TO‘RINCHI TARTIBLI INTEGRO-DIFFERENSIAL TENGLAMA UCHUN TO‘G‘RI VA TESKARI MASALA.

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So`ngi vaqtlarda noma`lum manbali differensial tengalamalar bilan shug`illanishga bo`lgan qiziqish ortib bormoqda. Bunga sabab ko`plab issiqlik taqalish va diffuziya jarayonlarini matematik modelini tuzish noma'lum manbali differensial tenglama uchun qo`yiladigan masalalarga keltiriladi. Odatda, bunday differensial tenglamalar uchun teskari masalalar ko`plab tadqiqotchilar tomonidan o`rganilgan (masalan, ushbu $[1]-[6]$ ishlarga qaralsin). Ammo yuqori tartibli tenglamalar uchun teskari masalalar kam o`rganilgan. Shu sababdan biz ushbu ishda to`rtinchi tartibli oddiy dierensial tenglama uchun bir teskari masalani bir qiymatli yechilishini ko`rsatamiz.

(0, 1) oraliqda ushbu

$$y^{(4)}(x)-\lambda I_{0x}^{\gamma}y(x)=f(x) \qquad (1)$$

to`rtinchi tartibli integro-differensial tenglamani qaraylik, bu yerda $y(x)-$ noma'lum funksiya; $I_{0x}^{\gamma}y(x)$ - Riman-Liuvill ma'nosida γ (kasr) tartibli integral; $f(x)-$ berilgan uzluksiz funksiya; λ,γ - o‘zgarmas haqiqiy sonlar bo'lib, $0<\gamma<1$:

$$I_{0x}^{\gamma}y(x)=\frac{1}{\Gamma(\gamma)}\int_0^x (x-t)^{\gamma-1}y(t)dt\,,\ x>0$$

A masala. Shunday $y(x)$ funksiya topilsinki, u quyidagi xossalarga ega bo'lsin:

1) (0, 1) oraliqda (1) tenglamani qanoatlantirsin;

2) $C^3[0,1]\text{I}\ C^3(0,4)$ sinfga tegishli bo‘lsin;

3) $x=0$ nuqtada esa

$$y(0)=A_1,\ y'(0)=A_2,\ y''(0)=A_3,\ y'''(0)=A_4 \qquad (2)$$

boshlang‘ich shartlarni qanoatlantirsin, bu yerda A_1,A_2,A_3,A_4 - berilgan o'zgarmas haqiqiy sonlar.

(1) tenglamani ketma-ket 4 marta integrallab, (2) shartdan foydalanib,

$$y(x)-\frac{\lambda}{\Gamma(\gamma+4)}\int_0^x (x-t)^{\gamma+4-1}y(t)dt=A_1+A_2x+A_3\frac{x^2}{2}+A_4\frac{x^3}{6}+\int_0^x\frac{(x-z)^3}{6}f(z)dz \qquad (3)$$

ko‘rinishdagi integral tenglamani hosil qilamiz.

(3) da ba’zi belgilashlarni kiritib, uni ko‘rinishda yozib olamiz, bu yerda

$$g(x)=A_1+A_2x+A_3\frac{x^2}{2}+A_4\frac{x^3}{6}+\int_0^x\frac{(x-z)^3}{6}f(z)dz\,,$$

$\beta=\gamma+4$, $K(x,t)=\dfrac{(x-t)^{\beta-1}}{\Gamma(\beta)}$

$$y(x)-\lambda\int_a^x K(x,t)y(t)dt=g(x) \qquad (4)$$

(4) Volltera ikkinchi tur tenglamasi bo‘lib, uni krtma-ket yaqinlashish usuli yordamida yechamiz.

$$K_1(x,t)=\frac{(x-t)^{\beta-1}}{\Gamma(\beta)} \text{ va } K_i(x,y)=\int_y^x K_1(x,t)K_{i-1}(t,y)dt$$

rekurent formuladan foydalanib, ba’zi hisoblashlarni amalga oshirib $K_n(x,t)=\dfrac{(x-t)^{n\beta-1}}{\Gamma(n\beta)}$, $n=1,2,3,\ldots$ topamiz.

Integral tenglamalar nazariyasiga ko‘ra, (4) tenglamani yechimi

$$y(x)=g(x)+\lambda\int_a^x R(x,t,\lambda)g(t)dt \qquad (5)$$

ko‘rinishda yozib olamiz, bu yerda

$$R(x,t;\lambda)=\sum_{n=1}^{\infty}\lambda^{n-1}K_n(x,t) \quad R(x,t,\lambda)=\sum_{n=1}^{\infty}\frac{\lambda^{n-1}(x-t)^{\beta n-1}}{\Gamma(\beta m)}$$

(5) da $g(x)=A_1+A_2x+A_3\dfrac{x^2}{2}+A_4\dfrac{x^3}{6}+\int_0^x\dfrac{(x-z)^3}{6}f(z)dz$ ekanligi e’tiborga olib, ba’zi soddalastirishlarni amalga oshirib A masalaning yechimini

$$y(x)=A_1E_{\beta,1}(\lambda x^\beta)+A_2E_{\beta,2}(\lambda x^\beta)+A_3E_{\beta,3}(\lambda x^\beta)+A_4E_{\beta,4}(\lambda x^\beta)+\int_0^x(x-z)^3E_{\beta,4}\left[\lambda(x-z)^\beta\right]f(z)dz$$

ko‘rinishda topamiz.

Endi,

$$y^{(4)}(x)-\lambda I_{0x}^{\gamma}y(x)=kf(x) \qquad (7)$$

tenglamani $(0,1)$ oraliqda qaraylik, bu yerda $y(x)$-no’malum funksiya, λ,γ-o‘zgarmas haqiqiy sonlar, $f(x)$-berilgan funksiya, k-no’malum son.

T masala Shunday $y(x)$-funksiya va k sonni topilsinki u quyidagi xossalarga ega bo‘lsin:

1) (0, 1) oraliqda (7) tenglamani qanoatlantirsin;

2) $C^3[0,1]$I $C^3(0,4)$ sinfga tegishli bo‘lsin;

3) $x=0$ nuqtada esa

$$y(0)=A_1,\ y'(0)=A_2,\ y''(0)=A_3,\ y'''(0)=A_4 \qquad (8)$$

$$y(1)=B \qquad (9)$$

boshlang‘ich shartlarni qanoatlantirsin, bu yerda A_1, A_2, A_3, A_4 - berilgan o'zgarmas haqiqiy sonlar.

T masala yechimini A masala yechimidan foydalanib,

$$y(x) = A_1 E_{\beta,1}\left(\lambda x^{\beta}\right) + A_2 E_{\beta,2}\left(\lambda x^{\beta}\right) + A_3 E_{\beta,3}\left(\lambda x^{\beta}\right) + A_4 E_{\beta,4}\left(\lambda x^{\beta}\right) + k\int_0^x (x-z)^3 E_{\beta,4}\left[\lambda (x-z)^{\beta}\right] f(z)\,dz$$

yozib olamiz.

(9) tenglikdan k

$$k = \frac{B - A_1 E_{\beta,1}(\lambda) - A_2 E_{\beta,2}(\lambda) - A_3 E_{\beta,3}(\lambda) - A_4 E_{\beta,4}(\lambda)}{\int_0^1 (1-z)^3 E_{\beta,4}(\lambda) f(z)\,dz} \qquad (10)$$

ko‘rinishda bo‘ladi.

Teorema. Agar $\int_0^1 (1-z)^3 E_{\beta,4}(\lambda) f(z)\,dz \neq 0$ bo‘lsa, u holda (7) tenglama yagona yechimga ega bo‘ladi.

1-izoh: $B - A_1 E_{\beta,1}(\lambda) - A_2 E_{\beta,2}(\lambda) - A_3 E_{\beta,3}(\lambda) - A_4 E_{\beta,4}(\lambda) = 0$, $\int_0^1 (1-z)^3 E_{\beta,4}(\lambda) f(z)\,dz = 0$ bo‘lsa, u holda (7) tenglama cheksiz ko‘p yechimga ega bo‘ladi.

2-izoh: $B - A_1 E_{\beta,1}(\lambda) - A_2 E_{\beta,2}(\lambda) - A_3 E_{\beta,3}(\lambda) - A_4 E_{\beta,4}(\lambda) \neq 0$, $\int_0^1 (1-z)^3 E_{\beta,4}(\lambda) f(z)\,dz = 0$ bo‘lsa, u holda (7) tenglama yechimga ega bo‘lmaydi.

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