



KOSHI TIPIDAGI INTEGRAL LIMITINING UZLUKSIZLIK XARAKTERI.

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Annotatsiya: Ko'p o'zgaruvchili funksiyaning Gyolder sinfi va uning xossalari, n – karrali Koshi tipidagi integralning polisilindirik soha chegarasidagi xarakteri.

KIRISH.

(z_k) ($k = \overline{1, n}$) – kompleks tekisligida γ^k – yopiq, to'g'rulanuvchi Jordan chizig'i (yo,t,j,ch) berilgan bo'lib, u (z_k) ($k = \overline{1, n}$) – tekislikni ikki ichki D_k^+ va tashqi D_k^- sohalarga ajratadi. Ishda ushbu

$$\Phi(z) = \frac{1}{(2\pi i)^n} \int_{\Delta} \frac{\varphi(\tau)}{\tau - z} d\tau \quad (1)$$

Koshi tipidagi integral qaraladi, bunda

$$\tau - z = \prod_{k=1}^n (\tau_k - z_k), \quad \tau = (\tau_1, \tau_2, \dots, \tau_n), \quad z = (z_1, z_2, \dots, z_n),$$

$$d\tau = d\tau_1 \cdot d\tau_2 \cdot \dots \cdot d\tau_n, \quad \Delta = \gamma^1 \times \gamma^2 \times \dots \times \gamma^n,$$

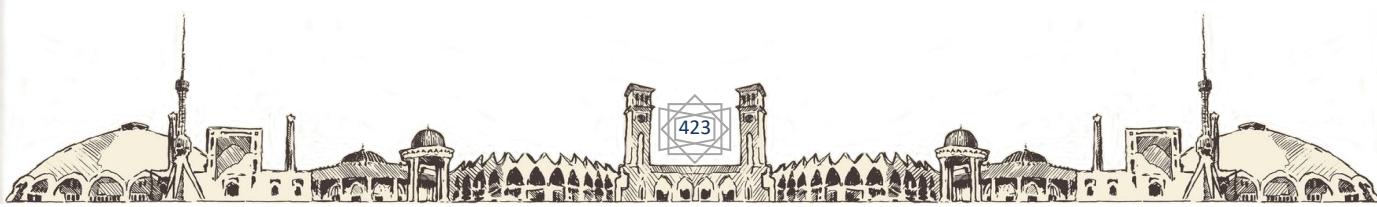
$\varphi \in C(\Delta) - \Delta$ da uzluksiz bo'lgan funksiyalar sinfi.

Biz kelgusida yozishni qisqartirish maqsadida hech qanday izohlarsiz [1], [2], [3] ishlardagi ba'zi belgilashlar va tasdiqlardan foydalanamiz.

Agar $\varphi(\tau) = \varphi(\tau_1, \tau_2, \dots, \tau_n)$ funksiya Δ – ostovda $H(\alpha_1, \alpha_2, \dots, \alpha_n)$ Gyolder shartini qanoatlantirsa, u holda (1) integralning $\Phi^{\pm\pm\dots\pm}(t)$ – limitik qiymatlari mavjud bo'ladi va limitik qiymatlar uchun Soxotskiy tipidagi formulalar o'rinali bo'ladi ([1], [5]). $\Phi(z)$ funksiyaning limitik qiymatlarini o'rganishda, uning tarkibida quydag'i maxsus integrallar paydo bo'ladi ([1], [4]).

$$\begin{aligned} & \int_{\Delta} \frac{\varphi_n(\tau; t)}{\tau - t} d\tau, \quad \int_{\Delta_{[p]}} \frac{\varphi_n(\tau_{t_p}; t)}{(\tau - t)_{[p]}} (d\tau)_{[p]} \quad (p = \overline{1, n}), \quad \int_{\Delta_{[pq]}} \frac{\varphi_n(\tau_{t_{pq}}; t)}{(\tau - t)_{[pq]}} (d\tau)_{[pq]} \\ & (p < q; p, q = \overline{1, n}), \dots, \int_{\gamma^p} \int_{\gamma^q} \frac{\varphi_n(t_{\tau_{pq}}; t)}{(\tau_p - t_p)(\tau_q - t_q)} d\tau_p d\tau_q \quad (p < q; p, q = \overline{1, n}), \\ & \int_{\gamma^p} \frac{\varphi_n(t_{\tau_p}; t)}{(\tau_p - t_p)} d\tau_p, \end{aligned} \quad (2)$$

bunda





$$\begin{aligned}\varphi_n(\tau; t) &= \varphi(\tau) - \sum_{p=1}^n \varphi(\tau_{t_p}) + \sum_{p=1}^n \sum_{\substack{q=1 \\ p < q}}^n \varphi(\tau_{t_{pq}}) - \dots + \\ &+ (-1)^{n-2} \sum_{p=1}^n \sum_{\substack{q=1 \\ p < q}}^n \varphi(t_{\tau_{pq}}) + (-1)^{n-1} \sum_{p=1}^n \varphi(t_{\tau_p}) + (-1)^n \varphi(t), \\ \varphi_n(\tau_{t_p}; t) &= \varphi(\tau_{t_p}) - \sum_{\substack{q=1 \\ q \neq p}}^n \varphi(\tau_{t_{pq}}) + \sum_{\substack{q=1 \\ q < r, q, r \neq p}}^n \sum_{r=1}^n \varphi(\tau_{t_{pqr}}) - \dots + \\ &+ (-1)^{n-3} \sum_{\substack{q=1 \\ q < r, q, r \neq p}}^n \sum_{r=1}^n \varphi(t_{\tau_{qr}}) + (-1)^{n-2} \sum_{\substack{q=1 \\ q \neq p}}^n \varphi(t_{\tau_q}) + (-1)^{n-1} \varphi(t), \\ \dots, \\ \varphi_n(t_{\tau_{pq}}; t) &= \varphi(t_{\tau_{pq}}) - \varphi(t_{\tau_q}) - \varphi(t_{\tau_p}) + \varphi(t), \\ \varphi_n(t_{\tau_p}; t) &= \varphi(t_{\tau_p}) - \varphi(t).\end{aligned}$$

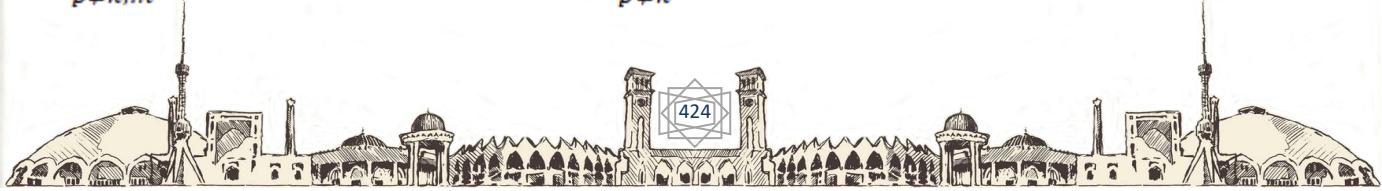
(2) maxsus integrallarni baholashda quydagи ayniyat va tengsizliklardan foydalaniлadi.

Osonlik bilan tekshirib ko'rish mumkinki, ushbu ayniyat o'riniлi:

$$\begin{aligned}\frac{1}{\prod_{k=1}^n (\tau_k - z_k)} - \frac{1}{\prod_{k=1}^n (\tau_k - t_k)} &= \frac{1}{\prod_{k=1}^n (\tau_k - t_k)} \left(\prod_{p=1}^n \frac{z_p - t_p}{\tau_p - z_p} + \right. \\ &+ \sum_{k=1}^n \prod_{\substack{p=1 \\ p \neq k}}^n \frac{z_p - t_p}{\tau_p - z_p} + \sum_{k=1}^n \sum_{m=1}^n \prod_{\substack{p=1 \\ k < m, p \neq k, m}}^n \frac{z_p - t_p}{\tau_p - z_p} + \dots + \\ &\quad \left. + \sum_{k=1}^n \sum_{m=1}^n \frac{z_k - t_k}{\tau_k - z_k} \cdot \frac{z_m - t_m}{\tau_m - z_m} + \sum_{k=1}^n \frac{z_k - t_k}{\tau_k - z_k} \right), \quad (3)\end{aligned}$$

bunda

$$\begin{aligned}\prod_{k=1}^n (\tau_k - t_k) &= \prod_{k=1}^n [(\tau_k - z_k) + (z_k - t_k)] = \prod_{p=1}^n (z_p - t_p) + \\ &+ \sum_{k=1}^n (\tau_k - z_k) \prod_{\substack{p=1 \\ p \neq k}}^n (z_p - t_p) + \sum_{k=1}^n \sum_{m=1}^n (\tau_k - z_k)(\tau_m - z_m) \cdot \\ &\cdot \prod_{\substack{p=1 \\ p \neq k, m}}^n (z_p - t_p) + \dots + \sum_{k=1}^n (z_k - t_k) \prod_{\substack{p=1 \\ p \neq k}}^n (\tau_p - z_p) + \prod_{k=1}^n (\tau_k - z_k).\end{aligned}$$





γ^k ($k = \overline{1, n}$) chiziqning silliqligidan

$$s(t_1, t_2) \leq m|t_1 - t_2| \quad (4)$$

tengsizlik o'rini bo'ladi ([3]), bunda m – musbat o'zgarmas son, $s = s(t_1, t_2) - \gamma^k$ chiziqning t_1, t_2 nuqtalarini birlashtiruvchi yoylarning eng kichigi.

Bu ishda quydagи teoremlar isbot qilinadi.

1-teorema. Agar (1) integralning zichligi $\varphi(\tau) \in H_\alpha(\Delta) = H_{\alpha_1, \alpha_2, \dots, \alpha_n}(\Delta)$ bo'lsa, u holda $\Phi(z)$ funksiyaning limitik qiymatlari $\Phi^{\pm\pm\dots\pm}(t)$ mavjud bo'ladi va ular Soxotskiy tipidagi formulalar o'rini bo'ladi ([5]).

2-teorema. Agar (1) integralning zichligi $\varphi \in H_{\alpha_1, \alpha_2, \dots, \alpha_n}(\Delta)$ bo'lsa, u holda $\Phi^{\pm\pm\dots\pm}(t) \in H_{\beta_1, \beta_2, \dots, \beta_n}(\Delta)$ bo'ladi,

bunda

$$\beta_k = \begin{cases} \alpha_k, & \alpha_k < 1, \text{ bo'lsa}, \\ 1 - \varepsilon, & \alpha_k = 1 \text{ bo'lsa}, \quad k = \overline{1, n}, \end{cases}$$

ε – istalgancha kichik musbat son. Takidlaymizki, bu teorema quydagи formulirovkada [1] ishida isbot qilingan.

3-teorema. Agar (1) integralning zichligi Δ – ostovda $\varphi \in H_{\alpha_1, \alpha_2, \dots, \alpha_n}(\Delta)$ bo'lsa, u holda $\Phi(z)$ funksiyaning limitik qiymatlari $\Phi^{\pm\pm\dots\pm}(t) \in H_{\alpha_1 - \varepsilon, \alpha_2 - \varepsilon, \dots, \alpha_n - \varepsilon}(\Delta)$ bo'ladi,

bunda ε – istalgancha kichik musbat son.

Takidlaymizki, 2-teorema 3-teoremaga nisbatan umumiy teorema bo'lib hisoblanadi.

Xulosa.

Ushbu maqola Gyolder fazosida n – karrali Koshi tipidagi integralning

$$\Phi(z) = \frac{1}{(2\pi i)^n} \int_{\Delta} \frac{\varphi(\tau)}{\prod_{k=1}^n (\tau_k - z_k)} d\tau$$

polisilindirik soha chegarasidagi xarakterini o'rganishga bag'ishlangan.

FOYDALANILGAN ADABIYOTLAR:

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