BITTA SINGULYAR KOEFFITSIENTGA EGA BO`LGAN TO`RTINCHI TARTIBLI TENGLAMA UCHUN GURSA MASALASI

Sh.T. Karimov
X.I. Botirova
Farg`ona davlat universtiteti, Farg`ona,O`zbekiston.

Annotatsiya: *ushbu maqolada to`rtinchi tartibli bitta singulyar koeffitsientga ega bo`lgan bir jinsli tenglama uchun to`g`ri to`rtburchak sohada Gursa masalasining klassik yechimi topilgan.*

Abstract: *In this article, a classical solution of the Gursa problem is found in a rectangular field for a homogeneous equation with one singular coefficients of the fourth order.*

Аннотация: *В данной статье найдено классическое решение задачи Гурсы в прямоугольном поле для однородного уравнения с одними сингулярными коэффициентами четвертого порядка.*

Kalit so`zlar: *Bessel differensial operatori, umumiy yechim, Gursa masalasi, klassik yechim.*

I. Kirish

$\Omega\{(x,y): 0<x<b, 0<y<h\}$ sohada ushbu

$$L_0^{\alpha} B_{x,\alpha-\frac{1}{2}} D_2 U(x,y) = \left(\frac{\partial^2}{\partial x^2}+\frac{2\alpha}{x}\frac{\partial}{\partial x}\right)\left(\frac{\partial^2}{\partial y^2}\right)U \qquad (1)$$

tenglamani qaraymiz,bu yerda

$B_{x,p} \equiv \frac{\partial^2}{\partial x^2}+\frac{2p+1}{x}\frac{\partial}{\partial x}$ $\alpha, p \in R, 0<\alpha$ – Bessel differensial operatori

(1) tenglamaning umumiy yechimi quyidagicha topiladi

$$L_0^\alpha B_{x,\alpha-\frac{1}{2}} D_2 U(x,y) = \left(\frac{\partial^2}{\partial x^2} + \frac{2\alpha}{x}\frac{\partial}{\partial x}\right)\left(\frac{\partial^2}{\partial y^2}\right)U = \frac{\partial^4 U}{\partial x^2 \partial y^2} + \frac{2\alpha}{x}\frac{\partial^3 U}{\partial x \partial y^2} = 0$$

$$\frac{\partial^2}{\partial x \partial y}\left(\frac{\partial^2}{\partial x \partial y} + \frac{2\alpha}{x}\frac{\partial}{\partial y}\right)U = 0$$

$$\frac{\partial^2}{\partial x \partial y} x^{-2\alpha}\left(\frac{x^{2\alpha}\partial^2 U}{\partial x \partial y} + \frac{x^{2\alpha} 2\alpha}{x}\frac{\partial U}{\partial y}\right) = \frac{\partial^2 x^{-2\alpha}}{\partial x \partial y}\frac{\partial}{\partial x}\left(x^{2\alpha}\frac{\partial U}{\partial y}\right) = 0$$

$$x^{-2\alpha}\frac{\partial}{\partial x}\left(x^{2\alpha}\frac{\partial U}{\partial y}\right) = g_1(x) + f_1(y)$$

$$\frac{\partial}{\partial x}\left(x^{2\alpha}\frac{\partial U}{\partial y}\right) = x^{2\alpha} g_1(x) + x^{2\alpha} f_1(y)$$

$$x^{2\alpha}\frac{\partial U}{\partial y} = \int_0^x t^{2\alpha} g_1(t)dt + \int_0^t t^{2\alpha} f_1(y)dt + f_2(y)$$

$$x^{2\alpha}\frac{\partial U}{\partial y} = \int_0^x t^{2\alpha} g_1(t)dt + \frac{t^{2\alpha+1}}{2\alpha+1} f_1(y)\Bigg|_{t=0}^{t=x} + f_2(y)$$

$$\frac{\partial U}{\partial y} = x^{-2\alpha}\int_0^x t^{2\alpha} g_1(t)dt + x^{-2\alpha}\frac{f_1(y)(x^{2\alpha+1}-1)}{2\alpha+1} + x^{-2\alpha} f_2(y)$$

$$U(x,y) = yx^{-2\alpha}\int_0^x t^{2\alpha} g_1(t)dt + \frac{(1-x^{-2\alpha})}{2\alpha+1}\int_0^y f_1(z)dz + x^{-2\alpha}\int_0^y f_2(z)dz + g_2(x)$$

$$U(x,y) = yg_1(x) + g_2(x) + \frac{x^{1-2\alpha}}{1-2\alpha}\int_0^y [y-t] f_1(t)dt + \int_0^y [y-t] f_2(t)dt \qquad (2)$$

bu yerda $f_k(y), g_k(x), (k=1,2)$ ixtiyoriy uzluksiz differensiallanuvchi funksiyalar.

(1) tenglama uchun Ω sohada Gursa masalasining analogini tadqiq qilamiz.

II **Asosiy qism.**

Gursa masalasi. Ω sohada (1) tenglamaning ushbu

$$U(0,y) = \varphi_1(y), \quad \lim_{x\to+0} x^{2\alpha} U_x(x,y) = \varphi_2(y), 0 \le y \le h, \qquad (3)$$

$$U(x,0) = \psi_1(x), \quad U_y(x,y) = \psi_2(x), 0 \le x \le l; \qquad (4)$$

shartlarni qanoatlantiruvchi klassik yechimi topilsin, bu yerda $\varphi_k(y), \psi_k(x)\, (k=1,2)$ berilgan funksiyalar.

Ushbu masalani yechish uchun (2) umumiy yechimni (3) va (4) shartlarga qo`yib ixtiyoriy bo`lgan $f_k(y), g_k(x), (k=1,2)$ funksiyalarni topamiz.

$$f_2(t) = \varphi_2''(y), \quad f_2(t)dt = \varphi_1''(y)$$

$$g_2(x) = \psi_1(x), \; g_1(x) = \psi_2(x).$$

Topilgan $f_k(y), g_k(x), (k=1,2)$ larni (2) umumiy yechimga qo`yib,

$$U(x,y) = \psi_1(x) + y\psi_2(x) + \frac{x^{1-2\alpha}}{1-2\alpha}\int_0^y [y-t]\varphi_2'' dt + \int_0^y [y-t]\varphi_1''(t)dt$$

yechimni hosil qilamiz. $\varphi_1(y)$ va $\varphi_2(y)$ xosilalari qatnashgan integrallarini hisoblaymiz.

Ushbu

$$U(x,y)=\frac{x^{1-2\alpha}}{1-2\alpha}\varphi_2(y)+\psi_1(x)+\psi_2(x)+\varphi_1(y)-y\psi_2(0)-\psi_1(0) \qquad (5).$$

yechimga ega bo`lamiz

III Natija.

Teorema. Agar $0<\alpha$, $\varphi_k(y)\in C[0,h]\cap C^2(0,h)$, $\psi_k(x)\in C[0,l]\cap C^2(0,l)$, bo`lib, $\psi_k(x)$ funksiya $x\to+0$ da $2\alpha$ dan kichik maxsuslikka ega bo`lishi mumkin. U holda ushbu

$$U(x,y)=\frac{x^{1-2\alpha}}{1-2\alpha}\varphi_2(y)+\psi_1(x)+\psi_2(x)+\varphi_1(y)-y\psi_2(0)-\psi_1(0) \qquad (5)$$

funksiya Gursa masalasini yagona yechimi bo`ladi.

Teoremani isbotlash uchun (5) funksiya (1) tenglamani va (3), (4) shartlarni qanoatlantirishini ko`rsatish lozim.

Buning uchun (5) funksiyani (3) va (4) shartlarni qanoatlantirishini tekshiramiz.

$\varphi_2(0)=0$ tenglikdan $U(0,y)=\varphi_1(y)$, $\lim\limits_{x\to0}x^{2\alpha}U_x=\varphi_2(y)-\varphi_2(0)=\varphi_2(y)$, shartlar o`rnli ekani kelib chiqadi.

$\varphi_2(0)=0\,,\psi_2(0)=0$ tengliklardan foydalanib

$$U(x,0)=\varphi_1(0)+\psi_1(x)+\frac{x^{1-2\alpha}}{1-2\alpha}[\varphi_2(0)-\varphi_2(0)]-\varphi_1(0)\,, \qquad U(x,0)=\psi_1(x) \qquad \lim_{y\to o}y^{2\beta}U_y=\psi_2(x)$$

shartlarning o`rnli ekani keltirib chiqaramiz.

(5) funksiyadan kerakli hosilalarni olib (1) tenglamani qanoatlantirishini tekshiramiz.

$$U_x(x,y)=\psi_1'(x)+x^{-2\alpha}\varphi_2(y)+\psi_2'(x)$$

$$U_{xx}(x,y)=-2\alpha x^{-2\alpha-1}\varphi_2(y)+\psi_1''(x)+y\psi_2''(x)$$

$$U_{xxy}(x,y)=-2\alpha x^{-2\alpha-1}\varphi_2'(y)+\psi_2''(x)$$

$$U_{xxyy}(x,y)=-2\alpha x^{-2\alpha-1}\varphi_2''(y)$$

$$U_y(x,y)=\varphi_1'(y)+\frac{x^{1-2\alpha}}{1-2\alpha}\varphi_2'(y)+\psi_2(x)$$

$$U_{yy}(x,y)=\varphi_1''(y)+\frac{x^{1-2\alpha}}{1-2\alpha}\varphi_2''(y)$$

$$U_{yyx}(x,y)=x^{-2\alpha}\varphi_2''(y)$$

$$U_{xxyy}+\frac{2\alpha}{x}U_{xyy}=-2\alpha x^{-2\alpha-1}\varphi_2''(y)+2\alpha x^{-2\alpha-1}\varphi_2''(y)=0$$

teorema isbotlandi.

IV. Xulosa

Maqolada (1) tenglama uchun Gursa masalasining yechimi topilgan. Buning uchun (1) tenglama umumiy yechimi ketma ket differensiallash hamda ko`paytmaning hosilasi formulalaridan foydalanib topilgan, maqolaning asosiy qismida yuqorida topilgan umumiy yechim yordamida (3),(4) shartlarni qanoatlantiruvchi Gursa masalasining yechimi topilgan. Maqolaning natija qismida yechimni aniqlovchi teorema va uning isboti keltirilgan.

FOYDALANILGAN ADABIYOTLAR RO’YXATI:

1. Операторы Эрдейи-Кобера и их приложения к дифференциальным уравнениям в частных производных: монография; научное издание; на русском языке; /А.К Уринов, Ш.Т. Каримовю – Фергана: изд «Фаргона»,2021. -202 стр.