KAPUTO MA’NOSIDAGI KASR TARTIBLI DIFFERENSIAL OPERATORNI O‘Z ICHIGA OLUVCHI BIR JINSLI BO‘LMAGAN CHIZIQLI DIFFERENSIAL TENGLAMALAR SISTEMASINI YECHISHNING DALAMBER USULI

O.A.Abdubannopova

Annotatsiya. *Ushbu ishda Kaputo ma’nosidagi kasr tartibli differensial operatorni o‘z ichiga oluvchi bir jinsli bo‘lmagan chiziqli differensial tenglamalar sistemasi uchun Koshi masalasi yechimi Dalamber usulidan foydalanib topilgan.*

Dastlab, biz ba’zi kerakli asosiy tushuncha va ta’riflarini keltiraylik.

1-ta’rif. [1] $f\in L(a,b)$ va $\alpha>0$ bo‘lsin. U holda quyidagi

$$I_{a+}^{\alpha}[f](t)=\frac{1}{\Gamma(\alpha)}\int_a^t (t-s)^{\alpha-1} f(s)ds$$

va

$$I_{b-}^{\alpha}[f](t)=\frac{1}{\Gamma(\alpha)}\int_t^b (s-t)^{\alpha-1} f(s)ds$$

integrallar mos holda Riman - Liuvillning chap tomonli va o‘ng tomonli kasr tartibli integrallari deb ataladi, bu yerda $\Gamma(z)$ – Eylerning gamma - funksiyasi[2].

2-ta’rif. [1] Riman - Liuvillning chap tomonli $\alpha(0<\alpha<1)$ kasr tartibli hosilasi[1] $D_{a+}^{\alpha}f$ quyidagicha aniqlanadi :

$$D_{a+}^{\alpha}[f](t)=\frac{d}{dt}I_{a+}^{1-\alpha}[f](t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_a^t (t-s)^{-\alpha} f(s)ds.$$

3-ta’rif. [1] Riman - Liuvillning o‘ng tomonli $\alpha(0<\alpha<1)$ kasr tartibli hosilasi $D_{b-}^{\alpha}f$ quyidagicha aniqlanadi :

$$D_{b-}^{\alpha}[f](t)=-\frac{d}{dt}I_{b-}^{1-\alpha}[f](t)=-\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_t^b (s-t)^{-\alpha} f(s)ds.$$

4-ta’rif. [1] Kaputoning $\alpha(0<\alpha<1)$ kasr tartibli hosilasi quyidagicha aniqlanadi:

$${}_C D_{at}^{\alpha} f(t)=\frac{1}{\Gamma(1-\alpha)}\int_a^t \frac{f'(\tau)}{(t-\tau)^{\alpha}}d\tau.$$

Bizga ushbu

(1) $$\begin{cases} {}_C D_{mt}^{\alpha}x(t)=ax+by+f_1(t), \\ {}_C D_{mt}^{\alpha}y(t)=cx+dy+f_2(t), \end{cases} \qquad t>m,\ 0<\alpha<1$$

chiziqli tenglamalar sistemasi berilgan bo‘lsin, bu yerda $\alpha, a, b, c, d, x_0, y_0$ – berilgan haqiqiy sonlar bo‘lib, $f_1(t)$ va $f_2(t)$ – berilgan funksiyalar, $x(t)$ va $y(t)$ lar esa noma’lum funksiyalar, ${}_C D_{mt}^{\alpha}$ esa 4 - ta’rif bo‘yicha aniqlangan Kaputo differensial operatori.

(1) sistemaning

(2) $$x(m) = x_0, \quad y(m) = y_0$$

shartlarni qanoatlantiruvchi yechimini topish masalasini qaraylik.

Bu masala Koshi masalasi deb ham ataladi. Koshi masalasining yechimini topish uchun Dalamber usulidan foydalanamiz. Shu maqsadda (1) sistemaning ikkinchi tenglamasini λ songa ko‘paytirib, so‘ngra birinchi tenglamaga hadma - had qo‘shamiz. Natijada quyidagi tenglikka ega bo‘lamiz:

(3) $${}_C D_{mt}^{\alpha}\left[x(t) + \lambda y(t)\right] = (a + \lambda c)x(t) + (b + \lambda d)y(t) + f_1(t) + \lambda f_2(t).$$

(3) ni quyidagi ko‘rinishda yozib olaylik:

(4) $${}_C D_{mt}^{\alpha}\left[x(t) + \lambda y(t)\right] = (a + \lambda c)\left[x(t) + \frac{b + \lambda d}{a + \lambda c} y(t)\right] + f_1(t) + \lambda f_2(t).$$

λ son ixtiyoriy son ekanligidan uni shunday tanlaylikki, u

(5) $$\frac{b + \lambda d}{a + \lambda c} = \lambda$$

tenglamani qanoatlantirsin. Ma’lumki oxirgi tenglama λ ga nisbatan kvadrat tenglama bo‘lib, uning diskriminanti $D = (a - d)^2 + 4bc$ ga teng. Bu yerda quyidagi hollarni qaraymiz:

a) $D > 0$;

b) $D = 0$;

c) $D < 0$.

Dastlab a) holni qaraylik. Bu holda (5) tenglama ikkita λ_1 va λ_2 ildizlarga ega bo‘lib, bu ildizlarni (4) tenglamaga qo‘ysak, ushbu tenglamalar hosil bo‘ladi:

(6) $${}_C D_{mt}^{\alpha}\left[x(t) + \lambda_i y(t)\right] = (a + \lambda_i c)\left[x(t) + \lambda_i y(t)\right] + f_1(t) + \lambda_i f_2(t),$$
$i = 1, 2$.

Soddalik maqsadida quyidagi belgilashlarni kiritaylik:

(7) $$x(t) + \lambda_i y(t) = z(t), \qquad f_1(t) + \lambda_i f_2(t) = f_{3,i}(t), \qquad a + \lambda_i c = \mu_i,$$
$i = 1, 2$,

U holda (6) tenglama $z(t)$ noma’lum funksiyaga nisbatan quyidagi ikkita tenglama ko‘rinishini oladi:

(8) $${}_C D_{mt}^{\alpha} z(t) = \lambda_i z(t) + f_{3,i}(t),$$
$i = 1, 2$.

Quyidagi formulani e’tiborga olgan holda

(9) $$I_{mt}^{\alpha}\left({}_{C}D_{mt}^{\alpha}\varphi(t)\right)=\varphi(t)-\varphi(m)$$

(8) tenglamaning har ikkala tomoniga I_{mt}^{α} operatorni tatbiq qilamiz. (2) shartlarni va kasr tartibli integral operator yoyilmasini e'tiborga olsak, $z(t)$ noma'lum funksiyaga nisbatan

(10) $$z(t)-\mu_i\int_m^t\frac{(t-\eta)^{\alpha-1}z(\eta)}{\Gamma(\alpha)}=f_{4,i}(t)$$

ko'rinishdagi 2-tur Volterra integral tenglamasiga[3] ega bo'lamiz. Bu yerda

(11) $$f_{4,i}(t)=I_{mt}^{\alpha}f_{3,i}(t)+z(m).$$

U holda (10) tenglama quyidagi ko'rinishga keladi:

(12) $$z(t)-\mu_i\int_m^t K(t,\eta)z(\eta)d\eta=f_{4,i}(t).$$

Oxirgi tenglama yechimini [4] ishdagi kabi ketma-ket yaqinlashish usulidan foydalanib topamiz. Natijada (11) tenglama yechimi ushbu formula orqali aniqlanadi:

(13) $$z(t)=f_{4,i}(t)+\mu_i\int_m^t(t-\eta)^{\alpha-1}E_{\alpha,\alpha}\left[\mu_i(t-\eta)^{\alpha}\right]f_{4,i}(\eta)d\eta,$$

bu yerda $E_{\alpha,\beta}(z)$ – ikki parametrli Mittag - Leffler funksiyasi bo'lib, u quyidagicha aniqlanadi[5]:

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)},\qquad \alpha>0,\ \beta>0.$$

Endi (7) va (13) tengliklarga asosan quyidagi tenglamalar sistemasini tuzamiz:

$$x(t)+\lambda_i y(t)=f_{4,i}(t)+$$

(14) $$+\mu_i\int_m^t(t-\eta)^{\alpha-1}E_{\alpha,\alpha}\left[\mu_i(t-\eta)^{\alpha}\right]f_{4,i}(\eta)d\eta,\qquad i=1,2.$$

Oxirgi tenglamalar sistemasini algebraik qo'shish usulidan foydalanib, $x(t)$ va $y(t)$ noma'lum funksiyalarni

$$x(t)=f_{4,1}(t)+(a+\lambda_1c)\int_m^t(t-\eta)^{\alpha-1}E_{\alpha,\alpha}\left[(a+\lambda_1c)(t-\eta)^{\alpha}\right]f_{4,1}(\eta)d\eta-\lambda_1y(t),$$

$$y(t)=\frac{1}{\lambda_1-\lambda_2}\left\{f_{4,1}(t)-f_{4,2}(t)+(a+\lambda_1c)\int_m^t(t-\eta)^{\alpha-1}E_{\alpha,\alpha}\left[(a+\lambda_1c)(t-\eta)^{\alpha}\right]f_{4,1}(\eta)d\eta-\right.$$

$$\left.-(a+\lambda_2c)\int_m^t(t-\eta)^{\alpha-1}E_{\alpha,\alpha}\left[(a+\lambda_2c)(t-\eta)^{\alpha}\right]f_{4,2}(\eta)d\eta\right\}$$

ko'rinishda bir qiymatli aniqlashimiz mumkin.

Endi b) holni qaraylik. Bu holda (14) tenglamalar sistemasidan quyidagi differensial tenglamaga ega bo‘lamiz:

(15) $$x(t)+\lambda y(t)=f_4(t)+\mu\int_m^t (t-\eta)^{\alpha-1}E_{\alpha,\alpha}\left[\mu(t-\eta)^{\alpha}\right]f_4(\eta)d\eta.$$

$y(t)$ noma’lum funksiyani (15) tenglikdan aniqlaylik:

$$y(t)=\frac{1}{\lambda}\left[f_4(t)+\mu\int_m^t (t-\eta)^{\alpha-1}E_{\alpha,\alpha}\left[\mu(t-\eta)^{\alpha}\right]f_4(\eta)d\eta\right]-\frac{x}{\lambda}.$$

$y(t)$ funksiyaning bu ifodasini (1) tenglamalar sistemaning birinchi tenglamasiga qo‘yib, $x(t)$ noma’lumga nisbatan

$${}_C D_{mt}^{\alpha}x(t)=\left(a-\frac{b}{\lambda}\right)x(t)+$$

(16) $$+\frac{b}{\lambda}\left[f_4(t)+\mu\int_m^t (t-\eta)^{\alpha-1}E_{\alpha,\alpha}\left[\mu(t-\eta)^{\alpha}\right]f_4(\eta)d\eta\right]+f_1(t)$$

ko‘rinishdagi tenglamaga ega bo‘lamiz. Soddalik maqsadida

$$f_5(t)=\frac{b}{\lambda}I_{mt}^{\gamma}\left[f_4(t)+\mu\int_m^t (t-\eta)^{\alpha-1}E_{\alpha,\alpha}\left[\mu(t-\eta)^{\alpha}\right]f_4(\eta)d\eta\right]+f_1(t),$$

(17) $$\lambda'=a-b\lambda^{-1}$$

belgilashlarni kiritganimizdan so‘ng, (16) tenglama quyidagi ko‘rinishni oladi:

$$D_{mt}^{\alpha}x(t)=\lambda' x(t)+f_5(t).$$

Oxirgi tenglamaning har ikki tarafiga (9) formulani qo‘llaymiz. Natijada quyidagi

(18) $$x(t)-\frac{\lambda'}{\Gamma(\alpha)}\int_m^t (t-\eta)^{\alpha-1}x(\eta)d\eta=f_6(t)$$

ko‘rinishdagi 2-tur Volterra integral tenglamasiga ega bo‘lamiz. Bu yerda

$$f_6(t)=x(m)+I_{mt}^{\alpha}f_5(t).$$

(18) tenglamaning yechimi (10) tenglamaning yechimi kabi topiladi.

c) holda ham {(1),(2)} Koshi masalasining yechimi a) holdagi kabi topiladi.

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