y)P_1(z)$ desak,

$$J_2=-\int_0^1 \left(\tau_2^+(y)\right)^2 P(y,y)dy+\int_0^1 \tau_2^+(y)\left[\int_y^1 \tau_2(z)P_1(y)P_1(z)dz\right]dy=-\int_0^1 \left(\tau_2^+(y)\right)^2 P(y,y)dy+$$

$$.+\frac{1}{2}\int_0^1 \frac{d}{dy}\left[\int_y^1 \tau_2^+(z)P_1(z)dz\right]^2 dy=-\int_0^1 \left(\tau_2^+(y)\right)^2 P(y,y)dy+\frac{1}{2}\left[\int_0^1 \tau_2^+(z)P_1(z)dz\right]^2 \ge 0$$

Agar $P(y,y)\le 0$ va $\frac{\partial}{\partial z}P(y,z)=-P_1(y)P_1(z)$ larni ta’minlay olsak $P(y,z)$ funksiya uchun $\tau_2(y)$ funksiyani nolga tengligi kelib chiqadi. Bundan esa $\nu_2(y)$ funksiyani ham nolga tengligi kelib chiqadi.

Agar $\tau_2(y)=0,\nu_2(y)=0$ bo‘lsa Dalamber formulasida $v(x,y)$ ni $v(x,y)\equiv 0$ ekanligi kelib chiqadi, bu esa Ω_2 sohada bizga yechim yagonaligini keltirib chiqaradi. sohada masala yechimi yagonaligi qaralayotgan tenglama uchun 1-chegaraviy Ω_0 masala yechimidan kelib chiqadi

.

Masala yechimining mavjudligi.

Dastlab (5) va (6) shartlarni quyidagicha ko‘rinishga keltirib olamiz:

$$v_1^{+}(x)=I_1(v_1^{-}(x)), \quad (5^*)$$

$$v_2^{+}(x)=I_2(v_2^{-}(x)). \quad (6^*)$$

sohada $y\to+0$ da limitga o‘tgan ifodamiz Ω_0

$$\tau_1^{+\prime\prime}(x)-v_1^{+}(x)=0 \quad (9)$$

va Ω_1 sohada olgan (7)

$$\psi_1'\left(\frac{x}{2}\right)=\tau_1^{-\prime}(x)-v_1^{-}(x),\quad 0<x<1$$

munosabatlarimiz hamda $\tau_1^{-}(0)=\psi_1(0)$, $\tau_1^{-}(1)=\varphi(0)$ shartlarga asosan quyidagi masalaga kelamiz:

$$\begin{cases}\tau_1^{+\prime\prime}(x)-I_1(\tau_1^{+\prime}(x))=-I_1\left(\psi_1'\left(\frac{x}{2}\right)\right)\\ \tau_1^{+}(0)=\psi_1(0),\quad \tau_1^{+}(1)=\varphi(0).\end{cases}$$

operatorning ushbu I_1

$$I_1(f(x))=\int_x^1 f(z)K(x,z)dz \quad (10)$$

ko‘rinishini hisobga olsak, quyidagi integral tenglamaga kelamiz:

$$\tau_1^{+\prime\prime}(x)+\int_x^1\tau_1^{+\prime}(x)K(x,z)dz=\tilde{\psi}_1(x), \quad (11)$$

bu yerda $\tilde{\psi}_1(x)=-\int_1^x\psi_1'\left(\frac{x}{2}\right)K(x,z)dz$.

Hosil bo‘lgan (11) integro-differensial tenglamani $(0,x)$ da integrallab

$$\int_0^x\tau_1^{+\prime\prime}(\xi)d\xi+\int_0^x d\xi\int_\xi^1\tau_1^{+\prime}(s)K(\xi,s)ds=\int_0^x\tilde{\psi}_1(\xi)d\xi$$

ifodaga ega bo‘lamiz va

$$\int_0^x\tau_1''(\xi)d\xi=\tau_1'(x)-\tau_1'(0)$$

ni hisobga olib integrallash tartibini o‘zgartirsak, quyidagi integral tenglamaga kelamiz:

$$\tau_1^{+\prime}(x)dx-\int_0^x\tau_1^{+\prime}(s)ds\int_\xi^x K(\xi,s)d\xi-\int_x^1\tau_1^{+\prime}(s)ds\int_0^x K(\xi,s)d\xi=\tilde{\tilde{\psi}}_1(x)+\tau_1^{+\prime}(0)$$

bu yerda $\tilde{\tilde{\psi}}_1(x)=\int_0^x\tilde{\psi}_1(\xi)d\xi$.

$$\tau_1^{+\prime}(x)dx-\int_0^1 \tau_1^{+\prime}(z)\tilde{K}(x,z)dz=\tilde{\psi}_1(x)+\tau_1^{+\prime}(0)$$

Hozircha $\tau_1'(0)$ ni ma’lum deb olib integral tenglamani yechishda davom etamiz va quyidagi belgilashlarni kiritib olamiz:

$$,\ \tilde{f}(x)=\tilde{\psi}_1(x)+\tau_1^{+\prime}(0)$$

$$\tilde{K}(x,z)=\begin{cases}\int_\xi^x K(\xi,z)d\xi, & 0\le z\le x\\ \int_0^x K(\xi,z)d\xi, & x\le z\le 1.\end{cases}$$

Agar $\tilde{K}(x,z)$ yadro uzluksiz yoki integrallanuvchi maxsuslikka ega bo‘lsa va $\tilde{f}(x)$ uzluksiz funksiya bo’lsa, shunda quyidagi 2-tur Fredgolm integral tenglamasini olamiz:

$$\tau_1^{+\prime}(x)dx-\int_0^1 \tau_1^{+\prime}(z)\tilde{K}(x,z)dz=\tilde{f}(x).$$

Yuqordagi integral tenglama yechimini rezolventa orqali quyidagicha yozish mumkin bo‘ladi:

$$.\ \tau_1'(x)=\tilde{f}(x)+\int_0^1 R(x,z)\tilde{f}(z)dz$$

Bu yerda $R(x,z)$ funksiya $\tilde{K}(x,z)$ yadroning rezolventasi.

Ushbu tenglikni $(0,x)$ intervalda integrallab $\tau_1(x)$ funksiyani topamiz.

$$\tau_1(x)-\tau_1(0)=\int_0^x \tilde{f}(\xi)d\xi+\int_0^x\int_0^1 R(x,z)\tilde{f}(z)dzd\xi$$

yoki

$$.\ \tau_1(x)=\int_0^x \tilde{f}(\xi)d\xi+\int_0^x\int_0^1 R(x,z)\tilde{f}(z)dzd\xi+\psi_1(0)$$

Endi yuqorida ma’lum deb olib davom etgan qiymat $\tau_1'(0)$ ni topish uchun quyidagi hisob kitoblarni amalga oshiramiz:

$$(12)\qquad \tau_1(x)=\int_0^x\left[\tilde{\psi}_1(\xi)+\tau_1'(0)\right]d\xi+\int_0^x\int_0^1 R(x,z)\left[\tilde{\psi}_1(z)+\tau_1'(0)\right]dzd\xi+\psi_1(0)$$

yoki

$$\tau_1(x)=\int_0^x\tilde{\psi}_1(\xi)d\xi+\int_0^x\tau_1'(0)d\xi+\int_0^x\int_0^1 R(x,z)\tilde{\psi}_1(z)dzd\xi+\int_0^x\int_0^1 R(x,z)\tau_1'(0)dzd\xi+\psi_1(0).$$

Ushbu belgilashni $\tilde{F}(x,z)=\int_0^x \tilde{\psi}_1(\xi)d\xi+\int_0^x\int_0^1 R(x,z)\tilde{\psi}_1(z)dzd\xi+\psi_1(0)$ kiritib olib hisob kitobni sodda ko‘rinishda tasvirlashimiz miumkin.

$$.\tau_1(x)=\int_0^x \tau_1'(0)d\xi+\int_0^x\int_0^1 R(x,z)\tau_1'(0)dzd\xi+\tilde{F}(x,z)$$

dagi $\tau_1(x)$ funksiyani qiymatidan foydalangan holda $\tau_1'(0)$ ni topishimiz $x=1$ mumkin bo‘ladi:

$$,\tau_1(1)=\tau_1'(0)+\tau_1'(0)\int_0^1\int_0^1 R(1,z)dzd\xi+\tilde{F}(1,z)$$

$$.\tau_1'(0)=\frac{\varphi(0)-\tilde{F}(1,z)}{1+\int_0^1\int_0^1 R(1,z)dzd\xi}$$

Bundan $\int_0^1\int_0^1 R(1,z)dzd\xi\neq -1$ qo‘shimcha shartga ega bo‘lamiz.

- (12) ko‘rinishda topilgandan so‘ng (7) tenglikdan foydalanib $\nu_1^+(x)$ ni $\tau_1(x)$ topish mumkin bo‘ladi.

Ulash shartiga ko‘ra esa $\nu_1^-(x)$ aniq ko‘rinishda topiladi.

Ω_2 sohada $U_{xx}-U_{yy}=0$ tenglamaning $U(-0,y)=\tau_2^-(y)$, $0\le y\le 1$, $U_x(-0,y)=\nu_2^-(y)$, $0<y<1$ boshlang‘ich shartlarni qanoatlantiruvchi yechimini Dalamber formulasi orqali yozib olamiz [4].

$$U(x,y)=\frac{1}{2}\left[\tau_2(y-x)+\tau_2(y+x)\right]-\frac{1}{2}\int_{y+x}^{y-x}\nu_2^-(z)dz.$$

Bu yechimni (3) shartga bo‘ysundirib olgan (8) tengligimizga I_2 operatorni qo‘llab

$I_2\left(\tau_2'(y)\right)-I_2\left[\psi_1'\left(\frac{y+1}{2}\right)\right]=-\nu_2^+(y)$ tenglikka ega bo‘lamiz.

Keyin esa

$$\nu_2^+(y)=\int_0^1 \tau_1(\xi)G_{1x}(0,y;\xi,0)d\xi+\tau_2(\eta)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\Bigg|_0^y-$$

$$-\int_0^y \tau_2'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)d\eta-\varphi(\eta)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n+1)}{\sqrt{y-\eta}}}\Bigg|_0^y+$$

$$+\int_0^y \varphi'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n+1)}{\sqrt{y-\eta}}}\right)d\eta$$

yoki

$$-I_2\left(\tau_2{}'(y)\right)+I_2\left[\psi_1'\left(\frac{y+1}{2}\right)\right]=\int_0^1\tau_1(\xi)G_{1x}(0,y;\xi,0)d\xi+\tau_2(\eta)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\Bigg|_0^y-$$

$$-\int_0^y \tau_2'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)d\eta-\varphi(\eta)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n+1)}{\sqrt{y-\eta}}}\Bigg|_0^y+$$

$$.+\int_0^y \varphi'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n+1)}{\sqrt{y-\eta}}}\right)d\eta$$

Shundan so'ng quyidagiga ega bo'lamiz:

$$-I_2\left(\tau_2{}'(y)\right)+I_2\left[\psi_1'\left(\frac{y+1}{2}\right)\right]=\int_0^1\tau_1(\xi)G_{1x}(0,y;\xi,0)d\xi-\tau_2(0)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y}}e^{-\frac{(2n)}{\sqrt{y}}}-$$

$$-\int_0^y \tau_2'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)d\eta-\varphi(0)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y}}e^{-\frac{(2n+1)}{\sqrt{y}}}+$$

$$+\int_0^y \varphi'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n+1)}{\sqrt{y-\eta}}}\right)d\eta.$$

Bu tenglikda ma'lum narsalarni bir tomonga olib o'tsak

$$I_2\left(\tau_2{}'(y)\right)-\int_0^y \tau_2'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)d\eta=\tilde{F}_2(y) \quad (13)$$

hosil bo'ladi. Bu yerda

$$\tilde{F}_2(y)=I_2\left[\psi_1'\left(\frac{y+1}{2}\right)\right]-\int_0^1\tau_1(\xi)G_{1x}(0,y;\xi,0)d\xi+\psi_1(0)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y}}e^{-\frac{(2n)}{\sqrt{y}}}-$$

$$+\varphi(0)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y}}e^{-\frac{(2n+1)}{\sqrt{y}}}-\int_0^y \varphi'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n+1)}{\sqrt{y-\eta}}}\right)d\eta$$

operatorni $I_2\left(g(y)\right)=\int\limits_y^1 g(y)P(y,z)dz$ ko‘rinishda tanlab olib, (13) tenglikni I_2 quyidagicha yozib olish mumkin:

$$\int\limits_y^1 \tau_2'(\eta)P(y,\eta)d\eta-\int\limits_0^y \tau_2'(\eta)\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}d\eta=\tilde{F}_2(y) \quad (14)$$

dagi qiymatni ajratib olib quyidagi Abel integrali ko‘rinishida yozib olamiz. $n=0$

$$\int\limits_0^y \frac{\tau_2'(\eta)}{\sqrt{y-\eta}}d\eta=\tilde{F}_2(y)-\int\limits_y^1 \tau_2'(\eta)P(y,\eta)d\eta+\int\limits_0^y \tau_2'(\eta)\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}d\eta$$

$$\int\limits_0^y \frac{\tau_2'(\eta)}{\sqrt{y-\eta}}d\eta=\tilde{F}_2(y) \quad (15)$$

bu yerda

$$\tilde{F}_2(y)=\sqrt{\pi}\left[\tilde{F}_2(y)-\int\limits_y^1 \tau_2'(\eta)P(y,\eta)d\eta+\int\limits_0^y \tau_2'(\eta)\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}d\eta\right]$$

(15) Abel integral tenglamasi bo‘lib uning yechimi quyidagicha aniqlanadi [3]:

$$\tau_2'(y)=\frac{1}{\sqrt{\pi}}\frac{d}{dy}\int\limits_0^y \frac{\tilde{F}_2(\eta)}{\sqrt{y-\eta}}d\eta \quad (16)$$

Endi (16) tenglikni $(0,y)$ intervalda integrallab quyidagi ifodalarga ega bo‘lamiz:

$$\int\limits_0^y \tau_2'(s)ds=\frac{1}{\sqrt{\pi}}\int\limits_0^y \frac{d}{ds}\left(\int\limits_0^s \frac{\tilde{F}_2(\eta)}{\sqrt{y-\eta}}d\eta\right)ds,$$

$$\tau_2(y)-\tau_2(0)=\frac{1}{\sqrt{\pi}}\int\limits_0^y \frac{\tilde{F}_2(\eta)}{\sqrt{y-\eta}}d\eta,$$

$$\tau_2(y)=\psi_1(0)+\int\limits_0^y \frac{\tilde{F}_2(\eta)}{\sqrt{y-\eta}}d\eta-\int\limits_0^y \frac{d\eta}{\sqrt{y-\eta}}\int\limits_\eta^1 \tau_2'(\eta)P(\eta,s)ds+$$

$$+\int\limits_0^y \frac{d\eta}{\sqrt{y-\eta}}\int\limits_0^\eta \tau_2'(s)\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}\frac{1}{\sqrt{\eta-s}}e^{-\frac{(2n)}{\sqrt{\eta-s}}}ds. \quad (17)$$

(17) tenglikda quyidagi integrallarda integrallash tartibini quyidagicha almashtiramiz:

$$\int_0^y \frac{d\eta}{\sqrt{y-\eta}} \int_\eta^1 \tau_2'(s)P(\eta,s)ds = \int_0^y \frac{d\eta}{\sqrt{y-\eta}} \int_y^1 \tau_2'(s)P(\eta,s)ds + \int_0^y \frac{d\eta}{\sqrt{y-\eta}} \int_\eta^y \tau_2'(s)P(\eta,s)ds =$$

$$= \int_0^y \tau_2'(s)ds \int_0^s \frac{P(\eta,s)}{\sqrt{y-\eta}}d\eta + \int_y^1 \tau_2'(s)ds \int_0^y \frac{P(\eta,s)}{\sqrt{y-\eta}}d\eta$$

$$\int_0^y \frac{d\eta}{\sqrt{y-\eta}} \int_0^\eta \tau_2'(s) \sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{1}{\sqrt{\eta-s}} e^{-\frac{(2n)}{\sqrt{\eta-s}}} ds = \int_0^y \tau_2'(s)ds \int_s^y \frac{\sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{1}{\sqrt{\eta-s}} e^{-\frac{(2n)}{\sqrt{\eta-s}}}}{\sqrt{y-\eta}} d\eta .$$

Hamda ushbu ko‘rinishdagi integral tenglamaga ega bo‘lamiz:

$$\tau_2(y) = \psi_1(0) + \int_0^y \frac{\tilde{F}_2(\eta)}{\sqrt{y-\eta}}d\eta - \int_0^y \tau_2'(s)ds \int_0^s \frac{P(\eta,s)}{\sqrt{y-\eta}}d\eta - \int_y^1 \tau_2'(s)ds \int_0^y \frac{P(\eta,s)}{\sqrt{y-\eta}}d\eta +$$

$$+ \int_0^y \tau_2'(s)ds \int_s^y \frac{\sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{1}{\sqrt{\eta-s}} e^{-\frac{(2n)}{\sqrt{\eta-s}}}}{\sqrt{y-\eta}} d\eta.$$

Buni quyidagi ko‘rinishda yozib olib:

$$\tau_2(y) + \int_0^1 \tau_2'(s)K(y,s)ds = f(y) \qquad (18)$$

bu yerda

$$Q(y,s) = \begin{cases} \int_0^s \frac{P(\eta,s)}{\sqrt{y-\eta}}d\eta - \int_s^y \frac{\sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{1}{\sqrt{\eta-s}} e^{-\frac{(2n)}{\sqrt{\eta-s}}}}{\sqrt{y-\eta}} d\eta, & 0 \le s \le y \\ \int_0^y \frac{P(\eta,s)}{\sqrt{y-\eta}}d\eta, & y \le s \le 1. \end{cases}$$

$$f(y) = \psi_1(0) + \int_0^y \frac{\tilde{F}_2(\eta)}{\sqrt{y-\eta}}d\eta$$

ko‘rinishida aniqlanadi.

Hamda (18) integral tenglamada quyidagi integralni bo‘laklab ko‘rinishini o‘zgartirib olamiz.

$$\int_0^1 \tau_2'(s)Q(y,s)ds = \tau_2(s)Q(y,s)\Big|_0^1 - \int_0^1 \tau_2(s)\frac{\partial}{\partial s}Q(y,s)ds =$$

$$= \tau_2(1)Q(y,1) - \tau_2(0)Q(y,0) - \int_0^1 \tau_2(s)\frac{\partial}{\partial s}Q(y,s)ds$$

$$\frac{\partial}{\partial s}Q(y,s) = \begin{cases} \dfrac{P(s,s)}{\sqrt{y-s}} + \displaystyle\int_0^s \frac{\frac{\partial}{\partial s}P(\eta,s)}{\sqrt{y-\eta}}d\eta - \int_s^y \frac{\sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \dfrac{e^{-\frac{(2n)}{\sqrt{\eta-s}}}(2n-\sqrt{\eta-s})}{2(\eta-s)^2}}{\sqrt{y-\eta}}d\eta, & 0 \le s \le y \\ \displaystyle\int_0^y \frac{\frac{\partial}{\partial s}P(\eta,s)}{\sqrt{y-\eta}}d\eta, & y \le s \le 1, \end{cases}$$

$$Q(y,1) = \int_0^y \frac{P(\eta,1)}{\sqrt{y-\eta}}d\eta,\ Q(y,0) = \int_0^\eta \frac{\sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{1}{\sqrt{\eta}} e^{-\frac{(2n)}{\sqrt{\eta}}}}{\sqrt{y-\eta}}d\eta,$$

$$\tau_2(y) - \int_0^1 \tau_2(s)\frac{\partial}{\partial s}Q(y,s)ds = f(y) - \psi_2(1)Q(y,1) + \psi_1(0)Q(y,0).$$

$$\tau_2(y) - \int_0^1 \tau_2(s)\frac{\partial}{\partial s}Q(y,s)ds = F(y), \qquad (19)$$

$$F(y) = f(y) - \psi_2(1)Q(y,1) + \psi_1(0)Q(y,0).$$

Bularni hisobga olib integral tenglama yechimini rezolventa orqali quyidagicha yozish mumkin bo‘ladi:

$$.\tau_2(y) = F(y) + \int_0^1 R(y,z)F(z)dz$$

Bu yerda $R(y,z)$ funksiya $\frac{\partial}{\partial s}Q(y,s)$ yadroning rezolventasi

ni (8) tenglikka olib borib qo‘yib $\nu_2^-(x)$ ni topish mumkin bo‘ladi. $\tau_2(y)$

Topilganlarga asosan qo‘yilgan {(1)-(6)} masala yechimini Ω_0 sohada 1-chegaraviy masala yechimi ko‘rinishda Ω_1 va Ω_2 da esa Dalamber formulasi orqali tiklab ko‘rsatish mumkin.

Shunday qilib quyidagi tasdiq o‘rinli ekanligini isbotladik:**Teorema.** Agar

$$\frac{\partial}{\partial z}K(x,z)=-K_1(x)K_1(z),\quad K(x,x)\le 0,\int_0^1\int_0^1 R(1,z)dzd\xi\ne -1,\quad \frac{\partial}{\partial z}P(y,z)=-P_1(y)P_1(z),$$

$$P(y,y)\le 0,$$

$$,\varphi(y),\psi_i(x)\in C^1[0,1],(i=1,2)$$

$$,\ I_2\big(g(y)\big)=\int_y^1 g(y)P(y,z)dz\ I_1\big(f(x)\big)=\int_x^1 f(z)K(x,z)dz$$

shartlar bajarilsa, u holda masalaning yechimi mavjud va yagona.

FOYDALANILGAN ADABIYOTLAR

1. А. Қ. Ўринов. Параболик типдаги дифференциал тенгламалар учун чегаравий масалалар. -Тошкент: MUMTOZ SO‘Z, 2015;
2. Салоҳиддинов М. С. Математик физика тенгламалари. -Тошкент: Ўзбекистон, 2002;
3. M. Salohiddinov, Integral tenglamalar. T.: "Yangiyul poligraph service". - 2007.
4. Каримов Э.Т. Краевые задачи для уравнений параболо-гиперболического типа со спектральным параметром. Автореферат. Кандидатской диссертации. Ташкент: - 2006.