A PURSUIT PROBLEM IN DIFFERENTIAL GAME UNDER GEOMETRICAL AND GRONWALL TYPE CONSTRAINTS

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Annotatsiya. *Ushbu ma’ruzada differensial o‘yinlar nazariyasida Gronuoll tipidagi tengsizliklarni qo‘llanishi ko‘rilgan. Bunda quvlovchining boshqaruv funksiyasiga geometrik chegaralanish va qochuvchining boshqaruv funksiyasiga Gronuoll tipidagi chegaralanish qo‘yilgan holda chiziqli differensial o‘yinlarda tutish masalasi o‘rganiladi.*

Kalit so‘zlar. *Differensial o‘yin, Gronuoll tengsizligi, quvlovchi, qochuvchi, geometrik chegaralanish, qochish, strategiya.*

Аннотация. *В данной работе рассматривается неравенства Грануолла в теории дифференциальных игр. Здесь рассматриваются задача преследования для линейных дифференциальных игр, которая для игроков задаются преследователя геометрическая ограничения и убегавшего ограничения Грануолла.*

Ключевые слова. *Дифференциальные игры, неравенство Гронуолла, преследователь, убегающий, геометрическое ограничение, убегания, стратегия.*

Abstract. *The main aim of this work is to present some natural applications of Gronwall type inequalities in the Differential Games. We study the pursuit problem for linear differential games of the when geometrical constraint imposed on control function of pursuer and Gronwall type constraint imposed on control function of evader.*

Key words. *Differential game, Gronwall’s inequality, pursuer, evader, geometrical constraint, evasion, strategy.*

In this paper, we study the pursuit problem for linear differential game when the controls of players are subjected to differential constraints of the form of integral inequality of Gronwall [1]. The proposed method substantiates the parallel approach and the new sufficient solvability conditions are obtained for problem of the pursuit.

Let the motions of the pursuer P and the evader E be described by the following differential equations, initial conditions and Gronwall constraints (briefly, Gr-constraint)

$$P: \dot{x} + ax = u, \quad x(0) = x_0, \qquad (1)$$

$$E: \dot{y} + ay = v, \quad y(0) = y_0, \qquad (2)$$

where $x, y, u, v \in R^n$, $n \geq 2$, and a is arbitrary; u is the velocity vector of the pursuer and here the temporal variation of u must be a measurable function

$u(\cdot):[0,\infty)\to R^n$. We denote by U_G the set of all measurable functions $u(\cdot)$ satisfying Geometric constraint (briefly, G-constraint):

$$|u(t)|\le\alpha \qquad (3)$$

Similarly, v is the velocity vector of the evader and here the temporal variation of v must be a measurable function $v(\cdot):[0,+\infty)\to\mathbf{R}^n$. We denote by $\mathbf{V}_{Gr}$ the set of all measurable functions $v(\cdot)$ satisfying Gr-constraint:

$$|v(t)|^2\le\sigma^2+2k\int_0^t|v(s)|^2\,ds, \qquad (4)$$

where $\sigma\ne k$ and ρ,σ,k are nonnegative numbers.

By virtue of the equations (1)-(2) each pair of $(x_0,u(\cdot))$ and $(y_0,v(\cdot))$ generates the trajectories of motion

$$x(t)=e^{-at}\left(x_0+\int_0^t u(\tau)e^{a\tau}d\tau\right),\qquad y(t)=e^{-at}\left(y_0+\int_0^t v(\tau)e^{a\tau}d\tau\right).$$

The goal of the pursuer P is capture the evader E i.e., achievement of the equality

$$x(t)=y(t) \qquad (5)$$

and evader E strives to avoid an encounter.

Definition. In the pursuit problem (1)-(4), the function

$$u_{GGr}(t,v)=v-\lambda_{GGr}(t,v)\xi_0 \qquad (6)$$

is called Π-strategy of the pursuer, where $\lambda_{GGr}(t,v)=<v,\xi_0>+\sqrt{<v,\xi_0>^2+\alpha^2-|v|^2}$, $\xi_0=\frac{z_0}{|z_0|}$ and $<v,\xi_0>$ is a scalar product of the vectors v and ξ_0 in the space R^n.

Property 1. If $\alpha>\sigma$ when $t\in\left[0,\frac{1}{k}\ln\frac{\alpha}{\sigma}\right]$ then for all v, $|v|\le\sigma e^{kt}$ the function $\lambda_{GGr}(t,v)$ is defined and non-negative.

Property 2. The function $\lambda_{GGr}(t,v)$ is bounded for all v, $|v|\le\sigma e^{kt}$ that is the following relation is true:

$$\alpha-|v|\le\lambda_{GGr}(t,v)\le\alpha+|v|.$$

Theorem. Let one of the following conditions holds

$$-k<a<0,\quad |z_0|<\frac{\sigma}{k+a}-\frac{\alpha}{a};$$

$$a<-k,\quad |z_0|\ge\frac{\sigma}{k+a}-\frac{\alpha}{a};$$

$$a>0,\ |z_0|\le\frac{\sigma\left[k(\alpha/\sigma)^{(k+a)/k}+a\right]-\alpha(k+a)}{a};$$

$a>0, \quad |z_0| = \frac{\sigma}{k+a} - \frac{\alpha}{a}$.

Then Π_{GGr}-strategy (6) is winning for the player P on the interval $[0, T_{GGr}]$ in the game (1)-(4), where T_{GGr} is the smallest positive root of the equation:

$$\sigma a e^{(k+a)t} = \alpha(k+a)e^{at} + \sigma a - (k+a)(\alpha + a|z_0|).$$

REFERENCES:

1. Gronwall T.H. Note on the derivatives with respect to a parameter of the solutions of a system of differential equations. Ann. Math., 1919, 20(2): 293-296.
2. Azamov A.A. About the quality problem for the games of simple pursuit with the restriction, Serdika. Bulgarian math. spisanie, 12, 1986, - P.38-43.
3. Azamov A.A., Samatov B.T. П-Strategy. An Elementary introduction to the Theory of Differential Games. - T.: National Univ. of Uzb., 2000. - 32 p.
4. Azamov A.A., Samatov B.T. The П-Strategy: Analogies and Appli-cations, The Fourth International Conference Game Theory and Management, June 28-30, 2010, St. Petersburg, Russia, Collected papers. - P.33-47.
5. Samatov B.T., Soyibboyev U.B., Akbarov A.Kh. Evasion problem in linear differential game with Gronwall type constraint. Scientific Bulletin of Namangan State University 1(10), 25-33, 2019.