YUKLANGAN ARALASH PARABOLIK TENGLAMA UCHUN IKKINCHI TUR INTEGRAL SHARTLI MASALA

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Annotatsiya. Ushbu maqolada vaqt yo'nalishlari perpendikulyar bo'lgan aralash parabolik tenglama uchun integral shartli masalaning bir qiymatli yechilishi tadqiq qilingan.

Kalit so'zlar. Aralash parabolik tenglama, integral shart.

D orqali $y=0$, $y=h$ ва $x=-T$ to'g'ri chiziqlar bilan chegaralangan yarim polosani belgilaylik, bu yerda $h=const>0$, $T=const>0$. Bu sohada quyidagi yuklangan tenglamani qaraylik:

$$0=Lu\equiv\begin{cases}L_1u\equiv u_{xx}-u_y+\lambda\dfrac{d}{dy}u(0,y)+\mu D_{0y}^{-\alpha}u(0,y), & (x,y)\in D_1=D\cap(x>0),\\ L_2u\equiv u_{yy}+u_x, & (x,y)\in D_2=D\cap(x<0),\end{cases}$$

bu yerda λ, μ – berilgan haqiqiy sonlar, D_{0y}^{α} – Riman-Liuvill ma'nosidagi α kasr tartibli differensial operator bo'lib,

$$D_{0y}^{\alpha}f(y)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dy}\int_0^y(y-t)^{-\alpha}f(t)dt,\ 0<\alpha<1.$$

$Lu=0$ – D sohada aralash parabolik tenglama bo'lib, D_1 sohada to'g'ri parabolik, D_2 sohada esa teskari parabolikdir. Aralash parabolik tenglamalar uchun chegaraviy masalalar birinchi bo'lib fransuz matematigi Mario Jevre [1] tomonidan o'rganilgan. Keyinchalik, tadqiqotchilar tomonidan aralash parabolik tenglamalar uchun lokal va nolokal masalalar o'rganishga bo'lgan qiziqish ortdi. Jumladan, [2] ishda vaqt yo'nalishlari almashinuvchi aralash parabolik tenglama uchun Jevre masalasi tadqiq qilingan bo'lsa, [3] ishda aralash parabolik tenglama uchun turli lokal va nolokal shartli masalalar qo'yilgan va o'rganilgan. A.M.Naxushevning [4] ishi esa xarakteristik formalari o'zgaruvchi ikkinchi tartibli parabolik tenglamalar uchun korrekt masalalar qo'yish va masalalar korrektligi uchun zaruriy va yetarli shartlar aniqlashga bag'ishlangan.

Dastlab, tadqiqotchilar tomonidan ikkinchi tartibli aralash parabolik tenglamalar qaralgan bo'lsa, keyinchalik yuqori tartibli tenglamalar uchun masalalar tadqiqoti T.D.Djuraev, S.A.Tersenev, D.Amanov, S.V. Popov va ularning shogirdlari tomonidan rivojlantirildi.

So'ngi vaqtlarda tadqiqotchilar tomonidan kasr tartibli differensial operatorlarni o'z ichiga oluvchi aralash parabolik tenglamalar uchun ham tadqiq otlar olib borilmoqda. Jumladan, [5] ishda ikkinchi tartibli aralash parabolik tenglama uchun Jevre masalasi o'rganilgan bo'lsa, [6] ishda to'rtinchi tartibli aralash parabolik tenglama uchun masalalar spektral analiz usuli bilan tadqiq qilingan.

Yuqoridagi ishlarda qaralayotgan tenglamalarning vaqt yo'nalishlari kollinear bo'lib, vaqt yo'nalishlari kollinear bo'lmagan aralash parabolik tenglamalar uchun masalalar kam o'rganilgan. [7] ishda vaqt yo'nalishlari perpendikulyar bo'lgan aralash parabolik tenglama uchun nolokal shartli masalalar tadqiq qilingan bo'lsa, [8],[9] ishlarda kasr tartibli differensial operatorlarni o'z ichiga oluvchi aralash parabolik tenglamalar uchun turli nolokal shartli masalalar o'rganilgan.

Mazkur maqolada vaqt yo'nalishlari perpendikulyar bo'lgan yuklangan aralash parabolik tenglama uchun bir integral shartli masalaning bir qiymatli yechilishi tadqiq qilinadi.

I **masala.** D sohaning yopig'ida aniqlangan, uzluksiz va chegaralangan shunday $u(x,y)$ funksiya topilsinki, u D_1 va D_2 sohalarda mos ravishda $L_1u=0$ va $L_2u=0$ tenglamalarni hamda quyidagi shartlarni qanoatlantirsin:

$$\lim_{x\to-0}u_x(x,y)=\lim_{x\to+0}u_x(x,y),\qquad 0<y<h;$$

(1) $$u(x,0)=\varphi_1(x),\qquad 0\le x<+\infty;$$

(2) $$u(x,0)=\varphi_2(x),\qquad -T\le x\le 0;$$

(3) $$u(x,h)=a(x)\int_0^h u(x,y)dy+\varphi_3(x),\ -T\le x\le 0,$$

bu yerda $a(x)$ va $\varphi_j(x)\ ,j=\overline{1}$ -berilgan funksiyalar bo'lib, $\varphi_1(0)=\varphi_2(0)$; $\varphi_1(x)\in C[0,+\infty)$ va chegaralangan; $a(x)$, $\varphi_2(x)$, $\varphi_3(x)\in C[-T,0]$.

Qo'yilgan masalaning yechimi mavjud va yagonaligini isbotlaymiz. Faraz qilaylik, $u(x,y)$ - I masalaning yechimi bo'lsin. Masala shartlariga asoslanib,

(4) $$u(-0,y)=u(+0,y)=\tau(y)\ ,\qquad 0\le y\le h;$$

(5) $$\lim_{x\to-0}u_x(x,y)=\lim_{x\to+0}u_x(x,y)=\nu(y),\quad 0<y<h$$

belgilashlarni kiritaylik.

Ma'lumki, $L_1u=0$ tenglamaning D_1 sohaning yopig'ida aniqlangan, uzluksiz, chegaralangan hamda (1) va $\lim_{x\to+0}u_x(x,y)=\nu(y),\ 0\le y\le h$ shartlarni qanoatlantiruvchi yechimi quyidagi ko'rinishda aniqlanadi [7]:

$$u(x,y)=\int_0^{+\infty}G(x;\xi,y)\varphi_1(\xi)d\xi-\int_0^y\nu(\eta)G(x,0,y-\eta)d\eta+$$

(6) $$-\int_0^y\int_0^{+\infty}\left[\lambda\tau'(\eta)+\mu D_{0\eta}^{\alpha}\tau(\eta)\right]G(x,\xi,y-\eta)d\xi d\eta,$$

bu yerda

$$G(x,\xi,y)=\frac{1}{2\sqrt{\pi y}}\left\{\exp\left[-\frac{(x-\xi)^2}{4y}\right]+\exp\left[-\frac{(x+\xi)^2}{4y}\right]\right\}.$$

(6) formulada $x\to+0$ da limitga o'tamiz. U holda

$$\int_0^{+\infty}G(0;\xi,y-\eta)d\xi=1$$

tenglikni e'tiborga olgan holda quyidagiga ega bo'lamiz:

$$\tau(y)=-\frac{1}{\sqrt{\pi}}\int_0^y\nu(\eta)(y-\eta)^{-1/2}d\eta+\int_0^{+\infty}G(0;\xi,y)\varphi_1(\xi)d\xi-$$

(7) $$-\int_0^y\left[\lambda\tau'(\eta)+\mu D_{o\eta}^{-\alpha}\tau(\eta)\right]d\eta.$$

(7) tenglikni kasr tartibli integral operator ko'rinishidan [7] foydalanib va $\tau(0)=\varphi_1(0)$ ekanligini e'tiborga olib, quyidagicha yozib olamiz:

(8) $$(1+\lambda)\tau(y)=-D_{0y}^{-1/2}\nu(y)-\int_0^y\mu D_{0\eta}^{-\alpha}\tau(\eta)d\eta+F(y),$$

bu yerda $F(y)=\lambda\varphi_1(0)+\int_0^{+\infty}G(0,\xi,y)\varphi_1(\xi)d\xi$.

(8) tenglikning har ikki tomoniga $D_{0y}^{1/2}$ differensial operatorni tatbiq qilib va $D_{0y}^{1/2}D_{0y}^{-1/2}g(y)=g(y)$ formulani e'tiborga olsak [7], ushbu tenglikka ega bo'lamiz:

(9) $$\nu(y)=-(1+\lambda)D_{0y}^{1/2}\tau(y)-\mu D_{0y}^{1/2}\left\{\int_0^y D_{0\eta}^{-\alpha}\tau(\eta)d\eta\right\}+D_{0y}^{1/2}F(y)$$

.

Kasr tartibli integral va differensial operatorlarning yoyilmalaridan [7] foydalanib, (9) tenglikning o'ng tomonidagi ikkinchi qo'shiluvchini soddalashtiramiz. Kasr tartibli integral operator yoyilmasidan foydalanib va karrali integralda integrallash tartibini o'zgartirib, ushbu natijaga ega bo'lamiz:

$$\int_0^y D_{o\eta}^{-\alpha}\tau(\eta)d\eta=\frac{1}{\Gamma(\alpha)}\int_0^y\int_0^\eta(\eta-z)^{\alpha-1}\tau(z)dzd\eta=$$

$$=\frac{1}{\Gamma(\alpha)}\int_0^y\tau(z)dz\int_z^y(\eta-z)^{\alpha-1}d\eta=\frac{1}{\Gamma(1+\alpha)}\int_0^y(y-z)^{\alpha}\tau(z)dz$$

Bu tenglikni e'tiborga olib, kasr tartibli differensial operator yoyilmasidan foydalansak, quyidagi tenglikka ega bo'lamiz:

$$D_{0y}^{1/2}\left\{\int_0^y D_{0\eta}^{-\alpha}\tau(\eta)d\eta\right\}=\frac{1}{\sqrt{\pi}\Gamma(1+\alpha)}\frac{d}{dy}\int_0^y (y-z)^{-1/2}\left\{\int_0^z (z-\eta)^{\alpha}\tau(\eta)d\eta\right\}dz.$$

Karrali integralda integrallash tartibini o'zgartirib, Eylerning beta va gamma funksiyalari xossalaridan foydalanib, ba'zi hisoblashlarni bajarsak, ushbu natijaga kelamiz:

$$D_{0y}^{\frac{1}{2}}\left\{\int_0^y D_{0\eta}^{-\alpha}\tau(\eta)d\eta\right\}=\Gamma^{-1}\left(\alpha+\frac{1}{2}\right)\int_0^y (y-\eta)^{\alpha-\frac{1}{2}}\tau(\eta)d\eta.$$

Bu tenglikni e'tiborga olsak, (9) quyidagi ko'rinishni oladi:

$$\text{(10)}\qquad \nu(y)=-(1+\lambda)D_{0y}^{1/2}\tau(y)-\mu\Gamma^{-1}\left(\alpha+\frac{1}{2}\right)\int_0^y (y-\eta)^{\alpha-\frac{1}{2}}\tau(\eta)d\eta+D_{0y}^{1/2}F(y).$$

(10) – noma'lum $\tau(y)$ va $\nu(y)$ funksiyalar orasidagi D_1 sohadan olingan funksional munosabatdir.

Endi $L_2 u=0$ tenglama va (3), (4) shartlarda x ni nolga intiltiramiz. Natijada ushbu

$$\text{(11)}\qquad \tau''(y)+\nu(y)=0,\qquad 0<y<h$$

Tenglamaga va quyidagi

$$\text{(12)}\qquad \tau(0)=\varphi_1(0),\qquad \tau(h)=a(0)\int_0^h \tau(y)dy$$

Shartlarga ega bo'lamiz.

(10) va (11) tengliklardan $\nu(y)$ ni chiqarib, $\tau(y)$ ga nisbatan quyidagi integro-differensial tenglama hosil bo'ladi:

$$\tau''(y)-\frac{1+\lambda}{\sqrt{\pi}}\frac{d}{dy}\int_0^y (y-\eta)^{-1/2}\tau(\eta)d\eta-$$

$$\text{(13)}\qquad -\mu\Gamma^{-1}(\alpha+1/2)\int_0^y (y-\eta)^{\alpha-\frac{1}{2}}\tau(\eta)d\eta=-D_{0y}^{1/2}F(y),\qquad 0<y<h.$$

Shunday qilib, $\tau(y)$ noma'lumni aniqlash uchun (13) tenglamaning (12) shartlarni qanoatlantiruvchi yechimini toppish masalasiga keldik. Agar bu masaladan $\tau(y)$ ni bir qiymatli aniqlasak, $\nu(y)$ funksiya (10) tenglik bilan aniqlanadi. Qo'yilgan masalaning yechimi esa D_1 sohada (6) formula bilan, D_2 sohada esa $u_{tt}+u_x=0$ tenglama uchun birinchi chegaraviy masalaning yechimi sifatida aniqlanadi. Shuning uchun bundan buyon (13) tenglamaning (12) shartlarni qanoatlantiruvchi yechimini topish masalasi bilan shug'ullanamiz.

(13) tenglamani $[0,y]$ oraliqda ketma-ket ikki marta integrallab, $\tau'(0)=C$ belgilash kiritib, ba‘zi hisoblashlardan so‘ng

$$\tau(y)-\frac{1+\lambda}{\Gamma(3/2)}\int_0^y (y-t)^{1/2}\tau(t)dt-$$

(14) $$-\mu\Gamma^{-1}(\alpha+5/2)\int_0^y (y-t)^{\alpha+3/2}\tau(t)dt=F_1(y),\ 0<y<h$$

ko‘rinishdagi integral tenglamaga ega bo‘lamiz, bu yerda

$$F_1(y)=\int_0^y\int_0^z D_{0t}^{1/2}F(t)dtdz+\varphi_1(0)+Cy.$$

Quyidagi lemma o‘rinli:

1-lemma. Agar $f(y)\in L_1[0,h]$ bo‘lsa, u holda

(15) $$\tau(y)-\frac{\lambda_1}{\Gamma(\alpha_1)}\int_0^y (y-t)^{\alpha_1-1}\tau(t)dt-\frac{\mu_1}{\Gamma(\beta_1)}\int_0^y (y-t)^{\beta_1-1}\tau(t)dt=f(y)$$

integral tenglamaning yechimi mavjud va yagona bo‘lib, u quyidagi tenglik bilan aniqlanadi:

(16) $$(\tau(y)=f(y)+\int_0^y R(y,s;\lambda_1,\mu_1)f(s)ds,$$

bu yerda α_1,β_1 – musbat haqiqiy sonlar, $\lambda_1,\mu_1\in R$;

$$R(y,s;\lambda_1,\mu_1)=\sum_{i=1}^{+\infty}\mu_1^i(y-s)^{i\beta_1-1}E_{\alpha_1,i\beta_1}^i\left[\lambda_1(y-s)^{\alpha_1}\right],$$

$E_{\alpha_1,\beta_1}^{\rho}(z)$ - Prabhakar funksiyasi bo‘lib [8] , $E_{\alpha_1,\beta_1}^{\rho}(z)=\sum_{n=0}^{+\infty}\frac{(\rho)_n z^n}{\Gamma(\alpha_1 n+\beta_1)n!}$.

Isbot. (16) tenglamani quyidagi ko‘rinishida yozib olamiz:

(17) $$\tau(y)-\frac{\lambda_1}{\Gamma(\alpha_1)}\int_0^y (y-t)^{\alpha_1-1}\tau(t)dt=\Phi(y),\quad 0<y<h,$$

bu yerda $\Phi(y)=\frac{\mu_1}{\Gamma(\beta_1)}\int_0^y (y-t)^{\beta_1-1}\tau(t)dt+f(y).$

(17) tenglikning o‘ng tomonini vaqticha ma‘lum funksiya deb hisoblasak, u holda uning yechimi

(18) $$\tau(y)=\Phi(y)+\lambda_1\int_0^y (y-t)^{\alpha_1-1}E_{\alpha_1,\alpha_1}\left[\lambda_1(y-t)^{\alpha_1}\right]\Phi(t)dt$$

formula bilan aniqlanadi, bu yerda $E_{\alpha_1,\beta_1}(z)=\sum_{n=0}^{+\infty}\frac{z^n}{\Gamma(\alpha_1 n+\beta_1)}$ – ikki parametrli Mittag-Leffler funksiyasi [9].

$\Phi(y)$ funksiyaning ifodasini (18) ga qo‘ysak, quyidagi tenglikka ega bo‘lamiz:

$$\tau(y)-\frac{\mu_1}{\Gamma(\beta_1)}\int_0^y (y-t)^{\beta_1-1}\tau(t)dt-$$

$$-\lambda_1\int_0^y (y-t)^{\alpha_1-1}E_{\alpha_1,\alpha_1}\left[\lambda_1(y-t)^{\alpha_1}\right]\times\left\{\frac{\mu_1}{\Gamma(\beta_1)}\int_0^t (t-s)^{\beta_1-1}\tau(s)ds+f(t)\right\}dt=f(y).$$

Karrali integralda integrallash tartibini o‘zgartirsak, oxirgi tenglik quyidagi ko‘rinishni oladi:

$$\tau(y)-\frac{\mu_1}{\Gamma(\beta_1)}\int_0^y (y-s)^{\beta_1-1}\tau(s)ds-$$

$$-\frac{\lambda\mu_1}{\Gamma(\beta_1)}\int_0^y \tau(s)ds\int_s^y (y-t)^{\alpha_1-1}E_{\alpha_1,\alpha_1}\left[\lambda_1(y-t)^{\alpha_1}\right](t-s)^{\beta_1-1}dt=$$

(19) $$=\lambda_1\int_0^y (y-t)^{\alpha_1-1}E_{\alpha_1,\alpha_1}\left[\lambda_1(y-t)^{\alpha_1}\right]f(t)dt+f(y).$$

Ichki integralda $t-s=\eta$ lamashtirish bajaramiz. Natijada

$$\int_s^y (y-t)^{\alpha_1-1}E_{\alpha_1,\alpha_1}\left[\lambda_1(y-t)^{\alpha_1}\right](t-s)^{\beta_1-1}dt=$$

$$=\int_0^{y-s}(y-s-\eta)^{\alpha_1-1}E_{\alpha_1,\alpha_1}\left[\lambda_1(y-s-\eta)^{\alpha_1}\right]\eta^{\beta_1-1}d\eta$$

tenglikka ega bo‘lamiz. Bu yerdan ushbu

$$\frac{1}{\Gamma(\nu)}\int_0^z (z-t)E_{\alpha_1,\beta_1}\left(\lambda_1 t^{\alpha_1}\right)t^{\beta_1-1}dt=z^{\beta_1+\nu-1}E_{\alpha_1,\beta_1+\nu}\left(\lambda z^{\alpha_1}\right)$$

formulani [9] qo‘llab, ushbu natijaga ega bo‘lamiz:

$$\frac{1}{\Gamma(\beta_1)}\int_s^y (y-t)^{\alpha_1-1}E_{\alpha_1,\alpha_1}\left[\lambda_1(y-t)^{\alpha_1}\right](t-s)^{\beta_1-1}dt=(y-s)^{\alpha_1+\beta_1-1}E_{\alpha_1,\alpha_1+\beta_1}\left[\lambda_1(y-s)^{\alpha_1}\right]$$

.

U holda (17) quyidagi ko‘rinishni oladi:

$$\tau(y)-\frac{\mu_1}{\Gamma(\beta_1)}\int_0^y (y-s)^{\beta_1-1}\tau(s)ds-$$

(20) $$-\lambda_1\mu_1\int_0^y (y-s)^{\alpha_1+\beta_1-1}E_{\alpha_1,\alpha_1+\beta_1}\left[\lambda_1(y-s)^{\alpha_1}\right]\tau(s)ds=\Phi_1(y),$$

bu yerda

$$\Phi_1(y)=\lambda_1\int_0^y (y-t)^{\alpha_1-1}E_{\alpha_1,\alpha_1}\left[\lambda_1(y-t)^{\alpha_1}\right]f(t)dt+f(y).$$

Mittag- Leffler funksiyasining qator koʻrinishdagi ifodasidan foydalansak,

$$\frac{1}{\Gamma(\beta_1)}(y-s)^{\beta_1-1}+\lambda_1(y-s)^{\alpha_1+\beta_1-1}E_{\alpha_1,\alpha_1+\beta_1}\left[\lambda_1(y-s)^{\alpha_1}\right]=$$

$$=\frac{1}{\Gamma(\beta_1)}(y-s)^{\beta_1-1}+\lambda_1(y-s)^{\alpha_1+\beta_1-1}\sum_{k=0}^{+\infty}\frac{\lambda_1^k(y-s)^{\alpha_1 k}}{\Gamma(\alpha_1 k+\alpha_1+\beta_1)}=$$

$$=(y-s)^{\beta_1-1}\left[\frac{1}{\Gamma(\beta_1)}+\sum_{k=0}^{+\infty}\frac{\lambda_1^{k+1}(y-s)^{\alpha_1 k+\alpha_1}}{\Gamma(\alpha_1 k+\alpha_1+\beta_1)}\right]=(y-s)^{\beta_1-1}E_{\alpha_1,\beta_1}\left[\lambda_1(y-s)^{\alpha_1}\right].$$

Buni eʻtiborga olib, (20) tenglamani quyidagicha yozib olishimiz mumkin:

(21) $$\tau(y)-\mu_1\int_0^y K(y,s)\tau(s)ds=\Phi_1(y),$$

bu yerda

$$K(y,s)=(y-s)^{\beta_1-1}E_{\alpha_1,\beta_1}\left[\lambda_1(y-s)^{\alpha_1}\right].$$

(21) – $\tau(y)$ noma'lum funksiyaga nisbatan ikkinchi tur Volterra integral tenglamasi boʻlib, u (15) integral tenglamaga ekvivalentdir. (21) tenglama yechimini ketma-ket yaqinlashish usulidan foydalanib topamiz.

Quyidagi formulalar boʻyicha iteratsiyalangan yadrolarni hisoblaymiz:

$$K_i(y,s)=\int_s^y K(y,t)K_{i-1}(t,s)dt,\ i=2,3,\dots\ .$$

$K_2(y,s)$ ni hisoblaymiz:

$$K_2(y,s)=\int_s^y (y-t)^{\beta-1}E_{\alpha_1,\beta_1}\left[\lambda_1(y-t)^{\alpha_1}\right](t-s)^{\beta_1-1}E_{\alpha_1,\beta_1}\left[\lambda_1(t-s)^{\alpha_1}\right]dt.$$

[8] ishdagi 5-teoremadan $\rho=\rho'=1,\ \beta'=\beta_1$ boʻlgan holda foydalanib, koʻrsatish mumkinki,

$$K_2(y,s)=\int_s^y (y-t)^{\beta_1-1}E_{\alpha_1,\beta_1}\left[\lambda_1(y-t)^{\alpha_1}\right](t-s)^{\beta_1-1}E_{\alpha_1,\beta_1}\left[\lambda_1(t-s)^{\alpha_1}\right]dt=$$

$$=(y-s)^{2\beta_1-1}E^2_{\alpha_1,2\beta_1}\left[\lambda(y-s)^{\alpha_1}\right].$$

Yuqoridagi kabi quyidagi tenglik oʻrinli ekanligini ham koʻrsatish mumkin:

$$K_3(y,s)=\int_s^y (y-t)^{\beta_1-1}E^1_{\alpha_1,\beta_1}\left[\lambda_1(y-t)^{\alpha_1}\right](t-s)^{2\beta_1-1}E^2_{\alpha_1,2\beta_1}\left[\lambda_1(t-s)^{\alpha_1}\right]dt=$$

$$=(y-s)^{3\beta_1-1}E^3_{\alpha_1,3\beta_1}\left[\lambda_1(y-s)^{\alpha_1}\right].$$

Bu jarayonni davom ettirib, matematik induksiya metodini qoʻllab, iteratsiyalangan yadrolar uchun quyidagi formulani hosil qilamiz:

$$K_i(y,s)=(y-s)^{i\beta_1-1}E^i_{\alpha_1,i\beta_1}\left[\lambda_1(y-s)^{\alpha_1}\right].$$

Buni e'tiborga olsak, (21) tenglamaning yechimi rezolventa yordamida

$$\tau(y)=\Phi_1(y)+\int_0^y R(y,s;\lambda_1,\mu_1)\Phi_1(s)ds,$$

bu yerda

$$R(y,s;\lambda_1,\mu_1)=\sum_{i=1}^{+\infty}\mu_1^i(y-s)^{i\beta_1-1}E^i_{\alpha_1,i\beta_1}\left[\lambda_1(y-s)^{\alpha_1}\right].$$

1- lemma isbotlandi.

1- lemma natijasidan $\alpha_1=3/2$, $\beta_1=\alpha+5/2$, $\lambda_1=1+\lambda$, $\mu_1=\mu$ bo'lgan holda (14) tenglama yechimini quyidagicha topamiz;

(22) $$\tau(y)=\Phi_1(y)+\int_0^y R(y,s;1+\lambda,\mu)\Phi_1(s)ds,$$

bu yerda

$$\Phi_1(y)=f_1(y)+C\left[(1+\lambda)\int_0^y t(y-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(y-t)^{3/2}\right]dt+y\right]$$

$$f_1(y)=(1+\lambda)\int_0^y\left[(y-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(y-t)^{3/2}\right]\times\left[\int_0^t\int_0^z D^{1/2}_{0x}F(x)dxdz+\varphi_1(0)\right]\right]dt+$$

$$+\int_0^y\int_0^z D^{1/2}_{0t}F(t)dtdz+\varphi_1(0).$$

(22) formula bo'yicha $\tau(h)$ va $\int_0^h\tau(y)dy$ larni hisoblaymiz:

$$\tau(h)=f_1(h)+C\left[(1+\lambda)\int_0^h t(h-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(h-t)^{3/2}\right]dt+h\right]+$$

$$+\int_0^h R(h,s;1+\lambda,\mu)\left[f_1(s)+C(1+\lambda)\int_0^s t(s-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(s-t)^{3/2}\right]dt+s\right]ds$$

$$,\int_0^h\tau(y)dy=\int_0^h\left[f_1(y)+C\left[(1+\lambda)\int_0^y t(y-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(y-t)^{3/2}\right]dt+y\right]\right]dy+$$

$$+\int_0^h R(y,s,1+\lambda,\mu)\left[f_1(s)+C\left[(1+\lambda)\int_0^s t(s-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(s-t)^{3/2}\right]dt+s\right]\right]dsdy.$$

Bularni (12) shartlarning ikkinchisiga qo'yib, quyidagi tenglikka kelamiz:

$$C\left[(1+\lambda)\int_0^h t(h-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(h-t)^{3/2}\right]dt+h+\right.$$

$$\left.+\int_0^h R(h,s;1+\lambda,\mu)\cdot\left[(1+\lambda)\int_0^s t(s-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(s-t)^{3/2}\right]dt+s\right]ds-\right.$$

$$-a(0)\int_0^h\left[(1+\lambda)\int_0^y t(y-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(y-t)^{3/2}\right]dt+y\right]dy-$$

$$-a(0)\int_0^h\int_0^y R(y,s;1+\lambda,\mu)\left[(1+\lambda)\int_0^s t(s-t)^{1/2}E_{3/3,3/2}\left[(1+\lambda)(s-t)^{3/2}\right]dt+s\right]dsdy\Bigg]=$$

$$=a(0)\int_0^h f_1(y)dy+a(0)\int_0^h\int_0^y R(y,s;1+\lambda,\mu)f_1(s)dsdy+$$

$$+\varphi_3(0)-f_1(h)-\int_0^h R(h,s;1+\lambda,\mu)f_1(s)ds.$$

Agar $a(0)$ va h sonlar uchun quyidagi tengsizlik

$$(1+\lambda)\int_0^h t(h-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(h-t)^{3/2}\right]dt+h+$$

$$+\int_0^h R(h,s;1+\lambda,\mu)\cdot\left[(1+\lambda)\int_0^s t(s-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(s-t)^{3/2}\right]dt+s\right]ds-$$

$$-a(0)\int_0^h\left[(1+\lambda)\int_0^y t(y-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(y-t)^{3/2}\right]dt+y\right]dy-$$

(23)

$$-a(0)\int_0^h\int_0^y R(y,s;1+\lambda,\mu)\left[(1+\lambda)\int_0^s t(s-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(s-t)^{3/2}\right]dt+s\right]dsdy\neq 0$$

bajarilgan bo‘lsa, (22) tenglikdan C noma’lum son bir qiymatli topiladi.

1-**izoh.** D_2 sohada $L_2u=0$ tenglamaning (2),(3) va $u(0,y)=\tau(y),0\leq y\leq h$ shartlarni qanoatlantiruvchi yechimi sifatida topiladi. Oxirgi masalani I_0 deb belgilaymiz va bir qiymatli echilishini isbotlaymiz.

Faraz qilaylik, $u(x,y)-I_0$ masalaning yechimi bo‘lsin. $u(x,h)=\varphi(x),-T\leq x\leq 0$ belgilash kiritaylik. U holda, $u(x,y)$ funksiyani D_2 sohada $L_2u=0$ tenglama uchun birinchi chegaraviy masalaning yechimi sifatida

$$u(x,y)=\int_0^h\tau(\eta)G(0,\eta;x,y)d\eta+$$

(24) $$+\int_x^0\varphi_2(\xi)G_\eta(\xi,0;x,y)d\xi-\int_x^0\varphi(\xi)G_\eta(\xi,h;x,y)d\xi$$

ko‘rinishda yozish mumkin bo‘ladi [13] , bu yerda

$$G(\xi,\eta;x,y)=\frac{1}{2\sqrt{\pi(\xi-x)}}\times$$

$$\times\sum_{n=-\infty}^{\infty}\left\{\exp\left[-\frac{(y-\eta-2n)^2}{4(\xi-x)}\right]-\exp\left[-\frac{(y+\eta-2n)^2}{4(\xi-x)}\right]\right\},\xi>x..$$

$u(x,h)=\varphi(x)$ belgilashni va (24) formulani e’tiborga olib, (3) shartda n

(25) $$\varphi(x)+\int_x^0 \varphi(\xi)\left[a(x)K(x,\xi)\right]d\xi = f_1(x), \qquad -T\le x\le 0$$

integral tenglamaga ega bo‘lamiz, bu yerda

$$K(x,\xi)=-\frac{1}{2\sqrt{\pi(\xi-x)}}\sum_{n=-\infty}^{+\infty}\left[e^{\frac{-n^2}{\xi-x}}-e^{\frac{-(2n+h)^2}{4(\xi-x)}}+e^{\frac{-(n-h)^2}{\xi-x}}-e^{\frac{-(2n-h)^2}{4(\xi-x)}}\right],\ \xi>x$$

$$f_1(x)=a(x)\int_0^h\left[\int_0^1\tau(\eta)G(0,\eta;x,y)d\eta+\int_x^0\varphi_2(\xi)G(\xi,0;x,y)d\xi\right]dy+\varphi_3(x).$$

(25) – Volterrning ikkinchi tur integral tenglamasidir. Uning yadrosi $(1/2)$ tartibli sust maxsuslikka ega bo‘lib, $x\to\xi$ da o‘zini $(x-\xi)^{-1/2}$ funksiya kabi tutadi, o‘ng tomoni esa $C[-T,0]$ sinfga tegishli, ya’ni

$$a(x)K(x,\xi)=O(1)(\xi-x)^{-1/2}, \qquad f_1(x)\in C[-T,0].$$

Shuning uchun (25) integral tenglama $[-T,0]$ oraliqda uzluksiz bo‘lgan yagona yechimga ega. Demak, I_0 masala ham yagona yechimga ega.

Shunday qilib, quyidagi teorema o‘rinli ekanligi isbotlandi.

Teorema. $a(x)\ne 0,\ x\in[-T,0],\ \varphi_1(x)\in C[0,+\infty)$ va chegaralangan; $a(x)$, $\varphi_2(x)$, $\varphi_3(x)\in C[-T,0]$ bo‘lib, $a(0)$ va h sonlar (23) tengsizlikni qanoatlantirsa, I masala yagona yechimga ega bo‘ladi.

2-**izoh.** $a(x)\equiv 0,\ x\in[-T,0]$ bo‘lganda ham I masalaning yechimi D_2 sohada (24) formula bilan aniqlanadi, faqat bunda $\varphi(x)=\varphi_3(x)$ deb olinadi.

FOYDALANILGAN ADABIYOTLAR:

1. Gevrey M. Sur les equations aux derivees partielles du type parabolique // J.Math. Appl.1913, T.9,Sec.6.-P. 305-475.

2. Керефов А.А. Об одной краевой задаче Жевре для параболического уравнения с знакопеременным разрывом первого рода у коэффициента при производной по времени // Дифференциальные уравнения.-Минск. 1974, Т.10, N1.-С.69-77.

3. Акбарова М.Х. Нелокальные краевые задачи для параболических уравнений смешанного типов. Автореферат на соискание ученой степени кандидата физико-математических наук. –Ташкент. 1995.-17с.;

4. Nakhushev A.M. The correct formulation of boundary value problems for parabolic equations with a characteristic form of variable sign. Differ. Uravn. 9, 130–135 (1973).

5. A.O. Mamanazarov. Gevrey problem for a mixed parabolic equation with singular coefficients. Itogi Nauki Tekh., Ser.: Sovrem. Mat. Pril. Temat. Obz. 156, 18–29 (2018).

6. D. Amanov. Boundary value problem for fourth order mixed parabolic equation. Uzb. Mathematical Jurnal. No. 2, 26-30.

7. A.O. Mamanazarov. Unique solvability of problems for a mixed parabolic equation in unbounded domain. Lobachevskii Journal of Mathematics, 2020, Vol. 41, No. 9, pp. 1837–1845.

8. T.R.Prabhakar, A singular integral equation with a generalized Mittag Leffler function in the kernel, Yokohama Math J. 19, (1971), 7-15.

9.I.Podlubny. Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications. Elsevier Science, 1998. 440 p.

10. Ўринов А.Қ. Махсус функциялар ва махсус операторлар.-Фарғона: Фарғона нашриёти, 2011.

11. A. D. Polyanin, Handbook of Linear Partial Differential Equations for Engineers and Scientists (Fizmatlit, Moscow, 2001; Chapman and Hall/CRC, New York, 2001).

12. Самко С. Г., Килбас А.А., Маричев О. И. Интегралы и производные дробного порядка и некоторые их приложения.- Минск: Наука и техника. 1987.

13. Джураев Т.Д. Уравнения смешанного-составного типов. –Ташкент: Фан. 1979.