GURUHLI DIFFERENSIAL O'YINDA BIR QOCHUVCHINI TUTIB OLISH MASALASI

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Annotatsiya. Ushbu ishda L.S.Pontryagin misolida bitta qochuvchini bir nechta quvuvchi ta'qib etish masalasi o'rganilgan. Barcha o'yin ishtirokchilarining dinamik imkoniyatlari bir xil. Bunday masalalar [1], [3], [4] adabiyotlarda tadqiq etilgan. Ushbu ishda sistemaga mos bir jinsli sistema uchun Koshi masalasi yechimi rekurrent deb faraz qilinganda tutib olish masalasi yechilgan.

Kalit so'zlar. Differensial o'yin, guruhli ta'qib etish, tutish masalasi, Pontryagin misoli.

$R^k (k \geq 2)$ fazoda $n+1$ ishtirokchili differensial o'yinni qaraymiz, bunda $P_1, \dots, P_n$ quvuvchilar va E qochuvchi.

Har bir P_i quvuvchining harakat qonuni

$$x_i^{(l)} + a_i(t)x_i^{(l-1)} + \cdots + a_l(t)x_i = u_i, \;\; u_i \in V \qquad (1)$$

ko'rinishga ega. E qochuvchining harakat qonuni

$$y_i^{(l)} + a_i(t)y_i^{(l-1)} + \cdots + a_l(t)y_i = v, \;\; v \in V \qquad (2)$$

ko'rinishga ega. Bu yerda va keying o'rinlarda

$$x_i, y, u_i, v \in R^k, \qquad i \in I = \{1, 2, \dots, n\}, \qquad a_1(t), \dots, a_l(t)$$

funksiyalar $[t_0, \infty)$ intervalda uzluksiz, V – qattiy qavariq silliq chegarali kompakt to'plam R^k fazoning qismi. $t = t_0$ da

$$x_i(t_0) = x_{i0}^0, \dot{x}_i(t_0) = x_{i0}^0, \dots, x_i^{(l-1)}(t_0) = x_{i,l-1}^0 \qquad (3)$$

$$y(t_0) = y_0^0, \dot{y}(t_0) = y_1^0, \dots, y^{(l-1)}(t_0) = y_{l-1}^0 \qquad (4)$$

boshlang'ich shart berilgan, bunda barcha i larda $x_{i0}^0 \neq y_i^0$.

Qaralayotgan o'yinni G orqali belgilaymiz.

(1) – (4) sistemalar o'rniga ushbu

$$z_i^{(l)} + a_i(t)z_i^{(l-1)} + \cdots + a_l(t)z_i = u_i - v, \;\; u_i, v \in V \qquad (5)$$

sistemani

$$z_i(t_0) = z_{i0}^0 = x_{i0}^0 - y_0^0, \dots, z_i^{l-1}(t_0) = z_{i,l-1}^0 = x_{i,l-1}^0 - y_{l-1}^0. \qquad (6)$$

boshlang'ich shart bilan qaraymiz. $z^0 = \{z_{i\alpha}^0, \alpha = 0, \dots, l-1, i = 1, \dots, n\}$ bo'lsin.

E qochuvchining $v(t)$ boshqaruvining $t, t \in [t_0, \infty)$ vaqt momentidagi tarixi deb

$$v_t(\cdot) = \{v(s), s \in [t_0, t], v - \text{o'lchovli funksiya}\}$$

to'plamga aytiladi.

1-ta'rif. Agar $z^0 = (z_1^0, \dots, z_n^0)$ boshlang'ich holatga, t momentga va E qochuvchining ixtiyoriy $v_t(\cdot)$ boshqaruvi tarixiga qiymatlarini V to'plamdan oluvchi $u_i(t) = U_i\big(t, z^0, v_t(\cdot)\big)$ o'lchovli funksiyani mos qo'yuvchi $U_i\big(t, z^0, v_t(\cdot)\big)$ akslantirish aniqlangan bo'lsa, u xolda P_i quvuvchining U_i kvazistrategiyasi berilgan deyiladi.

2-ta'rif. Agar shunday $T_0 = T(z^0)$ moment va $P_1, \dots, P_n$ quvuvchilarning shunday $U_1\big(t, z^0, v_t(\cdot)\big), \dots, U_n\big(t, z^0, v_t(\cdot)\big)$ kvazistrategiyalari topilsaki, ixtiyoriy $v(\cdot), v(t) \in V, t \in [0, T_0]$ o'lchovli funksiya uchun $z_\alpha(\tau) = 0$ tenglik bajariladigan $\alpha \in \{1, \dots, n\}$ nomer va $\tau \leq [t_0, T_0(z^0)]$ moment mavjud bo'lsa, u holda G_4 o'yinda tutib olish mumkin deymiz.

$\varphi_q(t, s), q = 0, 1, \dots, l-1, (t \geq s \geq t_0)$ orqali

$$\omega^{(l)} + a_1(t)\omega^{(l-1)} + \dots + a_l(t)\omega = 0$$

tenglamaning

$$\omega^{(j)}(s) = 0, \quad j = 0, \dots, q-1, q+1, \dots, l-1, \quad \omega^{(q)}(s) = 1$$

boshlang'ich shartlardagi yechimini belgilaylik. Quyidagi funksiyani

$$\xi_i(t) = \varphi_0(t, t_0)z_i^0 + \varphi_1(t, t_0)z_i^1 + \dots + \varphi_{l-1}(t, t_0)z_i^{l-1}.$$

va $H_i = \{\xi_i(t), t \in [t_0, \infty)\}$ to'plamni kiritib olamiz.

3-ta'rif. Agar ixtiyoriy $\varepsilon > 0$ uchun shunday $T(\varepsilon) > 0$ topilsaki, ixtiyoriy $t, a \in R^1$ lar uchun

$$\big|F\big(t + \tau(t)\big) - F(t)\big| < \varepsilon$$

tengsizlik o'rinli bo'ladigan $\tau(t) \in [a, a + T(\varepsilon)]$ mavjud bo'lsa, $F: R^1 \to R^k$ funksiya Zubov ma'nosida rekurrent (qisqacha rekurrent) deyiladi.

Agar barcha t lar uchun $\tau(t)$ ni barcha t ga bog'liq bo'lmagan holda tanlash mumkin bo'lsa, u holda $F(t)$ deyarli davriy deyiladi.

4-ta'rif. Agar barcha $t \in [t_0, \infty)$ nuqtalarda $f(t) = F(t)$ tenglikni qanoatlantiruvchi $F: R^1 \to R^k$ rekurrent funksiya mavjud bo'lsa, u holda $f: [t_0, \infty) \to R^k$ funksiya $[t_0, \infty)$ oraliqda Zubov ma'nosida rekurrent (qisqacha rekurrent) deyiladi.

1-lemma. Faraz qilaylik barcha $i \in I = \{1, 2, \dots, n\}$ lar uchun shunday $h_i^0 \in H_i, h_i^0 \neq 0$ vektorlar mavjudki $0 \in \text{Intco}\{h_i^0\}$ munosabat bajarilsin va $\xi_i(t)$ funksiyalar rekurrent bo'lsin. U holda shunday $\varepsilon > 0$ va $T(\varepsilon) > 0$ lar mavjudki, quyidagi tasdiqlar o'rinli bo'ladi:

1. $0 \notin D_\varepsilon\big(h_i^0\big)$ va barcha $h_i \in D_\varepsilon\big(h_i^0\big)$ larda $0 \in \text{Intco}\{h_i\}$ munosabat bajariladi, bu yerda $D_\varepsilon = \{z: \|z - a\| \leq \varepsilon\}$;

2. Har bir $t \geq t_0$ uchun shunday $\tau_i \in [t, t + T(\varepsilon)]$ momentlar topiladiki

$$\left\| \xi_i(\tau_i) - h_i^0 \right\| < \varepsilon$$

tengsizlik bajariladi.

Isbot. $\mathrm{co}\{h_i^0\}$ to'plam uchlari $h_j^0, j \in K \subset I$ nuqtalarda bo'lgan qavariq ko'pyoqdan iborat. Lemma shartidan $0 \in \mathrm{Intco}\{h_j^0\}$ kelib chiqadi. $\mathrm{Intco}\{h_j^0\}$ – ochiq to'plam. Demak shunday $\varepsilon > 0$ son mavjudki, ixtiyoriy $h_j \in D_\varepsilon(h_j^0)$ uchun $0 \in \mathrm{Intco}\{h_j\}$ munosabat o'rinli. $\mathrm{Intco}\{h_j\} \subset \mathrm{Intco}\{h_i\}$ munosabatdan lemmaning 1-tasdig'i to'g'ri ekanligi ko'rinadi.

ξ_i funksiyalar rekurrent, u holda ixtiyoriy $\varepsilon > 0$ uchun shunday $T(\varepsilon) > 0$ mavjudki, har bir $t \geq t_0$ uchun shunday $\tau_i \in [t, t + T(\varepsilon)]$ momentlar topiladiki $\left\| \xi_i(\tau_i) - h_i^0 \right\| < \varepsilon$ tengsizlik bajariladi. Lemma isbotlandi.

Keyingi o'rinlarda $\varepsilon > 0$ va T 2.1-lemma shartlariga mos tanlangan deb xisoblaymiz.

Quyidagi funksiyalarni aniqlab olamiz:

$$\rho(t,s) = \begin{cases} 1, & \varphi_{l-1}(t,s) \geq 0 \\ -1 & \varphi_{l-1}(t,s) < 0 \end{cases} \qquad (t_0 \leq s \leq t),$$

$$\lambda(v, \rho, h_i) = \sup\{\lambda : \lambda \geq 0, v - \lambda \rho h_i \in V\},$$

$$G(t, h_i) = \int_{t_0}^{t} |\varphi_{l-1}(t,s)| \lambda(v(s), \rho(t,s), h_i) ds.$$

Ushbu

$$h = (h_1, h_2, \dots, h_n), D = D_\varepsilon(h_1^0) \times D_\varepsilon(h_2^0) \times \dots \times D_\varepsilon(h_n^0)$$

belgilashlarni kiritamiz.

2-lemma. Quyidagi shartlar o'rinli bo'lsin:

1. $\xi_i(t)$ funksiyalar $[t_0, \infty)$ da rekurrent;
2. $\lim\limits_{t\to\infty} \int_{t_0}^{t} |\varphi_{l-1}(t,s)| ds = +\infty$;
3. Barcha $i \in I = \{1, 2, \dots, n\}$ lar uchun shunday $h_i^0 \in H_i, h_i^0 \neq 0$ vektorlar mavjudki $0 \in \mathrm{Intco}\{h_i^0\}$ munosabat o'rinli bo'ladi.

U holda shunday T_1 moment topiladiki, ixtiyoriy $v(t)$ joiz boshqaruv va ixtiyoriy $h \in D$ uchun shunday $\alpha \in I$ nomer mavjud bo'lib u uchun $G(T_1, h_\alpha) \geq 1$ tengsizlikni bajariladi.

Isbot. Lemma shartidan ixtiyoriy $h \in D$ uchun

$$\delta_{\pm 1}(h) = \min_{v \in V} \max_{i \in I} \lambda(v, \pm 1, h_i) > 0$$

tengsizlik bajarilishi kelib chiqadi.

Har bir $V \times \{\pm 1\} \times D_\varepsilon(h_i^0)$ to'plamda λ funksiya uzluksiz bo'ladi[3], bundan

$$\lim_{h^* \to h} \delta_{\pm 1}(h^*) = \lim_{h^* \to h} \min_{v \in V} \max_{i \in I} \lambda(v, \pm 1, h_i^*) = \min_{v \in V} \max_{i \in I} \lambda(v, \pm 1, h_i) = \delta_{\pm 1}(h).$$

O'z navbatida D to'plamda $\delta_{\pm 1}(h)$ funksiyalar uzluksiz bo'ladi. D to'plamning kompaktligini xisobga olib

$$\delta = \min_{h\in D}\min_{p\in\{-1,1\}}\min_{v\in V}\max_{i\in I}\lambda(v,p,h_i) = \min_{h\in D}\{\delta_{+1}(h),\delta_{-1}(h)\} > 0$$

tengsizlikni xosil qilamiz. Bundan

$$\max_{i\in I} G(t,h_i) = \max_{i\in I}\int_{t_0}^{t}|\varphi_{l-1}(t,s)|\lambda(v(s),p(t,s),h_i)ds \geq$$

$$\geq \frac{1}{n}\int_{t_0}^{t}|\varphi_{l-1}(t,s)|\sum_{i\in I}\lambda(v(s),p(t,s),h_i)\,ds \geq \frac{\delta}{n}\int_{t_0}^{t}|\varphi_{l-1}(t,s)|ds.$$

Shunday qilib

$$\frac{\delta}{n}\int_{t_0}^{T_1}|\varphi_{l-1}(T_1,s)|ds \geq 1$$

shartidan aniqlangan T_1 moment va biror $\alpha\in I$ nomer uchun $G(T_1,h_\alpha)\geq 1$ tengsizlik o'rinli bo'ladi. Lemma isbotlandi.

$$T(z^0) = \min\left\{t\geq 0:\ \inf_{v(\cdot)}\min_{h\in D}\max_{i\in I} G(t,h_i)\geq 1\right\}$$

bo'lsin. 2 lemmaga ko'ra $T(z^0) < \infty$ tengsizlik o'rinli.

1-Teorema. Quyidagi shartlar o'rinli bo'lsin:

1. $[t_0,\infty)$ da $\xi_i(t)$ funksiyalar rekurrent;

2. Shunday $h_i^0\in H_i, h_i^0\neq 0$ vektorlar mavjudki $0\in \text{Intco}\{h_1^0,h_2^0,\ldots,h_n^0\}$ munosabat bajariladi;

3. Shunday $\tau_i\geq T(z^0)$ momentlar mavjudki, ular uchun

$$(a)\ \ \xi_i(\tau_i)\in D_\varepsilon(h_i^0);$$

$$(b)\ \ \inf_{v(\cdot)}\max_{i} G\big(\tau_i,\xi_i(\tau_i)\big)\geq 1$$

munosabatlar bajariladi.

U holda G_4 o'yinda tutib olish mumkin.

Isbot. Ixtiyoriy joiz boshqaruvlarda (5),(6) masalaning yechimi Koshi formulasi boyicha

$$z_i(t) = \xi_i(t) + \int_{t_0}^{t}\varphi_{l-1}(t,s)\big(u_i(s)-v(s)\big)ds$$

ko'rinishga ega. Faraz qilaylik τ_i – momentlar teorema shartlarini qanoatlantirsin, $v(s), s\in[t_0,T_0]$ – esa E qochuvchining ixtiyoriy joiz boshqaruvi bo'lsin, bu yerda $T_0=\max_i \tau_i$.

Ushbu $f(t) = 1-\max_{i\in I}\int_{t_0}^{t}|\varphi_{l-1}(\tau_i,s)|\lambda\big(v(s),p(\tau_i,s),\xi_i(\tau_i)\big)ds$

funksiyani qaraymiz. $t_1 \geq t_0$ orqali bu funksiyaning eng kichik ildizini belgilaylik. Ta'kidlash joizki teoremaning 3.shartiga ko'ra t_1 moment mavjud va kamida bitta i uchun $t_1 \leq \tau_i$ tengsizlik o'rinli.

Bundan tashqari shunday $l \in I$ nomer mavjudki, u uchun

$$1 - \int_{t_0}^{t} |\varphi_{l-1}(\tau_i, s)| \lambda\big(v(s), p(\tau_i, s), \xi_i(\tau_i)\big) ds = 0$$

tenglik bajariladi. P_i quvuvchini barcha $t \in [t_0, T_0]$ larda quyidagicha boshqaramiz:

$$u_i(t) = v(t) - \lambda\big(v(t), p(\tau_i, t), \xi_i(\tau_i)\big) p(\tau_i, t), \xi_i(\tau_i),$$

bu yerda $t \in [t_1, T_0]$ larda $\lambda\big(v(t), p(\tau_i, t), \xi_i(\tau_i)\big) = 0$ deb xisoblaymiz.

U xolda Koshi formulasiga asosan

$$z_i(\tau_i) = \xi_i(\tau_i)\left(1 - \int_{t_0}^{t} |\varphi_{l-1}(t, s)| \lambda\big(v(s), p(\tau_i, s), \xi_i(\tau_i)\big) ds\right).$$

t_1 ning aniqlanishiga ko'ra, $l \in I$ nomer ustida qavs ichidagi ifoda nolga aylanadi, bundan $z_l(\tau_l) = 0$. Teorema isbotlandi.

1-Natija. Quyidagi shartlar o'rinli bo'lsin:

1. $[t_0, \infty)$ da $\xi_i(t)$ funksiyalar rekurrent;
2. $0 \in \mathrm{Intco}\{z_1^0, z_2^0, \dots, z_n^0\}$ munosabat bajariladi;

U holda G o'yinda tutib olish mumkin.

Misol. (4) sistema quyidagi ko'rinishda bo'lsin

$$\dot{z} + a(t) z_i = u_i - v, \ \ u_i, v \in V,$$

bu yerda

$$a(t) = \begin{cases} 0, & t \in [0, \pi], \\ \sin t, & t \notin [0, \pi]. \end{cases}$$

$a(t)$ funksiya rekurrent ekanligini ko'rsatish qiyin emas.

$t_0 = 0$ bo'lsin. $\xi_i(t) = g(t) z_{i0}^0$, bu yerda

$$g(t) = \begin{cases} 1, & t \in [0, \pi], \\ e^{-\cos t - 1}, & t \notin [0, \pi]. \end{cases}$$

Demak g funksiya $[0, +\infty)$ da rekurrent va o'z navbatida $\xi_i(t)$ funksiya rekurrent.

Tasdiq. Agar $0 \in \mathrm{Intco}\{z_1^0, \dots, z_n^0\}$ bo'lsa bu o'yinda tutib olish mumkin.

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