UCH O‘ZGARUVCHI GIPERGEOMETRIK FUNKSIYASINI REKURSIYA FORMULALARI.

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Annotatsiya. *Avvalroq ikki o’zgaruvchili Appell gipergeometrik funksiyalari uchun rekursiv formulalar X.Wang [1] tomonidan topilgan edi. X.Wangning ishlaridan ruhlanib, uch o’zgaruvchili* $F_A^{(3)}$ *Laurichella gipergeometrik funksiyasi sonli parametrlaridan birortasining o’zgarishini huddi shu funksiyalarning chekli yig’indisi orqali ifodalash imkonini beradigan rekursiv formulalar topilgan.*

Kalit so’zlar: *Laurichella funksiyasi; rekursiv formula;Poxgammer simvoli*

Annotation. *Inspired by the recent work of X.Wang [1], who gave the recursion formulas for Appell’s functions in two variables, we establish the recursion formulas for Laurichella function* $F_A^{(3)}$ *by the contiguous relations of hypergeometric series.*

Keywords: *Lauchella function; recursion formulas Poxgammer symbol*

Аннотация. *Вдохновляясь недавней работой Х. Ванга [1, 422], который дал рекурсивные формулы для функций Аппеля от двух переменных, мы устанавливаем рекурсивные формулы для функции Лауричелла* $F_A^{(3)}$ *по смежным соотношениям гипергеометрических рядов.*

Ключевые слова: *функция Лаучеллы; рекурсивные формулы; символ Поксгаммера*

Quyidagi Poxgammer simvolini tengliklaridan foydalandik.

$$(a+m)_k=\frac{(a+k)_m(a)_k}{(a)_m},$$

$$(a-m)_k=\frac{(1-a)_m(a)_k}{(1-a-k)_m}\qquad k,m\in N;$$

$$(a)_m=a(a+1)(a+2)\ldots(a+m-1);$$

$$F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\c_1,c_2,c_3;\end{matrix}X\right]=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p}\frac{x^my^nz^p}{m!n!p!},$$

$$F_B^{(3)}\left[\begin{matrix}a_1,a_2,a_3;b_1,b_2,b_3;\\c;\end{matrix}X\right]=\sum_{m,n,p=0}^{\infty}\frac{(a)_m(a_2)_n(a_3)_p(b_1)_m(b_2)_n(b_3)_p}{(c)_{m+n+p}}\frac{x^my^nz^p}{m!n!p!},$$

$$F_C^{(3)}\left[\begin{matrix}a;b;\\c_1,c_2,c_3;\end{matrix}X\right]=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b)_{m+n+p}}{(c_1)_m(c_2)_n(c_3)_p}\frac{x^my^nz^p}{m!n!p!},$$

$$F_D^{(3)}\left[\begin{matrix} a;b_1,b_2,b_3; \\ c; \end{matrix} X\right]=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c)_{m+n+p}}\frac{x^m y^n z^p}{m!n!p!},$$

Quyidagi uch o‘lchovli Laruchella gipergeometrik funksiyani qaraylik.

$$F_A^{(3)}\left[\begin{matrix} a;b_1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p}\frac{x^m y^n z^p}{m!n!p!}, \quad (1)$$

bu yerda $X:=(x,y,z)$ uch o’lchovli funksiyalar $a,b_1,b_2,b_3,c_1,c_2,c_3$ berilgan sonli paramertlar, $(\lambda)_0=1;\ (\lambda)_\nu=\lambda(\lambda+1)(\lambda+2)\ldots(\lambda+\nu-1);\quad \lambda=a,b,c;\quad \nu=1,2,3\ldots.$

Laruchella gipergeometrik funksiyasi $F_A^{(3)}$ ning sonli parametrlaridan birortasining o’zgarishini xuddi shu funksiyalarning chekli yig’indisi orqali ifodalash imkonini beradigan rekursiv formulalar topilgan.

Biz Laruchella funksiyasining ikkinchi parametr b_1 bo’yicha rekursiya formulalarini o’rganamiz.

1-teorema. Quyidagi tenglik o‘rinli:

$$F_A^{(3)}\left[\begin{matrix} a;b_1+n,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]=F_A^{(3)}\left[\begin{matrix} a;b_1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]+$$

$$+\frac{ax}{c_1}\sum_{k=1}^{n}F_A^{(3)}\left[\begin{matrix} a+1;b_1+k,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right], \quad (2)$$

$$F_A^{(3)}\left[\begin{matrix} a;b_1-n,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]=F_A^{(3)}\left[\begin{matrix} a;b_1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]-$$

$$-\frac{ax}{c_1}\sum_{k=0}^{n-1}F_A^{(3)}\left[\begin{matrix} a+1;b_1-k,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]; \quad (3)$$

Isbot. Dastlab, (2) ni $n=1$ uchun isbotlaylik.Bu holda Laruchella funksiyasining (1) ta’rifi va

$$(b_1+1)_m=(b_1)_m\left(1+\frac{m}{b_1}\right);$$

tenglikni e’tiborga olsak,quyidagiga ega bo’lamiz:

$$F_A^{(3)}\left[\begin{matrix} a;b_1+1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1+1)_m(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p}\frac{x^m y^n z^p}{m!n!p!}=$$

$$=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m\left(1+\frac{m}{b_1}\right)(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p}\frac{x^m y^n z^p}{m!n!p!}=$$

$$=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p}\frac{x^m y^n z^p}{m!n!p!}+\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m\frac{m}{b_1}(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p}\frac{x^m y^n z^p}{m!n!p!}=$$

$$F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\ c_1,c_2,c_3;\end{matrix}X\right]+\sum_{\substack{n,p=0\\ m=1}}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p b_1}\frac{x^m y^n z^p}{(m-1)!n!p!}=$$

$$=F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\ c_1,c_2,c_3;\end{matrix}X\right]+\sum_{\substack{n,p=0\\ m=1}}^{\infty}\frac{(a)_{m+n+p}\,b_1(b_1+1)(b_1+2)(b_1+m-1)(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p\,b_1}\frac{x^m y^n z^p}{(m-1)!n!p!};$$

$m-1$ ni m bilan almashtirib, (2) formulaning $n=1$ uchun isbotiga ega bo’lamiz:

$$F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\ c_1,c_2,c_3;\end{matrix}X\right]+\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+1+n+p}(b_1+1)(b_1+2)(b_1+m)(b_2)_n(b_3)_p}{(c_1)_{m+1}(c_2)_n(c_3)_p}\frac{x^{m+1}y^n z^p}{m!n!p!}=$$

$$=F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\ c_1,c_2,c_3;\end{matrix}X\right]+$$

$$+\sum_{m,n,p=0}^{\infty}\frac{a(a+1)(a+2)\ldots(a+m+n+p)(b_1+1)(b_1+2)(b_1+m)(b_2)_n(b_3)_p}{c_1(c_1+1)(c_1+2)\ldots(c_1+m)(c_2)_n(c_3)_p}\frac{x^{m+1}y^n z^p}{m!n!p!}=$$

$$=F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\ c_1,c_2,c_3;\end{matrix}X\right]+\frac{ax}{c_1}F_A^{(3)}\left[\begin{matrix}a+1;b_1+1,b_2,b_3;\\ c_1+1,c_2,c_3;\end{matrix}X\right];$$

$$F_A^{(3)}\left[\begin{matrix}a;b_1+1,b_2,b_3;\\ c_1,c_2,c_3;\end{matrix}X\right]=F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\ c_1,c_2,c_3;\end{matrix}X\right]+\frac{ax}{c_1}F_A^{(3)}\left[\begin{matrix}a+1;b_1+1,b_2,b_3;\\ c_1+1,c_2,c_3;\end{matrix}X\right];$$

(4) Endi (2) munosabatni $n=2$ uchun ko’rsatamiz. (4) tenglikni e’tiborga olsak, quyidagiga ega bo‘lamiz:

$$F_A^{(3)}\left[\begin{matrix}a;b_1+2,b_2,b_3;\\ c_1,c_2,c_3;\end{matrix}X\right]$$

$$=F_A^{(3)}\left[\begin{matrix}a;b_1+1,b_2,b_3;\\ c_1,c_2,c_3;\end{matrix}X\right]+\frac{ax}{c_1}F_A^{(3)}\left[\begin{matrix}a+1;b_1+2,b_2,b_3;\\ c_1+1,c_2,c_3;\end{matrix}X\right]=$$

$$=F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\ c_1,c_2,c_3;\end{matrix}X\right]+\frac{ax}{c_1}F_A^{(3)}\left[\begin{matrix}a+1;b_1+1,b_2,b_3;\\ c_1+1,c_2,c_3;\end{matrix}X\right]+\frac{ax}{c_1}F_A^{(3)}\left[\begin{matrix}a+1;b_1+2,b_2,b_3;\\ c_1+1,c_2,c_3;\end{matrix}X\right]=$$

$$F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\ c_1,c_2,c_3;\end{matrix}X\right]+\frac{ax}{c_1}\left\{F_A^{(3)}\left[\begin{matrix}a+1;b_1+1,b_2,b_3;\\ c_1+1,c_2,c_3;\end{matrix}X\right]+F_A^{(3)}\left[\begin{matrix}a+1;b_1+2,b_2,b_3;\\ c_1+1,c_2,c_3;\end{matrix}X\right]\right\};$$

Yuqoridagi kabi $F_A^{(3)}$ Loruchella funksiyasi parametr b_1+n bo‘lganda

(4) munosabatni n marta qo‘llab, (2) formulani isbotiga ega bo‘lamiz.

$$F_A^{(3)}\left[\begin{matrix} a;b_1+n,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]=$$

$$=F_A^{(3)}\left[\begin{matrix} a;b_1+n-1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]+\frac{ax}{c_1}F_A^{(3)}\left[\begin{matrix} a+1;b_1+n,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]=$$

$$=F_A^{(3)}\left[\begin{matrix} a;b_1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]+$$

$$\frac{ax}{c_1}\left\{F_A^{(3)}\left[\begin{matrix} a+1;b_1+1,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]+F_A^{(3)}\left[\begin{matrix} a+1;b_1+2,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]+\ldots+F_A^{(3)}\left[\begin{matrix} a+1;b_1+n,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]\right\}=$$

$$=F_A^{(3)}\left[\begin{matrix} a;b_1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]+\frac{ax}{c_1}\sum_{k=1}^{n}F_A^{(3)}\left[\begin{matrix} a+1;b_1+k,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right];$$

Laruchella funksiyasining ta’rifidan $F_A^{(3)}$ funksiyasini Poxgammer simvolining xossalaridan foydalanib (3) munosabatni $n=1$ uchun isbotlaymiz.

$$(b_1-1)_m=\frac{(1-b_1)(b_1)_m}{(1-b_1-m)}=(b_1)_m\left(1+\frac{m}{1-b_1-m}\right);$$

tenglikni e’tiborga olib, quyidagiga ega bo‘lamiz.

$$F_A^{(3)}\left[\begin{matrix} a;b_1-1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1-1)_m(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p}\frac{x^m y^n z^p}{m!n!p!}=$$

$$=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m\left(1+\dfrac{m}{1-b_1-m}\right)(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p}\frac{x^m y^n z^p}{m!n!p!}=$$

$$=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p}\frac{x^m y^n z^p}{m!n!p!}+$$

$$=F_A^{(3)}\left[\begin{matrix} a;b_1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]-$$

$$-\sum_{\substack{n,p=0\\m=1}}^{\infty}\frac{(a)_{m+n+p}(b_1+1)(b_1+2)\ldots(b_1+m-1)(b_2)_n(b_3)_p}{(m+b_1-1)(c_1)_m(c_2)_n(c_3)_p}\frac{x^m y^n z^p}{(m-1)!n!p!}=$$

$$+\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m\dfrac{m}{1-b_1-m}(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p}\frac{x^m y^n z^p}{m!n!p!}=$$

$$F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\c_1,c_2,c_3;\end{matrix}X\right]-\sum_{\substack{n,p=0\\m=1}}^{\infty}\frac{(a)_{m+n+p}(b_1)_{m-1}(b_2)_n(b_3)_p}{(c_1)_m(c_2)_n(c_3)_p b_1}\frac{x^m y^n z^p}{(m-1)!n!p!};$$

$m-1$ ni m bilan almashtirib, (2) formulaning $n=1$ uchun isbotiga ega bo’lamiz:

$$F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\c_1,c_2,c_3;\end{matrix}X\right]-$$

$$-\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+1+n+p}b_1(b_1+1)(b_1+2)(b_1+m-1)(b_2)_n(b_3)_p}{(c_1)_{m+1}(c_2)_n(c_3)_p}\frac{x^{m+1}y^n z^p}{m!n!p!}=$$

$$=F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\c_1,c_2,c_3;\end{matrix}X\right]-$$

$$-\sum_{m,n,p=0}^{\infty}\frac{a(a+1)(a+2)\ldots(a+m+n+p)\,b_1(b_1+1)(b_1+2)(b_1+m-1)(b_2)_n(b_3)_p}{c_1(c_1+1)(c_1+2)\ldots(c_1+m)(c_2)_n(c_3)_p}\frac{x^{m+1}y^n z^p}{m!n!p!}=$$

$$=F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\c_1,c_2,c_3;\end{matrix}X\right]-\frac{ax}{c_1}F_A^{(3)}\left[\begin{matrix}a+1;b_1,b_2,b_3;\\c_1+1,c_2,c_3;\end{matrix}X\right];$$

$$F_A^{(3)}\left[\begin{matrix}a;b_1-1,b_2,b_3;\\c_1,c_2,c_3;\end{matrix}X\right]=F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\c_1,c_2,c_3;\end{matrix}X\right]-\frac{ax}{c_1}F_A^{(3)}\left[\begin{matrix}a+1;b_1,b_2,b_3;\\c_1+1,c_2,c_3;\end{matrix}X\right];$$

(5)

Endi (2) munosabatni $n=2$ uchun ko’rsatamiz. (5) tenglikni e’tiborga olsak, quyidagiga ega bo‘lamiz:

$$F_A^{(3)}\left[\begin{matrix} a;b_1-2,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]$$

$$=F_A^{(3)}\left[\begin{matrix} a;b_1-1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]-\frac{ax}{c_1}F_A^{(3)}\left[\begin{matrix} a+1;b_1-1,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]=$$

$$=F_A^{(3)}\left[\begin{matrix} a;b_1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]-\frac{ax}{c_1}F_A^{(3)}\left[\begin{matrix} a+1;b_1,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]-\frac{ax}{c_1}F_A^{(3)}\left[\begin{matrix} a+1;b_1-1,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]=$$

$$F_A^{(3)}\left[\begin{matrix} a;b_1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]-\frac{ax}{c_1}\left\{F_A^{(3)}\left[\begin{matrix} a+1;b_1,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]+F_A^{(3)}\left[\begin{matrix} a+1;b_1-1,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]\right\};$$

Bu munosabatni $F_A^{(3)}$ Loruchella funksiyani parametrni b_1-n ni (5) munosabati bilan n marta qo‘llab, bu teoremadan (2) formulani isbotlashda qilganimiz kabi (3) rekursiya formulani olamiz.

$$F_A^{(3)}\left[\begin{matrix} a;b_1-n,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]=$$

$$=F_A^{(3)}\left[\begin{matrix} a;b_1-n-1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]-\frac{ax}{c_1}F_A^{(3)}\left[\begin{matrix} a+1;b_1-n-1,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]=$$

$$=F_A^{(3)}\left[\begin{matrix} a;b_1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]-$$

$$\frac{ax}{c_1}\left\{F_A^{(3)}\left[\begin{matrix} a+1;b_1,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]+F_A^{(3)}\left[\begin{matrix} a+1;b_1-1,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]+\ldots+F_A^{(3)}\left[\begin{matrix} a+1;b_1-n,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right]\right\}=$$

$$=F_A^{(3)}\left[\begin{matrix} a;b_1,b_2,b_3; \\ c_1,c_2,c_3; \end{matrix} X\right]-\frac{ax}{c_1}\sum_{k=0}^{n-1}F_A^{(3)}\left[\begin{matrix} a+1;b_1-k,b_2,b_3; \\ c_1+1,c_2,c_3; \end{matrix} X\right];$$