PARALLEL TIP O‘ZGARISH CHIZIG‘IGA EGA ARALASH TENGLAMA UCHUN INTEGRAL ULASH SHARTLI CHEGARAVIY MASALA

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Annotatsiya: *Ushbu ishda Riman-Liuvill kasr tartibli hosila ishtrok etgan aralash tenglama uchun aralash sohada umumiy integral shartli chegaraviy masalaning bir qiymatli yechilishi tadqiq qilinadi.*

Kalit so‘zlar: *Aralash tenglama, kasr tartibli hosila, integral ulash sharti.*

BOUNDARY PROBLEM WITH INTEGRAL GLUING CONDITION FOR MIXED EQUATION WITH PARALLEL LINE OF TYPE-CHANGING

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Abstract: *In this work a unique solvability of a boundary problem with general integral gluing condition for mixed equation involving the Riemann-Liouville fractional derivative has been proved.*

Keywords: *Mixed equation, fractional derivative, integral gluing condition.*

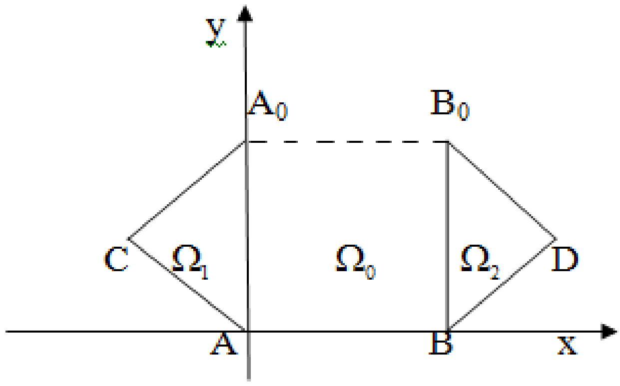
$$f(x,y)=\begin{cases} U_{xx}(x,y)-D_{0y}^{\alpha}U(x,y), & (x,y)\in\Omega_0 \\ U_{xx}(x,y)-U_{yy}(x,y), & (x,y)\in\Omega_i \quad (i=1,2) \end{cases} \tag{1}$$

tenglamani $\Omega=\Omega_0\cup\Omega_1\cup\Omega_2\cup AA_0\cup BB_0$ aralash sohada tadqiq qilamiz.

Bu yerda $f(x,y)$-berilgan funksiya , $D_{0y}^{\alpha}U$ esa α kasr tartibli Riman-Liuvill integro-differsial operatori bo‘lib, u $0<\alpha<1$ uchun quydagicha aniqlangan $[1]$:

$$D_{0y}^{\alpha}g(t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-z)^{-\alpha}g(z)dz.$$

(1) tenglama uchun Ω sohada



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quydagi masalani tadqiq etamiz:

1-Masala. (1) tenglamaning Ω sohada

$U(x,y)\in C(\bar{\Omega})\cap AC^1(\Omega_0)\cap C^2(\Omega_i),\quad U_{xx}\in C(\Omega_0)$

funksiyalar sinfiga tegishli quyidagi shartlarni qanoatlantiradigan regulyar yechimi topilsin:

$$U(x,0)=0,\qquad 0\le x\le 1,\tag{2}$$

$$U|_{AC}=\varphi(y),\qquad 0\le y\le\frac{1}{2},\tag{3}$$

$$U|_{BD}=\psi(y),\qquad 0\le y\le\frac{1}{2},\tag{4}$$

$$U_X(0+,y)=I_1(U(x,y)|_{x=0-}),\qquad 0<y<1,\tag{5}$$

$$U_X(1-0,y)=I_2(U_x(x,y)|_{x=1+0}),\qquad 0<y<1.\tag{6}$$

Bu yerda $\varphi(y)$, $\psi(y)$-berilgan funksiyalar, I_1, I_2 lar esa hozircha ixtiyoriy integral operatorlar.

Bunday tipdagi masalalar I_1 *va* I_2 integral operatorlarning maxsus ko‘rinishida $[2]$ da ($\alpha=1$ holda) hamda $0<\alpha<1$ uchun $[3]$ tadqiq etilgan.

(1) tenglamaning Ω_0 sohada (2) va

$$U_x(0+,y)=v_0^+(y),\qquad U_x(1-0,y)=v_1^-(y),\qquad 0<y<1\tag{7}$$

shartlarni qanoatlantiruvchi yechimi quydagicha yoziladi $[4]$:

$$U(x,y)=\int_0^y v_1^-(\eta)G(x,y;1,\eta)d\eta-\int_0^y v_0^+(\eta)G(x,y;0,\eta)d\eta-$$
$$-\int_0^y\int_0^1 f(\xi,\eta)G(x,y;\xi,\eta)d\xi d\eta,\tag{8}$$

bu yerda

$$G(x,y;\xi,\eta)=\frac{(y-\eta)^{\beta-1}}{2}\sum_{n=-\infty}^{+\infty}\left[e_{1,\beta}^{1,\beta}\left(-\frac{|x-\xi+2n|}{(y-\eta)^{\beta}}\right)+e_{1,\beta}^{1,\beta}\left(-\frac{|x+\xi+2n|}{(y-\eta)^{\beta}}\right)\right],$$

$$\beta = \frac{\alpha}{2} \qquad (9)$$

$$e_{1,\beta}^{1,\beta}(z) = \sum_{n=0}^{+\infty} \frac{z^n}{n!\Gamma(\beta - \beta n)}$$ $\to$ Rayt tipidagi funksiya $[4]$.

(1) tenglama uchun Ω_1 va Ω_2 sohalardagi Koshi masalasi yechimini Dalamber formulasi orqali yozib olamiz $[5]$:

$$U(x,y) = \frac{1}{2}\left\{\tau_0^-(y+x) + \tau_0^-(y-x) + \int_{y-x}^{y+x} \nu_0^-(t)dt + \int_{y-x}^{y+x}\int_0^1 f(\xi,\eta)d\xi d\eta\right\},$$

$$(x,y) \in \Omega_1 \qquad (10)$$

$$U(x,y) = \frac{1}{2}\left\{\tau_1^+(y-x+1) + \tau_1^+(y+x-1) + \int_{y+x-1}^{y-x+1} \nu_1^+(t)dt + \int_{y+x-1}^{y-x+1}\int_0^1 f(\xi,\eta)d\xi d\eta\right\},$$

$$(x,y) \in \Omega_2 \qquad (11)$$

Bu yerda $U(-0,y) = \tau_0^-(y)$, $U(1+0,y) = \tau_1^+(y)$.

(10) ni (3) ga qo‘yamiz:

$$U(-y,y) = \varphi(y) = \frac{1}{2}\left\{\tau_0^-(0) + \tau_0^-(2y) + \int_{2y}^{0} \nu_0^-(t)dt + \int_{2y}^{0}\int_0^1 f(\xi,\eta)d\xi d\eta\right\},$$

$$0 \le y \le \frac{1}{2}.$$

yoki (2) ni hisobga olsak $\tau_0^-(0) = 0$ va y bo‘yicha bir marta differensiallasak

$$2\varphi'(\frac{y}{2}) = \tau_0^{-\prime}(y) - \nu_0^-(y) - \int_0^1 f(y,\eta)d\eta\,, \qquad 0 < y < 1\,. \qquad (12)$$

Xuddi shuningdek, (11) ni (4) ga qo‘yib, yuqoridagidek amallarni bajarganda quyidagini olamiz:

$$2\psi'(\frac{y}{2}) = \tau_1^{+\prime}(y) - \nu_1^+(y) - \int_0^1 f(y,\eta)d\eta\,, \qquad 0 < y < 1\,. \qquad (13)$$

(5) va (6) ulash shartlarini quydagicha yozib olish mumkin:

$$\nu_0^+(y) = I_1(\nu_0^-(y)), \qquad \nu_1^-(y) = I_2(\nu_1^+(y)), \qquad 0 < y < 1, \qquad (14)$$

(12), (13) ni hisobga olsak, (14) dan

$$\nu_0^+(y) = I_1\left[\tau_0^{-\prime}(y) - \int_0^1 f(y,\eta)d\eta - 2\varphi'(\frac{y}{2})\right],$$

$$\nu_1^-(y) = I_2\left[\tau_1^{+\prime}(y) - \int_0^1 f(y,\eta)d\eta - 2\psi'(\frac{y}{2})\right]. \qquad (15)$$

(15) ni (8) ga qo'yamiz:

$$U(x,y)=\int_0^y I_2(\tau_1^{+\prime}(y)-\int_0^1 f(y,\eta)d\eta-2\psi'(\frac{y}{2}))G(x,y;1,\eta)d\eta-$$

$$-\int_0^y I_1(\tau_0^{-\prime}(y)-\int_0^1 f(y,\eta)d\eta-2\varphi'(\frac{y}{2}))G(x,y;0,\eta)d\eta- \qquad (16)$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(x,y;\xi,\eta)d\xi d\eta$$

(16) da $x\to 0+$ va $x\to 1-0$ limitlarga o'tamiz:

$$\tau_0^-(y)=\int_0^y G(0,y;1,\eta)(I_2\left[\tau_1^{+\prime}(\eta)-\int_0^1 f(\eta,\xi)d\xi-2\psi'(\frac{\eta}{2})\right])d\eta-$$

$$-\int_0^y G(0,y;0,\eta)(I_1\left[\tau_0^{-\prime}(\eta)-\int_0^1 f(\eta,\xi)d\xi-2\varphi'(\frac{\eta}{2})\right])d\eta- \qquad (17)$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(0,y;\xi,\eta)d\xi d\eta\ ,$$

$$\tau_1^+(y)=\int_0^y G(1,y;1,\eta)(I_2\left[\tau_1^{+\prime}(\eta)-\int_0^1 f(\eta,\xi)d\xi-2\psi'(\frac{\eta}{2})\right])d\eta-$$

$$-\int_0^y G(1,y;0,\eta)(I_1\left[\tau_0^{-\prime}(\eta)-\int_0^1 f(\eta,\xi)d\xi-2\varphi'(\frac{\eta}{2})\right])d\eta- \qquad (18)$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(1,y;\xi,\eta)d\xi d\eta\ .$$

Agar I_1 va I_2 integral operatorlarni quyidagi ko'rinishda olsak:

$$I_1(g)=\int_0^\eta g(z)K_1(\eta,z)dz,$$

$$I_2(g)=\int_0^\eta g(z)K_2(\eta,z)dz, \qquad (19)$$

(17) va (18) larni Volterra integral tenglamalar sistemasiga keltirsak bo'ladi.

$$\tau_0^-(y)=\int_0^y G(0,y;1,\eta)(\int_0^\eta K_2(\eta,z)dz\left[\tau_1^{+\prime}(z)-\int_0^1 f(z,\xi)d\xi-2\psi'(\frac{z}{2})\right])d\eta-$$

$$-\int_0^y G(0,y;0,\eta)(\int_0^\eta K_1(\eta,z)dz\left[\tau_0^{-\prime}(z)-\int_0^1 f(z,\xi)d\xi-2\varphi'(\frac{z}{2})\right])d\eta- \qquad (20)$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(0,y;\xi,\eta)d\xi d\eta\ ,$$

$$\tau_1^+(y)=\int_0^y G(1,y;1,\eta)(\int_0^\eta K_2(\eta,z)dz\left[\tau_1^{+\prime}(z)-\int_0^1 f(z,\xi)d\xi-2\psi'(\frac{z}{2})\right])d\eta-$$

$$-\int_0^y G(1,y;0,\eta)(\int_0^\eta K_1(\eta,z)dz\left[\tau_0^{-\prime}(z)-\int_0^1 f(z,\xi)d\xi-2\varphi'(\frac{z}{2})\right])d\eta- \qquad (21)$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(1,y;\xi,\eta)d\xi d\eta\,.$$

(20) ni soddalashtiramiz:

$$\tau_0^-(y)=\int_0^y G(0,y;1,\eta)(\int_0^\eta K_2(\eta,z)\tau_1^{+\prime}(z)dz)d\eta-\int_0^y G(0,y;1,\eta)(\int_0^\eta K_2(\eta,z)(\int_0^1 f(z,\xi)d\xi)dz)d\eta-$$

$$-2\int_0^y G(0,y;1,\eta)(\int_0^\eta K_2(\eta,z)\psi'(\frac{z}{2})dz)d\eta-\int_0^y G(0,y;0,\eta)(\int_0^\eta K_1(\eta,z)\tau_0^{-\prime}(z)dz)d\eta+$$

$$+\int_0^y G(0,y;0,\eta)(\int_0^\eta K_1(\eta,z)(\int_0^1 f(z,\xi)d\xi)dz)d\eta+2\int_0^y G(0,y;0,\eta)(\int_0^\eta K_1(\eta,z)\varphi'(\frac{z}{2})dz)d\eta-$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(0,y;\xi,\eta)d\xi d\eta.$$

(22)

(22) da $\tilde{K}_1(y,z)$ va $\tilde{F}(y)$ belgilashlar kiritsak u quyidagi ko'rinishga keladi.

$$\tau_0^-(y)+\int_0^y \tau_0^{-\prime}(z)\tilde{K}_1(y,z)dz=\tilde{F}(y), \qquad (23)$$

bu yerda

$$\tilde{K}_1(y,z)=\int_z^y G(0,y;0,\eta)K_1(\eta,z)d\eta,$$

$$\tilde{F}(y)=\int_0^y G(0,y;1,\eta)(\int_0^\eta K_2(\eta,z)\tau_1^{+\prime}(z)dz)d\eta-\int_0^y G(0,y;1,\eta)(\int_0^\eta K_2(\eta,z)(\int_0^1 f(z,\xi)d\xi)dz)d\eta-$$

$$-2\int_0^y G(0,y;1,\eta)(\int_0^\eta K_2(\eta,z)\psi'(\frac{z}{2})dz)d\eta+\int_0^y G(0,y;0,\eta)(\int_0^\eta K_1(\eta,z)(\int_0^1 f(z,\xi)d\xi)dz)d\eta+$$

$$+2\int_0^y G(0,y;0,\eta)(\int_0^\eta K_1(\eta,z)\varphi'(\frac{z}{2})dz)d\eta-\int_0^y\int_0^1 f(\xi,\eta)G(0,y;\xi,\eta)d\xi d\eta.$$

(23) ni bir marta bo'laklab integrallab olsak

$$\left\langle \begin{array}{ll} u=\tilde{K}_1(y,z) & dv=\tau_0^{-\prime}(z)dz \\ du=\dfrac{\partial}{\partial z}\tilde{K}_1(y,z), & v=\tau_0^-(z) \end{array} \right\rangle$$

$$\tau_0^-(y)+\tau_0^-(z)\tilde{K}_1(y,z)\Big|_0^y-\int_0^y \tau_0^-(z)\frac{\partial}{\partial z}\tilde{K}_1(y,z)dz=\tilde{F}(y)$$

$$\tau_0^-(y)+\tau_0^-(y)\underbrace{\tilde{K}_1(y,y)}_{=0}-\tau_0^-(0)\underbrace{\tilde{K}_1(y,0)}_{=0}-\int_0^y \tau_0^-(z)\frac{\partial}{\partial z}\tilde{K}_1(y,z)dz=\tilde{F}(y)$$

quyidagini olamiz:

$$\tau_0^-(y)-\int_0^y \tau_0^-(z)\frac{\partial}{\partial z}\tilde{K}_1(y,z)dz=\tilde{F}(y) \qquad (24)$$

$\tilde{F}(y)$ uzluksiz , $\int_0^y \tau_0^-(z)\frac{\partial}{\partial z}\tilde{K}_1(y,z)dz$ uzluksiz yoki 1 dan kichik maxsuslikka ega bo‘lsa u holda (25) integral tenglamaning yechimini rezolventa orqali yozib olish mumkin $[5]$:

$$\tau_0^-(y)=\tilde{F}(y)+\int_0^y \tilde{F}(z)R_1(y,z)dz, \qquad (25)$$

$R_1(y,z)$ – $\frac{\partial}{\partial z}\tilde{K}_1(y,z)$ yadroning rezolventasi.

Shuni ta’kidlash kerakki, $\tilde{F}(y)$ ifodaning ichida hozircha noma’lum bo’lgan $\tau_1^+(y)$ ifoda ham bor. Uni topish uchun esa (25)ni (21) ga qo‘yamiz. Shunda $\tau_1^+(y)$ funksiyaga nisbatan (24) ga o‘xshash 2-tur Volterra integral tenglamasini olamiz. Berilgan funksiyalarga ma’lum shartlar asosida bu tenglamaning bir qiymatli yechilishi ko‘rsatiladi.

Demak, quyidagi tasdiq o‘rinli:

Teorema. Agar $f(x,y)\in C(\bar{\Omega})$, $\varphi(y),\psi(y)\in C\left[0,\frac{1}{2}\right]\cap C^1(0,\frac{1}{2})$ bo‘lsa va I_1, I_2 integral operatorlar (19) ko‘rinishda berilsa, u holda 1-masalaning yechimi mavjud va yagona.

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