YUKLANGAN KASR TARTIBLI INTEGRO-DIFFERENSIAL TENGLAMA UCHUN MASALA

Omonova Dinora Dilshodjon qizi
Farg`ona davlat universiteti talabasi.

Annotatsiya. *Ushbu maqolada Riman-Liuvill kasr tartibli operator qatnashgan yuklangan tenglama uchun masala o`rganilgan. Bu masalalar yechimlari Koshi masalasi yechimdan foydalanib topilgan.*

Kalit so`zlar: *yuklangan integro-differensial tenglama, kasr tartibli operator, Koshi masalasi.*

ЗАДАЧА ДЛЯ НАГРУЖЕННЫХ ИНТЕГРО- ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИА ДРОБНОГО ПОРЯДКА

Аннотация: *В данной статье изучались две задачи для нагруженного уравнения с дробным оператором по Риман-Лиувилл Найдены решения этих задач с использованием решения задачи Коши.*

Ключевые слова: *нагруженное интегро-дифференциальное уравнение, оператор дробного порядка, задача Коши.*

PROBLEM FOR LOADED INTEGRO-DIFFERENTIAL EQUATION OF FRACTIONAL ORDER

Abstract: *In this article,the problem for the loaded equation involving the Riman-Liuvill fractional operator is studied.The solution of these problems are found using the solution of the Cauchy problem.*

Keywords: *loaded integro-differential equation, fractional order operator, Cauchy problem.*

I. Kirish. So‘ngi vaqtlarda noma`lum funksiyani biror qiymati qatnashgan differensial tengalamalar bilan shug‘ullanishga bo`lgan qiziqish ortib bormoqda. Bunga sabab ko`plab issiqlik tarqalish va diffuziya jarayonlarini matematik modelini tuzish funksiyani biror qiymati qatnashgan differensial tenglama uchun qo‘yiladigan masalalarga keltiriladi. Odatda, bunday turdagi tenglamalar yuklangan differensial tenglama deb yuritiladi. Yuklangan xususiy hosilali va oddiy differensial tenglamalar yuklangan differensial tenglama ko`plab tadqiqotchilar tomonidan o‘rganilgan (masalan, ushbu [1]–[3] ishlarga qaralsin).

II. Masalaning qo‘yilishi va tadqiqoti.

(0,1) oraliqda ushbu

$$D_{0x}^{\alpha} y(x) - \lambda I_{0x}^{\gamma} y(x) = f(x) \qquad (1)$$

kasr tartibli integro-differensial tenglamani qaraylik, bu yerda $y(x)$- noma'lum funksiya; α,γ,λ- o'zgarmas haqiqiy sonlar bo'lib, $\gamma>0$; $D_{0x}^{\alpha}y(x)$ - Riman-Liuvill ma'nosida α (kasr) tartibli hosila, $I_{0x}^{\gamma}y(x)$- Riman-Liuvill ma'nosida γ (kasr) tartibli integral:

$$D_{0x}^{\alpha}y(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int\frac{y(t)}{(x-t)^{\alpha}}dt$$

$$I_{0x}^{\gamma}y(x)=\frac{1}{\Gamma(\gamma)}\int_0^x(x-t)^{\gamma-1}y(t)dt\,,x>0$$

A **masala.** Shunday $y(x)$- funksiya topilsinki, u quyidagi xossalarga ega bo'lsin:

1) (0,1) oraliqda (1) tenglamani qanoatlantirsin;

2) $x=0$ nuqtada esa

$$I_{0x}^{1-\alpha}y(x)|_{x=0}=B_0 \qquad (2)$$

shartni qanoatlantirsin;

bu yerda B_0 - berilgan o'zgarmas haqiqiy son.

(1) tenglamani har ikkala tarafini 0 dan x gacha integrallab (2) shartdan foydalanib,

$$I_{0x}^{1-\alpha}y(x)-\lambda I_{0x}^{1+\gamma}y(x)=\int_0^x f(\tau)d\tau+B \qquad (3)$$

ko'rinishdagi tenglamani hosil qilamiz.

$$D_{0x}^{1-\alpha}I_{0x}^{1-\alpha}y(x)=y(x) \qquad (4)$$

(3) tenglamaga $D_{0x}^{1-\alpha}$ ni ta'sir ettiramiz va (4) xossadan foydalanib,

$$y(x)-\lambda D_{0x}^{1-\alpha}I_{0x}^{1+\gamma}y(x)=D_{0x}^{1-\alpha}\left(\int_0^x f(\tau)d\tau+B\right) \qquad (5)$$

ko'rinishda yozib olamiz.

$\lambda D_{0x}^{1-\alpha}I_{0x}^{1+\gamma}y(x)$ ni yoyib soddalashtirib,

$$\lambda D_{0x}^{1-\alpha}I_{0x}^{1+\gamma}y(x)=\frac{\lambda}{\Gamma(\alpha+\gamma)}\int_0^x(x-z)^{\alpha+\gamma-1}y(z)dz \qquad (6)$$

tenglikni hosil qilamiz.

$D_{0x}^{1-\alpha}\left(\int_0^x f(\tau)d\tau+B\right)$ ni yoyib soddalashtirib,

$$D_{0x}^{1-\alpha}\left(\int_0^x f(\tau)d\tau+B\right)=\frac{x^{1-\alpha}B}{\Gamma(\alpha)}+I_{0x}^{\alpha}f(x) \qquad (7)$$

tenglikni hosil qilamiz.

(5) ga (6) va (7) ni olib borib qo'yamiz va yechimni

$$y(x)=Bx^{\alpha-1}E_{\alpha+\gamma,\alpha}\left(\lambda x^{\alpha+\gamma}\right)+\int_0^x (x-\tau)^{\alpha-1} f(\tau)E_{\alpha+\gamma,\alpha}\left(\lambda (x-\tau)^{\alpha+\gamma}\right)d\tau \qquad (8)$$

ko'rinishda aniqlaymiz, bu yerda,

$$E_{\eta,\xi}(z)=\sum_{k=0}^{+\infty}\frac{z^k}{\Gamma(\xi k+\eta)} \qquad (9)$$

(9) Mittag-Leffler funksiyasi

Endi

(0,1) oraliqda ushbu

$$D_{0x}^{\alpha}y(x)-\lambda I_{0x}^{\gamma}y(x)=y(x_0) \qquad (10)$$

yuklangan kasr tartibli, integro-differensial tenglamani qaraylik.

B **masala.** Shunday $y(x)$ funksiya topilsinki, u quyidagi xossalarga ega bo'lsin:

1) $y(x)$ funksiya (10) tenglamani qanoatlantirsin;

2) $x=0$ nuqtada esa

$$I_{0x}^{1-\alpha}y(x)|_{x=0}=B_0$$

shartni qanoatlantirsin;

bu yerda B_0 - berilgan o'zgarmas haqiqiy son.

$y(x_0)$ ni vaqtincha ma'lum deb, A masalaning yechimidan foydalanib,

$$y(x)=B_0x^{\alpha-1}E_{\alpha+\gamma,\alpha}\left(\lambda x^{\alpha+\gamma}\right)+y(x_0)x^{\alpha}E_{\alpha+\gamma,\alpha+1}\left(\lambda x^{\alpha+\gamma}\right) \qquad (11)$$

B masalaning yechimini (11) ko'rinishda aniqlaymiz.

(11) dan $x=x_0$ deb $y(x_0)$ ni topamiz:

$$y(x_0)=\frac{B_0x_0^{\alpha-1}E_{\alpha+\gamma,\alpha}\left(\lambda x_0^{\alpha+\gamma}\right)}{1-x_0^{\alpha}E_{\alpha+\gamma,\alpha}\left(\lambda x_0^{\alpha+\gamma}\right)} \qquad (12)$$

(12) ni (11) ga olib borib qo'yib, B masalaning yechimini yechimini

$$y(x)=B_0x^{\alpha-1}E_{\alpha+\gamma,\alpha}\left(\lambda x^{\alpha+\gamma}\right)+\frac{Bx_0^{\alpha-1}E_{\alpha+\gamma,\alpha}\left(\lambda x_0^{\alpha+\gamma}\right)}{1-x_0^{\alpha}E_{\alpha+\gamma,\alpha}\left(\lambda x_0^{\alpha+\gamma}\right)}x^{\alpha}E_{\alpha+\gamma,\alpha+1}\left(\lambda x^{\alpha+\gamma}\right) \qquad (13)$$

ko'rinishda aniqlaymiz.

2-teorema. Agar bo'lsa, $x_0^{\alpha}E_{\alpha+\gamma,\alpha}\left(\lambda x_0^{\alpha+\gamma}\right)\neq 1$ u holda B masala yagona yechimga ega bo'lib, u (13) formula bilan aniqlanadi.

FOYDALANILGAN ADABIYOTLAR:

1. Urinov A.K., Azizov M.S. Boun dary value problems for a fourth order partial differential equation with an unknown right-hand part. Lobochevskii Journal of Mathematics, vol. 42, pp.632-640. 2021

2. Tillabayeva G.I. Birinchi tartibli oddiy differensial tenglama uchun nolakal shartli masalalar. NamDU ilmiy axborotnomasi. 2020-yil 1-son 3-6 betlar.

3. Tillabayeva G.I. O’ng tomoni noma’lum va koeffisiyenti uzulishga ega bo’lgan birinchi tartibli chiziqli oddiy differensial tenglama uchun Bitsadze-Samariskiy masalas.i NamDU ilmiy axborotnomasi. 2020-yil 2-son 20-26 betlar.

4. 4. Omonova D.D. Yuklangan kasr tartibli oddiy differensial tenglamalar uchun masalalar._ Ijodkor o'qituvchi. Vol. 2 No. 24 (2022). 252- 255- betlar.