KOSHI TIPIDAGI INTEGRAL LIMITINING UZLUKSIZLIK XARAKTERI.

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Annotatsiya: *Ko’p o’zgaruvchili funksiyaning Gyolder sinfi va uning xossalari,* $n-$ *karrali Koshi tipidagi integralning polisilindirik soha chegarasidagi xarakteri.*

KIRISH.

$(z_k)\ (k=\overline{1,n})-$ kompleks tekisligida γ^k- yopiq, to’g’rilanuvchi Jordan chizig’i (yo,t,j,ch) berilgan bo’lib, u $(z_k)\ (k=\overline{1,n})-$ tekislikni ikki ichki D_k^+ va tashqi D_k^- sohalarga ajratadi. Ishda ushbu

$$\Phi(z)=\frac{1}{(2\pi i)^n}\int\limits_{\Delta}\frac{\varphi(\tau)}{\tau-z}d\tau \qquad (1)$$

Koshi tipidagi integral qaraladi, bunda

$$\tau-z=\prod_{k=1}^{n}(\tau_k-z_k),\qquad \tau=(\tau_1,\tau_2,\dots,\tau_n),\qquad z=(z_1,z_2,\dots,z_n),$$

$$d\tau=d\tau_1\cdot d\tau_2\cdot\ldots\cdot d\tau_n,\qquad \Delta=\gamma^1\times\gamma^2\times\ldots\times\gamma^n,$$

$\varphi\in C(\Delta)-\Delta$ da uzluksiz bo’lgan funksiyalar sinfi.

Biz kelgusida yozishni qisqartirish maqsadida hech qanday izohlarsiz [1], [2], [3] ishlardagi ba’zi belgilashlar va tasdiqlardan foydalanamiz.

Agar $\varphi(\tau)=\varphi(\tau_1,\tau_2,\dots,\tau_n)$ funksiya $\Delta-$ ostovda $H(\alpha_1,\alpha_2,\dots,\alpha_n)$ Gyolder shartini qanoatlantirsa, u holda (1) integralning $\Phi^{\pm\pm\cdots\pm}(t)-$ limitik qiymatlari mavjud bo’ladi va limitik qiymatlar uchun Soxotskiy tipidagi formulalar o’rinli bo’ladi ([1], [5]). $\Phi(z)$ funksiyaning limitik qiymatlarini o’rganishda, uning tarkibida quydagi maxsus integrallar paydo bo’ladi ([1], [4]).

$$\int\limits_{\Delta}\frac{\varphi_n(\tau;t)}{\tau-t}d\tau,\qquad \int\limits_{\Delta_{[p]}}\frac{\varphi_n(\tau_{t_p};t)}{(\tau-t)_{[p]}}(d\tau)_{[p]}\ \ (p=\overline{1,n}),\qquad \int\limits_{\Delta_{[pq]}}\frac{\varphi_n(\tau_{t_{pq}};t)}{(\tau-t)_{[pq]}}(d\tau)_{[pq]}$$

$$(p<q;\ p,q=\overline{1,n}),\ \ \dots\ ,\int\limits_{\gamma^p}\int\limits_{\gamma^q}\frac{\varphi_n\left(t_{\tau_{pq}};t\right)}{(\tau_p-t_p)(\tau_q-t_q)}d\tau_p d\tau_q\ \ (p<q;\ p,q=\overline{1,n}),$$

$$\int\limits_{\gamma^p}\frac{\varphi_n\left(t_{\tau_p};t\right)}{(\tau_p-t_p)}d\tau_p, \qquad (2)$$

bunda

$$\varphi_n(\tau;t)=\varphi(\tau)-\sum_{p=1}^{n}\varphi\left(\tau_{t_p}\right)+\sum_{\substack{p=1\\ }}^{n}\sum_{\substack{q=1\\ p<q}}^{n}\varphi\left(\tau_{t_{pq}}\right)-\cdots+$$

$$+(-1)^{n-2}\sum_{p=1}^{n}\sum_{\substack{q=1\\ p<q}}^{n}\varphi\left(t_{\tau_{pq}}\right)+(-1)^{n-1}\sum_{p=1}^{n}\varphi\left(t_{\tau_p}\right)+(-1)^{n}\varphi(t),$$

$$\varphi_n\left(\tau_{t_p};t\right)=\varphi\left(\tau_{t_p}\right)-\sum_{\substack{q=1\\ q\neq p}}^{n}\varphi\left(\tau_{t_{pq}}\right)+\sum_{q=1}^{n}\sum_{\substack{r=1\\ q<r,\ q,r\neq p}}^{n}\varphi\left(\tau_{t_{pqr}}\right)-\cdots+$$

$$+(-1)^{n-3}\sum_{q=1}^{n}\sum_{\substack{r=1\\ q<r\ q,r\neq p}}^{n}\varphi\left(t_{\tau_{qr}}\right)+(-1)^{n-2}\sum_{\substack{q=1\\ q\neq p}}^{n}\varphi\left(t_{\tau_q}\right)+(-1)^{n-1}\varphi(t),$$

.... ,

$$\varphi_n\left(t_{\tau_{pq}};t\right)=\varphi\left(t_{\tau_{pq}}\right)-\varphi\left(t_{\tau_q}\right)-\varphi\left(t_{\tau_p}\right)+\varphi(t),$$

$$\varphi_n\left(t_{\tau_p};t\right)=\varphi\left(t_{\tau_p}\right)-\varphi(t).$$

(2) maxsus integrallarni baholashda quydagi ayniyat va tengsizliklardan foydalaniladi.

Osonlik bilan tekshirib ko'rish mumkinki, ushbu ayniyat o'rinli:

$$\frac{1}{\prod_{k=1}^{n}(\tau_k-z_k)}-\frac{1}{\prod_{k=1}^{n}(\tau_k-t_k)}=\frac{1}{\prod_{k=1}^{n}(\tau_k-t_k)}\Big(\prod_{p=1}^{n}\frac{z_p-t_p}{\tau_p-z_p}+$$

$$+\sum_{k=1}^{n}\prod_{\substack{p=1\\ p\neq k}}^{n}\frac{z_p-t_p}{\tau_p-z_p}+\sum_{k=1}^{n}\sum_{m=1}^{n}\prod_{\substack{p=1\\ k<m,\ p\neq k,m}}^{n}\frac{z_p-t_p}{\tau_p-z_p}+\cdots+$$

$$+\sum_{k=1}^{n}\sum_{\substack{m=1\\ k<m}}^{n}\frac{z_k-t_k}{\tau_k-z_k}\cdot\frac{z_m-t_m}{\tau_m-z_m}+\sum_{k=1}^{n}\frac{z_k-t_k}{\tau_k-z_k}\Big), \qquad (3)$$

bunda

$$\prod_{k=1}^{n}(\tau_k-t_k)=\prod_{k=1}^{n}[(\tau_k-z_k)+(z_k-t_k)]=\prod_{p=1}^{n}(z_p-t_p)+$$

$$+\sum_{k=1}^{n}(\tau_k-z_k)\prod_{\substack{p=1\\ p\neq k}}^{n}(z_p-t_p)+\sum_{k=1}^{n}\sum_{m=1}^{n}(\tau_k-z_k)(\tau_m-z_m)\cdot$$

$$\cdot\prod_{\substack{p=1\\ p\neq k,m}}^{n}(z_p-t_p)+\ldots+\sum_{k=1}^{n}(z_k-t_k)\prod_{\substack{p=1\\ p\neq k}}^{n}(\tau_p-z_p)+\prod_{k=1}^{n}(\tau_k-z_k).$$

$\gamma^k\ (k=\overline{1,n})$ chiziqning silliqligidan

$$s(t_1,t_2) \le m|t_1-t_2| \qquad (4)$$

tengsizlik o'rinli bo'ladi ([3]), bunda $m-$ musbat o'zgarmas son, $s=s(t_1,t_2)-\gamma^k$ chiziqning t_1,t_2 nuqtalarini birlashtiruvchi yoylarning eng kichigi.

Bu ishda quydagi teoremalar isbot qilinadi.

1-teorema. Agar (1) integralning zichligi $\varphi(\tau)\in H_\alpha\ (\Delta)=H_{\alpha_1,\alpha_2,\ldots,\alpha_n}(\Delta)$ bo'lsa, u holda $\Phi(z)$ funksiyaning limitik qiymatlari $\Phi^{\pm\pm\cdots\pm}(t)$ mavjud bo'ladi va ular Soxotskiy tipidagi formulalar o'rinli bo'ladi ([5]).

2-teorema. Agar (1) integralning zichligi $\varphi\in H_{\alpha_1,\alpha_2,\ldots,\alpha_n}(\Delta)$ bo'lsa, u holda $\Phi^{\pm\pm\cdots\pm}(t)\in H_{\beta_1,\beta_2,\ldots,\beta_n}(\Delta)$ bo'ladi,

bunda

$$\beta_k=\begin{cases}\alpha_k, & \alpha_k<1\ ,\ \text{bo'lsa,}\\ 1-\varepsilon, & \alpha_k=1\ \text{bo'lsa,}\qquad k=\overline{1,n},\end{cases}$$

$\varepsilon-$ istalgancha kichik musbat son. Takidlaymizki, bu teorema quydagi formulirovkada [1] ishida isbot qilingan.

3-teorema. Agar (1) integralning zichligi $\Delta-$ ostovda $\varphi\in H_{\alpha_1,\alpha_2,\ldots,\alpha_n}(\Delta)$ bo'lsa, u holda $\Phi(z)$ funksiyaning limitik qiymatlari $\Phi^{\pm\pm\cdots\pm}(t)\in H_{\alpha_1-\varepsilon,\alpha_2-\varepsilon,\ldots,\alpha_n-\varepsilon}(\Delta)$ bo'ladi,

bunda $\varepsilon-$ istalgancha kichik musbat son.

Takidlaymizki, 2-teorema 3-teoremaga nisbatan umumiy teorema bo'lib hisoblanadi.

Xulosa.

Ushbu maqola Gyolder fazosida $n-$ karrali Koshi tipidagi integralning

$$\Phi(z)=\frac{1}{(2\pi i)^n}\int_\Delta\frac{\varphi(\tau)}{\prod_{k=1}^n(\tau_k-z_k)}d\tau$$

polisilindirik soha chegarasidagi xarakterini o'rganishga bag'ishlangan.

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