YUKLANGAN KASR TARTIBLI ODDIY DIFFERENSIAL TENGLAMALAR UCHUN MASALALAR

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Annotatsiya. *Ushbu maqolada Hilfer ma'nosidagi hosila qatnashgan yuklangan kasr tartibli integro-differensial tenglama uchun bir masalani o`rganilgan. Ushbu masalaning yechimi Koshi masalaning yechiminidan foydalanib topilgan.*

ЗАДАЧИ ДЛЯ НАГРУЖЕННЫХ ОБЫКНОВЕННЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ДРОБНОГО ПОРЯДКА

Аннотация. *В статье исследована задача для нагруженного интегро-дифференциального уравнения дробного порядка с производной Хильферу. Решение этой проблемы было найдено с помощью решения задачи Коши.*

PROBLEMS FOR LOADED ORDINARY DIFFERENTIAL EQUATIONS OF FRACTIONAL ORDER

Abstract. *In the article, a problem for a loaded fractional-order integro-differential equation with a Hilfer derivative is studied. The solution to this problem was found by solving the Cauchy problem.*

Kalit so`zlar: *yuklangan differensial tenglama, kasr tartibli integro-differensial operator, integral tenglama ketma-ket yaqinlashish usuli, integral tenglama.*

Ключевые слова: *нагруженное дифференциальное уравнение, интегро-дифференциальный оператор дробного порядка, метод последовательного приближения интегрального уравнения, интегральное уравнение.*

Keywords: *loaded differential equation, fractional order integro-differential operator, integral equation successive approximation method, integral equation.*

I. Kirish.

So'ngi vaqtlarda noma`lum funksiyani biror qiymati qatnashgan differensial tengalamalar bilan shug'ullanishga bo`lgan qiziqish ortib bormoqda. Bunga sabab ko`plab issiqlik tarqalish va diffuziya jarayonlarini matematik modelini tuzish funksiyani biror qiymati qatnashgan differensial tenglama uchun qo'yiladigan masalalarga keltiriladi. Odatda, bunday turdagi tenglamalar yuklangan differensial tenglama deb yuritiladi.Xususiy hosilali va oddiy differensial tenglamalar differensial tenglama ko`plab tadqiqotchilar tomonidan o'rganilgan (masalan, ushbu [1]–[6] ishlarga qaralsin).

II. Masalaning qo`yilishi va tadqiqoti.

(0,1) oraliqda ushbu

$$D_{0x}^{\alpha,\beta}y(x)-\lambda I_{0x}^{\gamma}y(x)=f(x) \qquad (1)$$

kasr tartibli oddiy differensial tenglamani qaraylik, bu yerda $y(x)$- noma’lum funksiya; $\alpha,\beta,\lambda,\gamma$ - o‘zgarmas haqiqiy sonlar bo‘lib, $0<\alpha<1, 0\le\beta\le1, \gamma>0;$; $D_{0x}^{\alpha,\beta}y(x)-$ Hilfer ma’nosidagi kasr tartibli hosila bo‘lib [7],

$$D_{0x}^{\alpha,\beta}y(x)=I_{0x}^{\beta(1-\alpha)}\frac{d}{dx}I_{0x}^{(1-\beta)(1-\alpha)}y(x), \qquad (2)$$

$I_{0x}^{\gamma}y(x)-$ Riman-Liuvill ma’nosida γ (kasr) tartibli integral operator bo‘lib [8],

$$I_{0x}^{\gamma}y(x)=\frac{1}{\Gamma(\gamma)}\int_0^x (x-t)^{\gamma-1}y(t)dt$$

ko‘rinishda aniqlanadi.

A **masala.** Shunday $y(x)$ funksiya topilsinki, u quyidagi xossalarga ega bo‘lsin:

1) $x^{(1-\beta)(1-\alpha)}y(x)\in C[0,1]$, $D_{0x}^{\alpha,\beta}y(x)\in C(0,1)$ sinfga tegishli;

2) $y(x)$ funksiya (1) tenglamani qanoatlantirsin;

3) x=0 nuqtada esa

$$\lim_{x\to0} I_{0x}^{(1-\alpha)(1-\beta)}y(x)=A \qquad (3)$$

shartni qanoatlantirsin, bu yerda, A - berilgan o‘zgarmas haqiqiy son.

(1) tenglamani

$$I_{0x}^{\beta(1-\alpha)}\frac{d}{dx}I_{0x}^{(1-\beta)(1-\alpha)}y(x)-\lambda I_{0x}^{\gamma}y(x)=f(x) \qquad (4)$$

ko‘rinishda yozib olamiz.

(4) tenglamaga $D_{0x}^{\beta(1-\alpha)}$ ni ta'sir ettirib,

$$D_{0x}^{\beta(1-\alpha)}I_{0x}^{\gamma}y(x)=\frac{1}{\Gamma(1-\beta(1-\alpha)+\gamma)}\frac{d}{dx}\int_0^x (x-z)^{\gamma-\beta(1-\alpha)}y(z)dz \qquad (5)$$

$$D_{0x}^{\beta(1-\alpha)}y(x)=\frac{1}{\Gamma(1-\beta(1-\alpha))}\frac{d}{dx}\int_0^x (x-z)^{-\beta(1-\alpha)}y(z)dz \qquad (6)$$

dan foydalanib,uni quyidagicha yozib olamiz;

$$\frac{d}{dx}I_{0x}^{(1-\beta)(1-\alpha)}y(x)-\frac{\lambda}{\Gamma(1-\beta(1-\alpha)+\gamma)}\frac{d}{dx}\int_0^x (x-z)^{\gamma-\beta(1-\alpha)}y(z)dz=$$

$$=\frac{1}{\Gamma(1-\beta(1-\alpha))}\frac{d}{dx}\int_0^x (x-z)^{-\beta(1-\alpha)}y(z)dz \qquad (7)$$

(7) ni 0 dan x gacha integrallab, uni quyidagicha yozib olamiz:

$$I_{0x}^{(1-\beta)(1-\alpha)}y(x)-\frac{\lambda}{\Gamma(1-\beta(1-\alpha)+\gamma)}\int_0^x (x-z)^{\gamma-\beta(1-\alpha)}y(z)dz=$$

$$=\frac{1}{\Gamma(1-\beta(1-\alpha))}\int_0^x (x-z)^{-\beta(1-\alpha)}y(z)dz+A \qquad (8)$$

ga $D_{0x}^{(1-\alpha)(1-\beta)}$ ni ta’sir ettirib, ba'zi hisoblashlarni amalga oshirib, quyidagi

$$y(x)-\frac{\lambda}{\Gamma(\alpha+\gamma)}\int_0^x (x-z)^{\alpha+\gamma-1}y(z)dz=\frac{1}{\Gamma(\alpha)}\int_0^x (x-z)^{\alpha-1}f(z)dz+\frac{Ax^{-(1-\alpha)(1-\beta)}}{\Gamma(1-(1-\alpha)(1-\beta))} \quad (9)$$

ko'rinishdagi ikkinchi tur Volterra integral tenglamasini hosil qilamiz.

(9) ni yechish uchun ba'zi belgilashlarni kiritamiz:

$$g(x)=\frac{1}{\Gamma(\alpha)}\int_0^x (x-z)^{\alpha-1}f(z)dz+\frac{Ax^{-(1-\alpha)(1-\beta)}}{\Gamma(1-(1-\alpha)(1-\beta))},\ K(x,z)=\frac{(x-z)^{\alpha+\gamma-1}}{\Gamma(\alpha+\gamma)} \quad (10)$$

(9) tenglamani ketma-ket yaqinlashish usuli orqali yechamiz.

$$K_1(x,z)=\frac{(x-z)^{\alpha+\gamma-1}}{\Gamma(\alpha+\gamma)} \quad \text{va} \quad K_i(x,y)=\int_y^x K_1(x,t)K_{i-1}(t,y)dt$$

formulalardan foydalanib,ba'zi hisoblashlarni amalga oshirib, n- iteratsionalgan yadroni

$$K_n(x,z)=\frac{(x-z)^{n(\alpha+\gamma)-1}}{\Gamma(n(\alpha+\gamma))}$$

ko'rinishda topamiz. $K_n(x,z)$ orqali $R(x,z,\lambda)$ rezolventani

$$R(x,z,\lambda)=\sum_{n=1}^{+\infty}\frac{\lambda^{n-1}(x-z)^{n(\alpha+\gamma)-1}}{\Gamma(n(\alpha+\gamma))}$$

ko'rinishda topamiz.

Integral tenglamalar nazariyasiga ko'ra (9) tenglamani yechimini,

$$y(x)=g(x)-\lambda\int_0^x R(x,z,\lambda)g(z)dz$$

ko'rinishda yozib olamiz, bu yerda $R(x,z,\lambda)=\sum_{n=1}^{+\infty}\frac{\lambda^{n-1}(x-z)^{n(\alpha+\gamma)-1}}{\Gamma(m(\alpha+\gamma))}$.

(10) belgilashlarni e'tiborga olib, ba'zi hisoblashlarni amalga oshirib, A masalaning yechimini

$$y(x)=Ax^{-(1-\alpha)(1-\beta)}E_{\alpha+\gamma,1-(1-\alpha)(1-\beta)}\left(\lambda x^{\alpha+\beta}\right)+\int_0^x (x-z)^{\alpha-1}f(z)E_{\alpha+\gamma,\alpha}\left[\lambda(x-z)^{\alpha+\gamma}\right]dz$$

formal yechimini ko'rinishda topamiz, bu yerda

$$E_{p,q}(z)=\sum_{n=0}^{+\infty}\frac{z^n}{(pn+q)} \quad (11)$$

Mittag-Leffer funksiyasi [9].

Endi

$$D_{0x}^{\alpha,\beta}y(x)-\lambda I_{0x}^{\gamma}y(x)=y(x_0) \quad (12)$$

yuklangan kasr tartibli oddiy differensial tenglamani qaraylik.

B **masala.** Shunday $y(x)$ funksiya topilsinki, u quyidagi xossalarga ega bo'lsin:

1) $x^{(1-\beta)(1-\alpha)}y(x)\in C[0,1]$, $D_{0x}^{\alpha,\beta}y(x)\in C(0,1)$ sinfga tegishli

2) $y(x)$ funksiya (1) tenlamani qanoatlantirsin;

3) x=0 nuqtada esa

$$\lim_{x\to 0} I_{0x}^{(1-\alpha)(1-\beta)} y(x) = A$$

shartni qanoatlantirsin, bu yerda, A - berilgan o‘zgarmas haqiqiy son.

$y(x_0)$ ni vaqtincha ma’lum deb, A masalaning yechimidan foydalanib, B masalaning yechimini

$$y(x) = Ax^{-(1-\alpha)(1-\beta)} E_{\alpha+\gamma,1-(1-\alpha)(1-\beta)}\left(\lambda x^{\alpha+\beta}\right) + y(x_0) x^{\alpha} E_{\alpha+\gamma,\alpha+1}(\lambda x^{\alpha+\gamma}) \qquad (13)$$

ko‘rinishda yozib olamiz.

(13) dan $x = x_0$ deb $y(x_0)$ ni

$$y(x_0) = \frac{A x_0^{-(1-\alpha)(1-\beta)} E_{\alpha+\gamma,1-(1-\alpha)(1-\beta)}\left(\lambda x_0^{\alpha+\beta}\right)}{1 - x_0^{\alpha} E_{\alpha+\gamma,\alpha+1}(\lambda x_0^{\alpha+\gamma})} \qquad (14)$$

ko‘rinishda topamiz

(14) ni (13) ga olib borib qo‘yib, B masalaning yechimini

$$y(x) = Ax^{-(1-\alpha)(1-\beta)} E_{\alpha+\gamma,1-(1-\alpha)(1-\beta)}\left(\lambda x^{\alpha+\beta}\right) + \frac{A x_0^{-(1-\alpha)(1-\beta)} E_{\alpha+\gamma,1-(1-\alpha)(1-\beta)}\left(\lambda x_0^{\alpha+\beta}\right)}{1 - x_0^{\alpha} E_{\alpha+\gamma,\alpha+1}(\lambda x_0^{\alpha+\gamma})} x^{\alpha} E_{\alpha+\gamma,\alpha+1}(\lambda x^{\alpha+\gamma}) \qquad (15)$$

ko‘rinishda topamiz.

Teorema. Agar $x_0^{\alpha} E_{\alpha+\gamma,\alpha+1}(\lambda x_0^{\alpha+\gamma}) \neq 1$ bo‘lsa ,u holda, B masala yagona yechimga ega bo‘lib, u (15) formula bilan aniqlanadi.

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