IKKINCHI TARTIBLI INTEGRO-DIFFERENSIAL TENGLAMA UCHUN TO‘G‘RI VA TESKARI MASALALAR

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Annotatsiya: *Ushbu ishda ikkinchi tartibli integro-differensial tenglama uchun bir teskari masala bayon qilingan va tadqiq etilgan.*

Kalit so‘zlar: *ikkinchi tartibli integro-differensial tenglama, Riman-Liuvill ma'nosidagi kasr tartibli integral, teskari masala*.

ПРЯМАЯ И ОБРАТНАЯ ЗАДАЧИ ДЛЯ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ВТОРОГО ПОРЯДКА

Аннотация: *В этой работе была сформулирована и исследована обратная задача для интегро-дифференциального уравнения второго порядка*.

Ключевые слова: *Интегро-дифференциальное уравнение второго порядка, Интеграл Римана-Лювилля в смысле дроби, обратная задача*.

DIRECT AND INVERSE PROBLEMS FOR AN INTEGRO-DIFFERENTIAL EQUATION OF THE SECOND ORDER

Annotation: *In this paper, the inverse problem for a second order integro-differential equation was formulated and investigated*.

Keywords: *Integro-differential equation of the second order, Riemann-Liouville integral in the sense of (fractions), inverse problem*.

I Kirish. So‘ngi vaqtlarda noma’lum manbali differensial tengalamalar bilan shug‘illanishga bo‘lgan qiziqish ortib bormoqda. Bunga sabab ko‘plab issiqlik taqalish va diffuziya jarayonlarini matematik modelini tuzish noma’lum manbali differensial tenglama uchun qo`yiladigan masalalarga keltiriladi. Bunday differensial tenglamalar uchun teskari masalalar ko‘plab tadqiqotchilar tomonidan o‘rganilgan (masalan, ushbu $[1]-[10]$ ishlarga qaralsin).

II Masalani qo‘yilishi.

(0, 1) oraliqda ushbu

$$y''(x)-\lambda I_{0x}^{\gamma}y(x)=f(x) \qquad (1)$$

ikkinchi tartibli integro-differensial tenglamani qaraylik, bu yerda $y(x)-$ noma’lum funksiya; $f(x)-$ berilgan uzluksiz funksiya; λ,γ - o‘zgarmas haqiqiy sonlar bo'lib; $I_{0x}^{\gamma}y(x)$ - Riman-Liuvill ma'nosida γ (kasr) tartibli integral [11]

$$I_{0x}^{\gamma}y(x)=\frac{1}{\Gamma(\gamma)}\int_0^x(x-t)^{\gamma-1}y(t)dt.$$

Koshi masalasi. Shunday $y(x)$ funksiya topilsinki, u quyidagi xossalarga ega bo'lsin:

1) (0, 1) oraliqda (1) tenglamani qanoatlantirsin;

2) $C^1[0,1]\text{I } C^2(0,1)$ sinfga tegishli bo‘lsin;

3) $x=0$ nuqtada esa

$$y(0)=A_1,\ y'(0)=A_2, \qquad (2)$$

chegaraviy shartlarni qanoatlantisin, bu yerda A_1,A_2-berilgan o‘zgarmas haqiqiy sonlar.

(1) tenglamani ketma-ket 2 marta integrallab, (2) shartdan foydalanib,

$$y(x)-\frac{\lambda}{\Gamma(\gamma+2)}\int_0^x(x-t)^{\gamma+2-1}y(t)dt=A_1+A_2x+\int_0^x(x-z)f(z)dz \qquad (3)$$

ko‘rinishdagi integral tenglamani hosil qilamiz.

(3) da ba’zi belgilashlarni kiritib, uni ko‘rinishda yozib olamiz, bu yerda

$$g(x)=A_1+A_2x+\int_0^x(x-z)f(z)dz,\ \ \beta=\gamma+2,\ \ K(x,t)=\frac{(x-t)^{\beta-1}}{\Gamma(\beta)}$$

$$y(x)-\lambda\int_a^x K(x,t)y(t)dt=g(x) \qquad (4)$$

(4) Volltera ikkinchi tur tenglamasi bo‘lib, uni ketma-ket yaqinlashish usuli yordamida yechamiz.

$$K_1(x,t)=\frac{(x-t)^{\beta-1}}{\Gamma(\beta)}\ \text{ va }\ K_i(x,y)=\int_y^x K_1(x,t)K_{i-1}(t,y)dt$$

rekurent formuladan foydalanib, ba’zi hisoblashlarni amalga oshirib $K_n(x,t)=\frac{(x-t)^{n\beta-1}}{\Gamma(n\beta)}$, $n=1,2,3,\ldots$ topamiz.

Integral tenglamalar nazariyasiga ko‘ra, (4) tenglamaning yechimi

$$y(x)=g(x)+\lambda\int_a^x R(x,t,\lambda)g(t)dt \qquad (5)$$

ko‘rinishda yozib olamiz, bu yerda

$$R(x,t;\lambda)=\sum_{n=1}^{\infty}\lambda^{n-1}K_n(x,t),\ \ R(x,t,\lambda)=\sum_{n=1}^{\infty}\frac{\lambda^{n-1}(x-t)^{\beta n-1}}{\Gamma(\beta m)}$$

(5) da $g(x)=A_1+A_2x+\int_0^x (x-z)f(z)dz$ ekanligi e’tiborga olib, ba’zi soddalastirishlarni amalga oshirib Koshi masalasining yechimini

$$y(x)=A_1E_{\beta,1}\left(\lambda x^\beta\right)+A_2xE_{\beta,2}\left(\lambda x^\beta\right)+\int_0^x (x-z)E_{\beta,4}\left[\lambda(x-z)^\beta\right]f(z)dz \qquad (6)$$

ko‘rinishda topamiz, bu yerda $E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)}$ – Mittag-Lefler funksiyasi [12].

Endi

$$y''(x)-\lambda I_{0x}^{\gamma}y(x)=kf(x) \qquad (7)$$

tenglamani $(0,1)$ oraliqda qaraylik, bu yerda $y(x)$-noma’lum funksiya, λ,γ - o‘zgarmas haqiqiy sonlar, $f(x)$-berilgan funksiya, k-noma’lum son.

T₁ masala Shunday $y(x)$-funksiya va k sonni topilsinki u quyidagi xossalarga ega bo‘lsin:

1) (0, 1) oraliqda (7) tenglamani qanoatlantirsin;

2) $C^1[0,1]$ I $C^2(0,1)$ sinfga tegishli bo‘lsin;

3) $x=0$, $x=1$ nuqtalarda esa

$$y(0)=A_1,\ y(1)=B_1 \qquad (8)$$

$$y(\xi_0)=D_1 \qquad (9)$$

shartlarni qanoatlantirsin, bu yerda A_1,B_1,D_1 - berilgan o'zgarmas haqiqiy sonlar.

T_1 masala yechimini (6) formuladan foydalanib,

$$y(x)=A_1E_{\beta,1}\left(\lambda x^\beta\right)+A_2xE_{\beta,2}\left(\lambda x^\beta\right)+k\int_0^x (x-z)E_{\beta,4}\left[\lambda(x-z)^\beta\right]f(z)dz \qquad (10)$$

ko‘rinishda yozib olamiz, bu yerda A_2 noma’lum son.

(10) tenglikda $y(1)=B_1$ shartdan foydalanib, noma’lum A_2 ni

$$A_2=\frac{B_1-A_1E_{\beta,1}(\lambda)-k\int_0^1(1-z)E_{\beta,1}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,2}(\lambda)} \qquad (11)$$

ko‘rinishda topamiz.

Endi, $y(\xi_0)=D_1$ nolokal shartdan k ni

$$k=\frac{A_1E_{\beta,1}\left(\lambda\xi_0^\beta\right)+\frac{B_1-A_1E_{\beta,1}(\lambda)}{E_{\beta,2}(\lambda)}\xi_0E_{\beta,2}\left(\lambda\xi_0^\beta\right)-D_1}{\xi_0E_{\beta,2}\left(\lambda\xi_0^\beta\right)\frac{\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,2}(\lambda)}-\int_0^{\xi_0}(\xi_0-z)E_{\beta,2}\left[\lambda(\xi_0-z)^\beta\right]f(z)dz}$$

(12)

ko‘rinishda topamiz.

(11) va (12) larni (10) ga qo'yib, T_1 masalaning yechimi hosil qilamiz.

1-teorema. Agar

$$\xi_0 E_{\beta,2}\left(\lambda\xi_0^\beta\right)\frac{\int_0^1 (1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,2}(\lambda)} \neq \int_0^{\xi_0}(\xi_0-z)E_{\beta,2}\left[\lambda(\xi_0-z)^\beta\right]f(z)dz$$

bo'lsa, u holda T_1 masala yagona yechimga ega bo'ladi.

T_2 masala (7) tenglamada shunday $y(x)$-funksiya va k sonni topilsinki u quyidagi xossalarga ega bo'lsin:

1) (0, 1) oraliqda (7) tenglamani qanoatlantirsin;

2) $C^1[0,1]\mathrm{I}\ C^2(0,1)$ sinfga tegishli bo'lsin;

3) $x=0$, $x=1$ nuqtalarda esa

$$y'(0)=A_2,\ y(1)=B_1 \tag{13}$$

va (9) shartlarni qanoatlantirsin, bu yerda A_2, B_1, D_1 - berilgan o'zgarmas haqiqiy sonlar.

T_2 masala yechimini (6) formuladan foydalanib,

$$y(x)=A_1E_{\beta,1}\left(\lambda x^\beta\right)+A_2xE_{\beta,2}\left(\lambda x^\beta\right)+k\int_0^x (x-z)E_{\beta,4}\left[\lambda(x-z)^\beta\right]f(z)dz \tag{14}$$

ko'rinishida yozib olamiz, bu yerda A_1 noma'lum son

(12) formulada $y(1)=B_1$ shartdan foydalanib, noma'lum A_1 ni

$$A_1=\frac{B_1-A_2E_{\beta,2}(\lambda)-k\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,1}(\lambda)} \tag{15}$$

ko'rinishda topamiz.

Endi, $y(\xi_0)=D_1$ nolokal shartdan k ni

$$k=\frac{A_2\xi_0E_{\beta,2}\left(\lambda\xi_0^\beta\right)+\dfrac{B_1-A_2E_{\beta,2}(\lambda)}{E_{\beta,1}(\lambda)}E_{\beta,1}\left(\lambda\xi_0^\beta\right)-D_1}{E_{\beta,1}\left(\lambda\xi_0^\beta\right)\dfrac{\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,1}(\lambda)}-\int_0^{\xi_0}(\xi_0-z)E_{\beta,2}\left[\lambda(\xi_0-z)^\beta\right]f(z)dz} \tag{16}$$

ko'rinishda topamiz.

(15) va (16) larni (14) ga qo'yib, T_2 masalaning yechimi hosil qilamiz.

2-teorema. Agar

$$E_{\beta,1}\left(\lambda\xi_0^\beta\right)\frac{\int_0^1 (1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,1}(\lambda)} \neq \int_0^{\xi_0}(\xi_0-z)E_{\beta,2}\left[\lambda(\xi_0-z)^\beta\right]f(z)dz$$

bo'lsa, u holda T_2 masala yagona yechimga ega bo'ladi.

T_3 masala (7) tenglamada shunday $y(x)$-funksiya va k sonni topilsinki u quyidagi xossalarga ega bo'lsin:

1) (0, 1) oraliqda (7) tenglamani qanoatlantirsin;

2) $C^1[0,1]\text{I } C^2(0,1)$ sinfga tegishli bo'lsin;

3) $x=0$, $x=1$ nuqtalarda esa

$$y(0)=A_1,\ y'(1)=B_2 \qquad (17)$$

va (9) shartlarni qanoatlantirsin, bu yerda A_1, B_2, D_1 - berilgan o'zgarmas haqiqiy sonlar.

T_3 masala yechimini (6) formuladan foydalanib,

$$y(x)=A_1E_{\beta,1}(\lambda x^\beta)+A_2xE_{\beta,2}(\lambda x^\beta)+k\int_0^x (x-z)E_{\beta,4}\left[\lambda(x-z)^\beta\right]f(z)dz \qquad (18)$$

yozib olamiz, bu yerda A_2 noma'lum son.

(17) tenglikda $y'(1)=B_2$ shartdan foydalanib, noma'lum A_2 ni

$$A_2=\frac{B_2-A_1E_{\beta,\beta}(\lambda)-k\int_0^1 E_{\beta,1}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,1}(\lambda)} \qquad (19)$$

ko'rinishda topamiz.

Endi, $y(\xi_0)=D_1$ nolokal shartdan k ni

$$k=\frac{A_1E_{\beta,1}(\lambda\xi_0^\beta)+\xi_0E_{\beta,1}(\lambda\xi_0^\beta)\left[\frac{B_2-A_1E_{\beta,\beta}(\lambda)}{E_{\beta,1}(\lambda)}\right]-D_1}{\xi_0E_{\beta,2}(\lambda\xi_0^\beta)\frac{\int_0^1 E_{\beta,1}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,1}(\lambda)}-\int_0^{\xi_0}(\xi_0-z)E_{\beta,2}\left[\lambda(\xi_0-z)^\beta\right]f(z)dz} \qquad (20)$$

ko'rinishda topamiz.

(19) va (20) larni (18) ga qo'yib, T_3 masalaning yechimi hosil qilamiz.

3-teorema. Agar

$$\xi_0E_{\beta,2}(\lambda\xi_0^\beta)\frac{\int_0^1 E_{\beta,1}\left[\lambda(1-z)^\beta\right]f(z)dz}{E_{\beta,1}(\lambda)}\neq\int_0^{\xi_0}(\xi_0-z)E_{\beta,2}\left[\lambda(\xi_0-z)^\beta\right]f(z)dz$$

bo'lsa, u holda T_3 masala yagona yechimga ega bo'ladi.

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