H_φ FAZOSI VA UNING KLASSIFIKATSIYASI

https://doi.org/10.5281/zenodo.7843803

O'razaliyev shirinboy bo'ron o'g'li
samarqand iqtisodiyot va servis instituti, shirinboy.urazaliyev@mail.ru

Annotatsiya*:* *$f(x)$ funksiyaning $[a,b]$ dagi modul uzluksizligi, H_φ fazosi va uning xarakteristikasi .*

Аннотация: Н*епрерывность по модулю функции f(x) в [a,b], пространстве и H_φ ее характеристика.*

Annotation: *Modulo continuity of function f(x) in [a,b], space H_φ and its characteristic.*

1-ta'rif. $\omega(\delta) \in \Phi$ bo'lsin. Agar $[a,b]$ da aniqlangan uzluksiz funksiya quyidagi

$$\sup_{0<\delta\leq b-a} \frac{\omega(f,\delta)}{\omega(\delta)} = H_\omega(f) < \infty$$

Shartini qanoatlantirsa (Bunda $\omega(f,\delta) - f(x)$ funksiyaning $[a,b]$ dagi modul uzluksizligi), u holda $f \in H_\omega(f)$ deyiladi. Bundan keyin qisqacha $H_\omega(f)$ ning o'rniga H_ω deb ishlatib ketayveramiz.

$\omega(\delta) -$ funksiya H_ω sinfning xarakteristikasi deyiladi.

2-ta'rif. $\omega(\delta) \in \Phi$ bo'lsin. Agar $\exists C_f > 0$ bo'lib, $\omega(f,\delta) \leq C_f\omega(\delta)$ tengsizlik bajarilsa, u holda $f \in H_\omega$ deyiladi.

1-lemma. H_ω −to'plam, kompleks sonlar maydonida chiziqli sistemani tashkil etadi.

Isbot. Lemmani isbot qilish uchun

1). $\forall f, g \in H_\omega \Rightarrow f + g \in H_\omega$,

2).$\forall \lambda \in (z), f \in H_\omega \Rightarrow \lambda f \in H_\omega$

ekanligini ko'rsatish yetarli.

$f, g \in H_\omega$ bo'lsin $\Rightarrow$ $\exists C_f, C_g > 0$, $\forall \delta \in (0, l_0]$ uchun

$$\omega(f,\delta) \leq C_f\omega(\delta), \qquad \omega(g,\delta) \leq C_g\omega(\delta)$$

bu tengsizliklardan

$$\omega(f+g,\delta) \leq \omega(f,\delta) + \omega(g,\delta) \leq C_f\omega(\delta) + C_g\omega(\delta) = \left(C_f + C_g\right)\omega(\delta) \Rightarrow$$
$$\Rightarrow f + g \in H_\omega$$
$$\omega(\lambda f,\delta) \leq |\lambda|\omega(f,\delta) \leq |\lambda|C_f\omega(\delta) \Rightarrow \lambda f \in H_\omega.$$

2-lemma. H_ω − cheksiz o'lchovli chiziqli sistema.

3-lemma. $H_\omega \in C(\Gamma)$.

$C(\Gamma) -$ Γ da uzluksiz bo'lgan funksiyalar sinfi

1-teorema. H_ω fazosi (1) norma bo'lishicha Banax fazosi bo'ladi.

3-ta'rif. B_1 va B_2 lar Banax fazosi bo'lib, $B_1 \subseteq B_2$ bo'lsin. B_1 ning B_2 da yotishi to'la uzluksiz deyildi, agarda $\|f_n - f_0\|_{B_1} \longrightarrow 0 \Longrightarrow \|f_n - f_0\|_{B_2} \longrightarrow 0,\ f_n, f_0 \in B_1$ bo'lsa.

4-lemma. B_1 va B_2 fazolar Banax fazosi bo'lib, $B_1 \subseteq B_2$ yotishi to'la uzluksiz bo'lsin, agar B_1 cheksiz o'lchovli bo'lsa, u holda $B_1 \subset B_2$ ($B_1, B_2 -$ ning to'g'ri qismi bo'ladi)

4-ta'rif. B_1 va B_2 fazolar- Banax fazosi bo'lib, $B_1 \subseteq B_2$ bo'lsin. Agar B_1 ning ixtiyoriy chegaralangan qismi B_2 da kompakt bo'lsa, u holda B_1 ning B_2 da yotishi kompakt deyildi.

5-ta'rif. Uzluksiz va kompakt vlojeniya (yotishi) to'la uzluksiz deb ataladi.

uzluksiz va kompakt bo'lsa, ya'ni u B_1 ning ixtiyoriy chegaralangan to'plamini B_2 ning kompakt qismiga ko'chirsa, u holda A operator to'la uzluksiz operator deyiladi.

5-lemma. $\varphi \in \Phi$ bo'lsin. H_φ fazo, C_Γ da to'la uzluksiz bo'ladi.

2-teorema. $\varphi \epsilon \Phi$ bo'lsa H_φ fazo C_Γ ning to'g'ri qismi bo'ladi.

Isbot. Bu teoremaning isboti H_φ fazoning cheksiz o'lchovliligidan, H_φ ning C_Γ da yotishining (vlosheniya) to'la uzluksizligi va yuqoridagi 3-lemmadan kelib chiqadi.

6-lemma. $\varphi_1, \varphi_2 \in \Phi$ va $\lim_{\delta\to 0}\frac{\varphi_1(\delta)}{\varphi_2(\delta)} = 0$ bo'lsa, u holda $H_{\varphi_1} \subset H_{\varphi_2}$ va vlosheniya (H_{φ_1}ning H_{φ_2} da yotishi) to'la uzluksiz bo'ladi.

3-teorema. $\varphi_1, \varphi_2 \in \Phi$ bo'lsin. Agar: 1) $\varphi_1(\delta) \sim \varphi_2(\delta)$ bo'lsa, u holda
$H_{\varphi_1} = H_{\varphi_2}$ bo'ladi.

2) $\lim\limits_{\delta\to 0}\frac{\varphi_1(\delta)}{\varphi_2(\delta)} = 0$ bo'lsa, u holda $H_{\varphi_1}, H_{\varphi_2}$ ning to'g'ri qismi bo'ladi.

Isbot. [$\varphi_1(\delta),\ \varphi_2(\delta) > 0, \varphi_1(\delta) \sim \varphi_2(\delta)$ deyildi agarda $\exists A_1, A_2 > 0, \forall\delta$ uchun
$A_1\ \varphi_2(\delta) \le \varphi_1(\delta) \le A_2\ \varphi_2(\delta)$ bo'lsa]

Bu ta'rifga asosan teorema birinchi qismining isboti birdaniga kelib chiqadi :
$H_{\varphi_1} \subseteq H_{\varphi_2}$ ekanligini ko'rsatamiz.

Agar $\varphi_1 \le m\ \varphi_2$ bo'lsa $\Longrightarrow \left||f|\right|_{\varphi_2} \le m\left||f|\right|_{\varphi_1}$.

$\varphi_2 \le n\ \varphi_1$ bo'lsa $H_{\varphi_2} \subseteq H_{\varphi_1}$ va $\left||f|\right|_{H_{\varphi_1}} \le n\left||f|\right|_{H_{\varphi_2}}$

teoremaning ikkinchi qismini H_{φ_1} ning H_{φ_2} da etishi to'la uzluksizligi va ularning cheksiz o'lchovida, hamda 2-3 lemmalardan kelib chiqadi.

H_φ da normani quyidagicha kiritamiz:

$$\left||f|\right|_{H_\varphi} = \max_{t\in\gamma}|f(t)| + \sup_{0<\delta\le l}\frac{\omega(f,\delta)}{\varphi(\delta)}, \quad \varphi \in \Phi_{[0,d]} \qquad (1)$$

4-teorema. (1) norma bo'yicha H_φ fazosi to'la bo'ladi.

Isbot. $\forall\{f_n\} \in H_\varphi,\ \ \{f_n\} -$ fundamental ketma-ketlik bo'lsin $\Longrightarrow$

$\forall\varepsilon > 0, \exists N,\ m, n > N$ boshlab $\left||f_m - f_n|\right|_{H_\varphi} < \varepsilon \qquad (2)$

tengsizlik o'rinli bo'ladi. (2) ni (1) ni e'tiborga olganda

$$\left||f_m - f_n|\right|_{H_\varphi} = \max_{t\in\gamma}|f_m - f_n| + \sup_{0<\delta\le l}\frac{\omega(f_m - f_n,\delta)}{\varphi(\delta)} < \varepsilon \qquad (3)$$

(3) dan $\{f_n\}$ ketma-ketlik $C(\gamma)$ da ham fundamental bo'ladi

$||f_m - f_n||_{C_\Gamma} < \varepsilon$ (4)

Ma'lumki, $C(\gamma)$ – to'liq fazo. Shuning uchun $\exists f_0 \in C(\gamma)$

$||f_m - f_n||_{C_\Gamma} \longrightarrow_{n\to\infty} 0$ (5)

Endi $f_0 \in H_\varphi$ ekanligini ko'rsatamiz.

Ma'lumki, fundamental ketma-ketlik chegaralangan ketma-ketlik bo'ladi

$\exists r > 0, \forall n$ uchun $||f_n||_{H_\varphi} \le r \implies \frac{\omega(f_n,\delta)}{\varphi(\delta)} \le r, \ \forall n, \delta \implies \omega(f_n,\delta) \le r\varphi(\delta).$

$[\ \omega(f-g,\delta) \ge |\omega(f,\delta) - \omega(g,\delta)|, \qquad \omega(f+g,\delta) \ge |\omega(f,\delta) + \omega(g,\delta)|$

$\omega(f,\delta) = \omega(f-g+g,\delta) \le \omega(f-g,\delta) + \omega(g,\delta) \implies$

$\omega(f,\delta) - \omega(g,\delta) \le \omega(f-g,\delta)$ xuddi shunday

$\omega(g,\delta) - \omega(f,\delta) \le \omega(g-f,\delta)$

$$|\omega(f,\delta) - \omega(g,\delta)| \le \omega(f-g,\delta)$$

Agar $f_n \rightrightarrows f_0$ bo'lsa $\omega(f,\delta) \rightrightarrows \omega(f_0,\delta)$ bo'ladi. $f_n \rightrightarrows f_0$ bo'lsin, $\forall\delta$

$\omega(f_0,\delta) \le r\varphi(\delta) \implies f_0 \in H_\varphi$

Endi $||f_n - f_0||_{H_\varphi} \longrightarrow_{n\to\infty} 0$ ekanligini ko'rsatamiz. $||f_n - f_0||_{C_\Gamma} \longrightarrow_{n\to\infty} 0$ bo'lganligi uchun $\lim_{n\to\infty} sup \frac{\omega(f_n - f_0,\delta)}{\varphi(\delta)} = 0$ ekanligini ko'rsatish yetarli (2) dan $\forall\varepsilon > 0, \exists N, \ m, n > N \quad \sup_\delta \frac{\omega(f_m - f_n,\delta)}{\varphi(\delta)} < \varepsilon \implies \sup_\delta \omega(f_m - f_n,\delta) < \varepsilon\varphi(\delta) \implies$ Bundan

$n > N$ ni belgilab $m \to \infty \ \ \omega(f_m - f_n,\delta) < \varepsilon\varphi(\delta)$ da limitga o'tamiz:

$\omega(f_0 - f_n,\delta) \le \varepsilon\varphi(\delta) \implies \frac{\omega(f_0 - f_n,\delta)}{\varphi(\delta)} \le \varepsilon \implies \sup_\delta \frac{\omega(f_0 - f_n,\delta)}{\varphi(\delta)} \le \varepsilon$ (5)

(4) bilan (5) dan H_φ ning to'liqligi kelib chiqadi.

7-lemma. Agar $||U_n - U_0||_{C_\Gamma} \longrightarrow 0$ bo'lsa, $||\omega(U_n,\delta) - \omega(U_0,\delta)||_{C_\Gamma} \longrightarrow 0$ bo'ladi.

5-teorema. H_φ fazo to'liq fazo.

Isbot. $\{f_n\}$ –fundamental ketma-ketlik $\{f_n\} \in H_\varphi$ bo'lsin. $\{f_n\} - C_\Gamma$ da fundamental. C_Γ – to'liq fazo bo'lganligi uchun $\exists f_0, \ f_0 \in C_\Gamma$, ya'ni

$||f_n - f_0||_{C_\Gamma} \longrightarrow 0$ (6)

$\{f_n\}$ ketma-ketlik H_φ da fundamental bo'lganligi uchun u chegaralangan bo'ladi, ya'ni $\forall n$ uchun $||f_n||_{H_\varphi} \le r. \quad 0 < r < +\infty$

$$||f_n||_{H_\varphi} = ||f_n||_{C_\Gamma} + \sup_\delta \frac{\omega(f_n,\delta)}{\varphi(\delta)} \le r$$

bo'lgani uchun $\omega(f_n,\delta) \le r\varphi(\delta)$ (1) va 1-lemmaga asosan

$\omega(f_n,\delta) \le r\varphi(\delta) \implies f_0 \in H_\varphi$

Endi $||f_n - f_0||_{H_\varphi} \longrightarrow 0$ bo'lishini ko'rsatamiz

$$||f_n - f_0||_{H_\varphi} = ||f_n - f_0||_{C_\Gamma} + \sup_\delta \frac{\omega(f_n - f_0,\delta)}{\varphi(\delta)}$$

(1) ga asosan $||f_n - f_0||_{C_\Gamma} \longrightarrow 0$ va $\sup_\delta \frac{\omega(f_n - f_0,\delta)}{\varphi(\delta)} \longrightarrow 0$ ko’rsatish yetarli $\{f_n\} \in H_\varphi$ da fundamental bo’lganligi uchun $\exists N$ nomer topiladiki $m, n > N$ uchun $||f_n - f_m||_\varphi < \varepsilon$ bo’ladi. Bundan $sup\frac{\omega(f_n - f_0,\delta)}{\varphi(\delta)} < \varepsilon$, $\frac{\omega(f_n - f_0,\delta)}{\varphi(\delta)} < \varepsilon$,

$\omega(f_n - f_0, \delta) < \varepsilon\varphi(\delta)$ bo’ladi $\Longrightarrow$ $sup\frac{\omega(f_n - f_0,\delta)}{\varphi(\delta)} \leq \varepsilon$ $\Longrightarrow \lim_{n\to\infty}\frac{\omega(f_n - f_0,\delta)}{\varphi(\delta)} \longrightarrow 0$

Demak, H_φ fazo to’liq fazo ekan.

Agar $f \in H_\varphi$ bo’lsa, $\forall m > 0$ uchun $mf \in H_\varphi$ bo’ladi va $H_\varphi = H_{m\varphi}$ bo’ladi.Endi $\forall H_{\varphi_1}$ va H_{φ_2} sinflar qaysi vaqtda bir-biridan farq qiladi.

6-ta’rif. $B_1 \subseteq B_2$ to’la uzluksiz deyiladi: B_1 dagi har bir chegralangan ketma-ketlik B_2 da kompakt bo’lsa va B_1 da yaqinlashuvchiligidan B_2 da yaqinlashuvchiligi kelib chiqsa.

Teorema (Riss). Har bir B fazo chekli o’lchovli bo’lishi uchun uning chegaralangan qismi kompakt bo’lishi zarur va yetarli.

Xulosa.

Ushbu maqola H_φ fazosi va uning xarakteristikasini o’rganishga bag’ishlangan.

FOYDALANILGAN ADABIYOTLAR:

1. В. А. Какичев. «Граничные свойства интеграла типа Коши многих переменных». Уч. зап. Шахт. пед. ин-та. 2,вып. 6,1959-г,25-90 с.
2. Ф. Д. Гахов. «Краевые задачи». Москва. Наука. 1977-г.
3. Н. И. Мусхелишвили. «Сингулярные интегральные уравнения». Москва. Наука. 1968-г.
4. Б. А. Фукс. «Теория аналитических функций многих комплексных переменных». М. Л. 1948-г.
5. A. G’oziyev. «Analitik funksiyaning chegaraviy masalalari va singuliyar integral tenglamalar». Samarqand-2018.