GEOMETRIYA KURSINING MUNTAZAM KO`PYOQLAR MAVZUSINI O'QITISHDA AYRIM KO`RSATMALAR

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Annotatsiya. *Ushbu maqolada pedagogika oliy o`quv yurtlari talabalari va akademik litseylari talabalariga geometriya kursining muntazam ko'pyuqlar nazariyasini o'qitishda ayrim metodik ko'rsatmalar keltirilgan.*

Аннотация. *В данной статье представлены некоторые методические указания по преподаванию теории правильных многогранников по курсу геометрии студентов педагогических вузов и академических лицеев.*

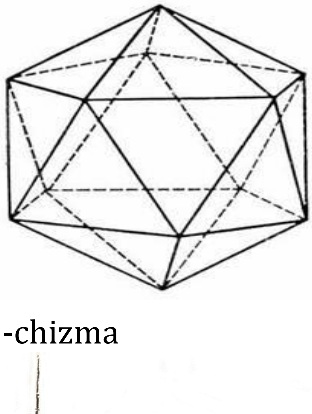
Abstract. *This article presents some guidelines for teaching the theory of regular polyhedra in the course of geometry for students of pedagogical universities and academic lyceums.*

**Kalit so`zlar.** *Ko`pyoqlar, muntazam ko`pyoqlar, muntazam tetraedr, oktaedr, ikosaedr,dodekaedr, ikki yoqli burchak, muntazam uchburchak, muntazam beshburchak.*

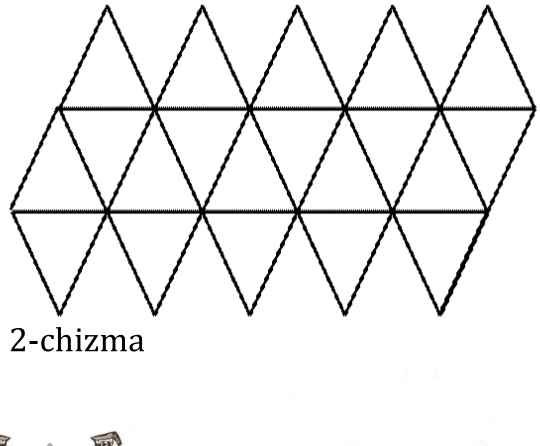
Ko`pyoqning barcha yoqlari kongurent muntazam ko`pburchaklardan iborat bo`lib, hamma ko`p yoqli burchaklari ham kongurent bo`lsa, u *muntazam ko`pyoq* deb ataladi [1-2].

Demak, yoqlari muntazam uchburchakdan iborat faqatgina uch xil muntazam ko`pyoq mavjud bo`lishi mumkin. Bular quyidagilar:

1. *Muntazam to`rtyoq, odatda muntazam tetraedr* deb yuritlib, uning 4 ta yog`i, 4ta uchi va 6 ta qirrasi bor.
2. *Muntazam sakkizyoq,* ba`zan *oktaedr* deb atalib, uning 8 yog`i, 6 ta uchi va 12 ta qirrasi bor.
3. *Muntazam yigirmayoq, ikosaedr* deb atalib, uning 20 ta yog`i, 12 ta uchi va 30 ta qirrasi bor (1-chizma),



1-chizma



2-chizma

Ikosaedrning yoyilmasi ushbu ko'rinishga ega[3].(2-chizma)

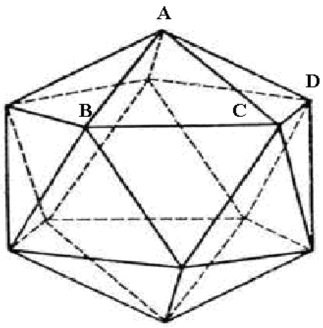
Tomonig a ga teng bo'lgan ikosaedrning quydagi elementlarini a ga nisbatan hisoblash qiziqarli hisoblanadi.

1.Ikosaedrning ikki yonli burchagini hisoblaymiz.

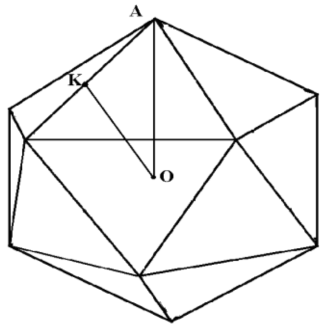
Ikosaedrning qirrasidagi ikki yoqli burchagini hisoblash uchun uning A uchidagi $ABCD$ uch yoqli burchagini qaraymiz. Bu uch yoqli burchagining ikkita yassi burchagi $BAC = CAD = 60^0$ ga teng. Ikki yoqli burchak qarshisidagi uchinchi yassi burchagi 108 ga teng (3-chizma).

Uch yoqli burchaklar uchun kosinuslar teoremasiga asosan $ABCD$ uch yoqli burchak uchun $\cos 108^0 = \cos^2 60^0 + \sin^2 60^0 \cos\varphi$ ga ega bo'lamiz.

$\cos 108^0 = -\sin 18^0 = -\dfrac{\sqrt{5}-1}{4}$, yoki $-\dfrac{\sqrt{5}-1}{4} = \dfrac{1}{4} + \dfrac{3}{4}\cos\varphi$, bundan esa $\cos\varphi = -\dfrac{\sqrt{5}}{2}$, yoki $\varphi = \pi - \arccos\dfrac{\sqrt{5}}{3}$.



3-chizma



4-chizma

2. Ikosaedr yon tamoni a ga teng bo'lgan muntazam uchburchakdan iborat. Shu sababli

$$S_{yon\,tomon} = \frac{\sqrt{3}}{4}a^2$$

3. To'la sirti esa

$$S_{to'la\,sirt} = 20S_{yonsirt} = \frac{20\sqrt{3}}{4}a^2 = 5\sqrt{3}a^2$$

4. Ixtiyori muntazam ko'pyoqning hajmini ushbu formula bilan hisoblash mumkin

$$V = \frac{G(S_{yon})^2 tg\frac{\alpha}{2}}{3P}$$

Bu yerda G -yon tamonlari soni, α -ikki yoqli burchagi, P - yon tamoni peremetri yarmi.

Unga asosan

$$V=\frac{20(\frac{\sqrt{3}}{4}a^2)^2 tg(\frac{\pi}{2}-\frac{1}{2}\arccos\frac{\sqrt{5}}{3})}{3\cdot\frac{3a}{2}}=\frac{\frac{15}{4}a^4}{\frac{9}{2}a}ctg(\frac{1}{2}\arccos\frac{\sqrt{5}}{3})=\frac{5}{6}\sqrt{\frac{1+\cos(\arccos\frac{\sqrt{5}}{3})}{1-\cos(\arccos\frac{\sqrt{5}}{3})}}\,a^3=$$

$$=\frac{5}{6}\sqrt{\frac{3+\sqrt{5}}{3-\sqrt{5}}}a^3=\frac{5(3+\sqrt{5})}{12}a^3.$$

5. Ikkosaedrga ichki chizilgan sfera radiusi uni ushbu formuladan toppish mumkin.

$$V=\frac{1}{3}r\,S_{to'lasirt}$$

yoki $$r=\frac{3V}{S_{to'lasirt}}=\frac{3\cdot\frac{5(3+\sqrt{5})}{12}a^3}{5\sqrt{3}a^2}=\frac{(3+\sqrt{5})}{4\sqrt{3}}a$$

6. Ikosaedrga tashqi chizilgan sfera radiusi (4-chizma) ΔOKA to‘g‘ri burchakli uchburchakning OA gipotenuzaga teng OK katet esa ichki chizilgan sfera radiusi $KA=\frac{\sqrt{3}}{3}a^2$ ga teng.

$$R=\sqrt{r^2+\left(\frac{\sqrt{3}}{3}a\right)^2}=\sqrt{\frac{(3+\sqrt{5})^2a^2}{16\cdot 3}+\frac{a^2}{3}}=\sqrt{\frac{9+6\sqrt{5}+5+16}{48}}\,a=\sqrt{\frac{30+6\sqrt{5}}{48}}\,a=\frac{a}{4}\sqrt{2(5+\sqrt{5})}$$

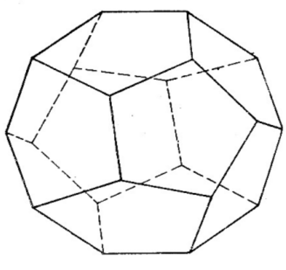
7. Ikosaedr barcha qirralariga urunuvchi sfera radiusini R^x hisoblaymiz. Ikosaedr chizmasidan (4-chizma) ΔOBL to‘g‘ri burchakli uchburchakni qaraymiz. Bu yerda $OB=r,\ BL=\frac{\sqrt{3}}{6}a$ ga teng

$$R^x=\sqrt{r^2+\left(\frac{\sqrt{3}}{6}a\right)^2}=\sqrt{\frac{(3+\sqrt{5})^2a^2}{48}+\frac{a^2}{12}}=\frac{a}{\sqrt{12}}\sqrt{\frac{18+6\sqrt{5}}{4}}=\frac{a}{\sqrt{12}}\sqrt{\frac{3(3+\sqrt{5})}{2}}=\frac{a}{2}\sqrt{\frac{3+\sqrt{5}}{2}}$$

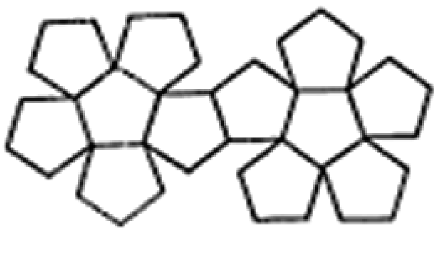
8. Ichki va tashqi chizilgan sfera radiuslar nisbatini hisoblaymiz.

$$\frac{R}{r}=\frac{\frac{a}{4}\sqrt{2(5+\sqrt{5})}}{\frac{(3+\sqrt{5})a}{4\sqrt{3}}}=\frac{\sqrt{3}\sqrt{2(5+\sqrt{5})}}{(3+\sqrt{5})}=\frac{\sqrt{6}\sqrt{(5+\sqrt{5})(3-\sqrt{5})^2}}{4}=\sqrt{15-6\sqrt{5}}$$

Endi yoqlari muntazam beshburchaklardan iborat muntazam ko`pyoqlarning ham turi bittadir uni ba`zan *dodekaedr* deb aytilib, 12 ta yoqdan, 20 ta uchdan va 30 qirradan iborat. (5- chizma)



5-chizma



6-chizma

Dodekaedrning yoyilmasi ushbu ko‘rinishga ega.(6-chizma)

Tomoni *a* ga teng bo‘lgan dodekaedrning quyidagi elementlarini qirrasi-*a* ga nisbatan hisoblash keltiriladi.

1. Dodekaedrning ikki yoqli burchagini hisoblaymiz.

Dodekaedrning har bir uchidagi yassi burchagi 108^0 ga teng, chunki bu burchak muntazam beshburchakning ichki burchagiga teng.

$$\cos 108^0 = \cos^2 108^0 + \sin^2 108^0 \cdot \cos\beta$$

Bu tenglikdan $\cos\beta$ ni topamiz

$$\cos\beta = \frac{\cos 108^0}{1+\cos 108^0} = \frac{-\sin 18^0}{1-\sin 18^0} = \frac{\frac{-1+\sqrt{5}}{4}}{1-\frac{-1+\sqrt{5}}{4}} = \frac{-1+\sqrt{5}}{5-\sqrt{5}} = \frac{-\sqrt{5}}{5}$$

Demak,

$$\cos\beta = -\frac{\sqrt{5}}{5} \quad \text{yoki} \quad \beta = \pi - \arccos\frac{\sqrt{5}}{5};$$

2. Yon tomonining yuzi.

Dodekaedr yon tomoni muntazam beshburchakning yuziga teng. Muntazam besh burchak yuzi ushbu formula bilan hisoblanadi.

$$S_{yon} = \frac{5}{4}a^2 ctg\frac{\pi}{5} = \frac{a^2\sqrt{25+10\sqrt{5}}}{4};$$

3. To‘la sirtini yuzi.

Dodekaedrning 12 tomoni bulgani uchun to‘la sirti yon sirtini 20 ga ko‘paytirganiga teng

$$S_{to'la} = 20 \cdot S_{yon} = 12 \cdot \frac{a^2\sqrt{25+10\sqrt{5}}}{4} = 3a^2\sqrt{25+10\sqrt{5}};$$

4. Dodekaedr hajmi.

Ixtiyori muntazam ko‘pyoqning hajmini ushbu formula bilan hisoblash mumkin

$$V = \frac{G(S_{yon})^2 tg\frac{\alpha}{2}}{3P}$$

bu yerda G -yon tamonlari soni, α -ikki yoqli burchagi, P - yon tamoni peremetri yarmi.

Unga asosan

$$V = \frac{12\left(\frac{a^2\sqrt{(25+10\sqrt{5})}}{4}\right)^2 tg\left(\frac{\pi}{2} - \frac{1}{2}\arccos\frac{\sqrt{5}}{5}\right)}{3 \cdot \frac{5a}{2}} = \frac{1}{4}\left(15+7\sqrt{5}\right)a^3$$

5. Dodekaedrga ichki chizilgan sfera radiusi.

Dodekaedr hajmidan foydalanib, ushbu formulaga

$$V = \frac{1}{3} \cdot r \cdot S_{t.s}$$

ko‘ra radiusni topish mumkin.

$$r=\frac{3\cdot V}{S_{t.s}}=\frac{\frac{3\cdot a^3}{4}\left(15+7\sqrt{5}\right)}{3\sqrt{25+10\sqrt{5}}\,a^2}=\frac{a}{4}\cdot\sqrt{10+\frac{22}{\sqrt{5}}}\cdot$$

6. Dodekaedrga tashqi chizilgan sfera radiusi.

Bu radiusni katetlari ichki sfera radiusi va tomoniga tashqi chizilgan aylana radiusidan iborat uchburchak gipotenuzasi deb qarash mumkin.

Muntazam beshburchakka tashqi chizilgan aylana radiusi ushbu formula yordamida hisoblanadi.

$$R^*=\sqrt{\frac{5+\sqrt{5}}{10}}a$$

Ichki chizilgan sfera radiusi esa bizga ma'lum Pifagor teoremasiga asosan

$$R=\sqrt{R^*+r^2}=\sqrt{\frac{\left(5+\sqrt{5}\right)}{10}a^2+\frac{a^2}{16}\left(\frac{50+22\sqrt{5}}{5}\right)}=\frac{a}{4}(1+\sqrt{5})\sqrt{3};$$

7. Dodekaedrning barcha qirralariga urunuvchi sfera radiusi.

Xuddi yuqoridagi usul bilan, tomoniga ichki chizilgan aylana radiusi va ichki chizilgan sfera radiuslarini katetlar sifatida qarab, gipotenuzani hisoblaymiz. Muntazam beshburchakka ichki chizilgan aylana radiusi:

$$r^*=\frac{\sqrt{5}\cdot\sqrt{5+2\sqrt{5}}}{10}a$$

ga teng.

$$R^{**}=\sqrt{\left(r^*\right)^2+r^2}=\sqrt{\frac{(5+2\sqrt{5})}{20}\cdot a^2+\frac{a^2}{16}(\frac{50+22\sqrt{5}}{5})}=\frac{a}{4}\cdot(3+\sqrt{5})\ ;$$

8. Dodekaedrga ichki va tashqi chizilgan sfera radiuslarini nisbatini hisoblaymiz.

$$\frac{R}{r}=\frac{\frac{a}{4}(1+\sqrt{5})\sqrt{3}}{\frac{a}{4}\sqrt{10+\frac{22}{\sqrt{5}}}}=\sqrt{15-6\sqrt{5}}.$$

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