IKKI ZARRACHALI SHREDINGER OPERATORINING XOS QIYMATLARI MAVJUDLIGI VA ULARNING SONI

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Annotatsiya: *Ikki zarrachali sistemaga mos diskret Shredinger operatori uchun (?) operatorning xos qiymatlari hosil bo'lish o'rni va ularning soni*

Kalit so'zlar: *Fredgolm determinanti, muhim spektr, virtual sath, xos qiymat, ko'paytirish operatori, qo'zg'atuvchi operator.*

THE EXISTENCE AND NUMBERS OF EIGENVALUES OF TWO PARTICAL SHREDINGER OPERATOR

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Annotation: *Fredholm determinant of discrete Shredinger operator suitable for two-particle system, a necessary and sufficient condition for $z \in \mathbb{C}$ to be eigenvalue of operator $H_{\mu,\gamma}(K)$, the place of eigenvalues and its numbers of operator $H_{\mu,\gamma}(K)$*

Key words: *Fredholm determinant, essential spectrum, virtual level, eigenvalue, multiplication operator, instigator operator.*

$L^2(\mathbb{T}^3)$ Hilbert fazosidagi chegaralangan o'z-o'ziga qo'shma operator va shu fazodagi ikki zarrachali Diskret Shrodinger operatorini quyidagicha aniqlaymiz:

$$H_{\mu,\gamma}(K) = H_{\gamma}^{0}(K) + \mu V$$

$H_{\gamma}^{0}(K)$ operator $L^2(\mathbb{T}^3)$ fazodagi $\xi_{K,\gamma}(p)$ funksiyaga ko'paytirish operatori quyidagicha aniqlanadi:

$$(H_{\gamma}^{0}(K)f)(p) = \xi_{K,\gamma}(p)f(p),\ f \in L^2(\mathbb{T}^3)$$

bunda

$$\xi_{K,\gamma}(p) = \varepsilon(p) + \gamma\varepsilon(K-p),\ \gamma > 0$$

Ushbu $\varepsilon(\cdot)$ funksiya quyidagi ko'rinishga ega:

$$\varepsilon(p) = \sum_{i=1}^{3}(1 - cosp^{(i)}),\ p = (p^{(1)}, p^{(2)}, p^{(3)}) \in \mathbb{T}^3$$

Qo'zg'atuvchi V integral operator:

$$(Vf)(p)=\frac{1}{(2\pi)^3}\int_{\mathbb{T}^3} f(q)dq,\quad f\in L^2(\mathbb{T}^3)$$

$H_{\gamma}^{0}(K)$ ko’paytirish operatorining qo’zg’atuvchisi V chegaralangan o’z-o’ziga qo’shma operator bo’lib, rangi birga teng. Shuning uchun Weyl teoremasiga asosan $H_{\mu,\gamma}(K)$, $K\in\mathbb{T}^3$, opratorning muhim spektri haqiqiy o’qdagi quyidagi kesmadan iborat:

$$\sigma_{ess}(H_{\mu,\gamma}(K))=\sigma_{ess}(H_{\gamma}^{0}(K))=[\xi_{min,\gamma}(K),\xi_{max,\gamma}(K)],$$

bunda

$$\xi_{min,\gamma}(K)=\min_{p\in\mathbb{T}^3}\xi_{K,\gamma}(p),\quad \xi_{max,\gamma}(K)=\max_{p\in\mathbb{T}^3}\xi_{K,\gamma}(p)$$

va $\xi_{K,\gamma}(p)$ funksiya (3) formula bilan aniqlangan. $\xi_{K,\gamma}(p)$ funksiyaning maksimum va minimumini hisoblaymiz. Buning uchun uni quyidagicha o’zgartiramiz:

$$\xi_{K,\gamma}(p)=\xi(p)+\gamma\xi(K-p)=$$

$$\sum_{i=1}^{3}(1-cosp^{(i)})+\gamma\sum_{i=1}^{3}(1-cos(K^{(i)}-p^{(i)}))=$$

$$=3(1+\gamma)-\sum_{i=1}^{3}(cosp^{(i)}+\gamma cos(K^{(i)}-p^{(i)}))=$$

$$=3(1+\gamma)-\sum_{i=1}^{3}(cosp^{(i)}+\gamma cosK^{(i)}cosp^{(i)}+\gamma sinK^{(i)}sinp^{(i)})=$$

$$=3(1+\gamma)-\sum_{i=1}^{3}[(1+\gamma cosK^{(i)})cosp^{(i)}+\gamma sinK^{(i)}sinp^{(i)}]$$

$$acosx+bsinx=\sqrt{a^2+b^2}cos(x-y)$$

$$cosy=\frac{a}{\sqrt{a^2+b^2}},\quad siny=\frac{b}{\sqrt{a^2+b^2}}$$

Ushbu

$$(1+\gamma K^{(i)})^2+(\gamma sinK^{(i)})^2=1+2\gamma cosK^{(i)}+\gamma^2$$

tenglikdan foydalanib $\xi_{K,\gamma}(\cdot)$ ni quyidagi ko’rinishga keltiramiz:

$$3(1+\gamma)-\sum_{i=1}^{3}[(1+\gamma cosK^{(i)})cosp^{(i)}+\gamma sinK^{(i)}sinp^{(i)}]=$$

$$=3(1+\gamma)-\sum_{i=1}^{3}\sqrt{1+2\gamma cosK^{(i)}+\gamma^2}\cdot$$

$$\cdot[\frac{1+\gamma cosK^{(i)}}{\sqrt{1+2\gamma cosK^{(i)}+\gamma^2}}cosp^{(i)}+\frac{\gamma sinK^{(i)}}{\sqrt{1+2\gamma cosK^{(i)}+\gamma^2}}sinp^{(i)}]=$$

$$=3(1+\gamma)-$$

$$-\sum_{i=1}^{3}\sqrt{1+2\gamma cosK^{(i)}+\gamma^2}\,[cosp_\gamma K^{(i)}cosp^{(i)}+sinp_\gamma K^{(i)}sinp^{(i)}]=$$

$$3(1+\gamma)-\sum_{i=1}^{3}\sqrt{1+2\gamma cosK^{(i)}+\gamma^2}\,cos(p^{(i)}-p_\gamma K^{(i)})$$

bunda

$$cosp_\gamma K^{(i)}=\frac{1+\gamma cosK^{(i)}}{\sqrt{1+2\gamma cosK^{(i)}+\gamma^2}},$$

$$sinp_\gamma K^{(i)}=\frac{\gamma sinK^{(i)}}{\sqrt{1+2\gamma cosK^{(i)}+\gamma^2}},$$

$$p_\gamma(K^{(i)})=arcsin\frac{\gamma sinK^{(i)}}{\sqrt{1+2\gamma cosK^{(i)}+\gamma^2}}.$$

Natijada $\xi_{K,\gamma}$ funksiya quyidagi ko'rinishga keltiriladi:

$$\xi_{K,\gamma}=3(1+\gamma)-\sum_{i=1}^{3}\sqrt{1+2\gamma cosK^{(i)}+\gamma^2}\,cos(p^{(i)}-p_\gamma(K^{(i)}))$$

Agar $p^{(i)}=p_\gamma(K^{(i)})$ bo'lsa,

$$\xi_{\min,\gamma}(K)=\min_{p\in\mathbb{T}^3}\xi_{K,\gamma}(p)=3(1+\gamma)-\sum_{i=1}^{3}\sqrt{1+2\gamma cosK^{(i)}+\gamma^2}$$

tenglikka va $p^{(i)}=(\pi+p_\gamma(K^{(i)}))$ bo'lsa,

$$\xi_{\max,\gamma}(K)=\max_{p\in\mathbb{T}^3}\xi_{K,\gamma}(p)=3(1+\gamma)+\sum_{i=1}^{3}\sqrt{1+2\gamma cosK^{(i)}+\gamma^2}$$

tenglikka ega bo'lamiz.

$\mathbb{C}$ komleks sonlar maydoni bo'lsin ixtiyoriy $K\in\mathbb{T}^3$ va $z\in\mathbb{C}\backslash[\xi_{\min,\gamma}(K),\xi_{\max,\gamma}(K)]$ uchun $\nu_\gamma(K,z)$ analitik funksiyani aniqlaymiz:

$$\nu_\gamma(K,z)=\frac{1}{(2\pi)^3}\int_{\mathbb{T}^d}\frac{1}{z-\xi_{K,\gamma}(q)}dq.$$

Eslatma 1. Ixtiyoriy $K\in\mathbb{T}^3$ da $\xi_{K,\gamma}(\cdot)$ funksiya $p=\vec{\pi}+p_\gamma(K)$ nuqtada aynimagan maksimumga ega va

$$\nu_\gamma(K)=\nu_\gamma(K,\xi_{\max,\gamma}(K))=\frac{1}{(2\pi)^d}\int_{\mathbb{T}^d}\frac{dq}{\xi_{\max,\gamma}(K)-\xi_{K,\gamma}(q)}$$

integral mavjud va silliq funksiyani aniqlaydi.

Ixtiyoriy $K\in\mathbb{T}^3$ va $z\in\mathbb{C}\backslash[\xi_{\min,\gamma}(K),\xi_{\max,\gamma}(K)]$, uchun $H_{\mu,\gamma}(K)$ operatorga mos $\Delta_{\mu,\gamma}(K,z)$ Fredgolm determinantini aniqlaymiz:

$$\Delta_{\mu,\gamma}(K,z)=1-\mu\nu_\gamma(K,z).$$

Aytib o'tish kerakki $\mathbb{T}^d\times(\mathbb{C}\backslash[\xi_{\min,\gamma}(K),\xi_{\max,\gamma}(K)])$ da $\Delta_{\mu,\gamma}(K,z)$ haqiqiy analitik funksiya bo'ladi.

Lemma 1. $z\in\mathbb{C}\backslash[\xi_{\min,\gamma}(K),\xi_{\max,\gamma}(K)]$ soni $H_{\mu,\gamma}(K)$ operatorning xos qiymati bo'lishi uchun $\Delta_{\mu,\gamma}(K,z)=0$ bo'lishi zarur va yetrali

Lemma 2. Quyidagi tasdiqlar ekvivalent:

(i) birorta $K\in\mathbb{T}^3$ uchun $H_{\mu,\gamma}(K)$ operator $z=\xi_{\max,\gamma}(K)$ da virtual sathga ega va mos virtual holat quyidagi ko'rinishga ega:

$$\psi^\gamma_{\mu_0,K}(\cdot)=\frac{\mu_0\cdot c}{\xi_{\max,\gamma}(K)-\xi_{K,\gamma}(\cdot)}\in L^2(\mathbb{T}^3);$$

$$(ii)\ \Delta_{\mu,\gamma}(K,\xi_{\max,\gamma}(K))=0;$$

$$(iii)\ \mu=\frac{1}{\nu_\gamma(K)}.$$

Lemma 3. Ixtiyoriy $K\in\mathbb{T}^3_0=\mathbb{T}^3\backslash\{0\}$ da $\Delta_{\mu,\gamma}(K,\xi_{\max,\gamma}(0))$ qat'iy musbat.

Ixtiyoriy $\mu>0$ son uchun quyidagi to'plamlarni aniqlaymiz:

$$M^\gamma_<(\mu)=\{K\in\mathbb{T}^3:1-\mu\nu_\gamma(K)<0\}$$
$$M^\gamma_=(\mu)=\{K\in\mathbb{T}^3:1-\mu\nu_\gamma(K)=0\}$$
$$M^\gamma_>(\mu)=\{K\in\mathbb{T}^3:1-\mu\nu_\gamma(K)>0\}$$

Eslatma 2. Aytib o'tish kerakki $M^\gamma_<(\mu)$ to'plam $\mathbb{T}^3$ dagi koo'lchami 1 ga teng bo'lgan chiziqli ko'pxillik bo'ladi.

Eslatma 3. Agar $M^\gamma_>(\mu)\neq\emptyset$ bo'lsa, $K=0\in M^\gamma_>(\mu)$ bo'ladi.

Eslatma 4. $M^\gamma_<(\mu)$ $(M^\gamma_>(\mu))$ to'plam $\mathbb{T}^3$ da simmetrik va ochiq to'plam,

$K\in M^\gamma_<(\mu)$ $(resp.\, K\in M^\gamma_>(\mu))$ bo'lsa, u holda $-K\in M^\gamma_<(\mu)$ $(-K\in M^\gamma_>(\mu))$

Teorema. Birorta $K\in\mathbb{T}^3$ da $\mu_0=\frac{1}{\nu_\gamma(K)}$ bo'lsin. U holda:

(i) Ixtiyoriy $K\in M^\gamma_=(\mu_0)$ uchun $H_{\mu_0,\gamma}(K)$ operatorning muhim spektri $\sigma_{ess}(H_{\mu_0,\gamma}(K))$ ning yuqori chekkasi $E_{\mu_0,\gamma}(K)=\xi_{\max,\gamma}(K)$ da virtual sathga ega va mos virtual holat quyidagi ko'rinishda yoziladi:

$$\psi^\gamma_{\mu_0,K}(\cdot)=\frac{\mu_0\cdot c}{E_{\mu_0,\gamma}(K)-\xi_{K,\gamma}(\cdot)}\in L^1(\mathbb{T}^3)\backslash L^2(\mathbb{T}^3),$$

bunda $c=constant$

(ii) Ixtiyoriy $K\in M^\gamma_<(\mu_0)$ uchun $H_{\mu_0,\gamma}(K)$ operatorning muhim spektri $\sigma_{ess}(H_{\mu_0,\gamma}(K))$ dan tashqarida yagona $E_{\mu_0,\gamma}(K)$ xos qiymatga ega va bu xos qiymat $M^\gamma_<(\mu_0)$ da juft, haqiqiy analitik bo'lib, ushbu

$\xi_{\max,\gamma}(K) < E_{\mu_0,\gamma}(K) < \xi_{\max,\gamma}(0) \neq 0$ munosabatlarni qanoatlantiradi. Bundan tashqari mos xos funksiya

$$\psi^{\gamma}_{\mu_0,K}(\cdot) = \frac{\mu_0 \cdot c}{E_{\mu_0,\gamma}(K) - \xi_{K,\gamma}(\cdot)}, \quad c = constant$$

ham $\psi^{\gamma}_{\mu_0,K}: M^{\gamma}_{<}(\mu_0) \to L^2(\mathbb{T}^3)$ funksiya sifatida haqiqiy analitik bo'ladi.

(iii) Ixtiyoriy $\mu > \mu_0$ va $K \in M^{\gamma}_{=}(\mu_0) \cup M^{\gamma}_{<}(\mu_0)$ uchun $H_{\mu,\gamma}(K)$ operatorning muhim spektri $\sigma_{ess}(H_{\mu,\gamma}(K))$ dan tashqarida yagona $E_{\mu,\gamma}(K)$ xos qiymatga ega. Shu bilan birga

$$E_{\mu,\gamma}(K) > E_{\mu_0,\gamma}(K) > \xi_{\max,\gamma}(K), \; K \in \mathbb{T}^3, \; K \neq 0$$

va

$$E_{\mu,\gamma}(0) > E_{\mu_0,\gamma}(0) = \xi_{\max,\gamma}(0)$$

munosabatlar o'rinli.

(iv) Ixtiyoriy $K \in M^{\gamma}_{>}(\mu_0)$ uchun $H_{\mu_0,\gamma}(K)$ operatorning muhim spektri $\sigma_{ess}(H_{\mu_0,\gamma}(K))$ ning yuqori chekkasi $\xi_{\max,\gamma}(K)$ dan yuqorida xos qiymatga ega emas.

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