KOMPLEKS O‘ZGARUVCHILI KOSHI TIPIDAGI INTEGRALNING MAVJUDLIK SHARTI

O‘razaliyev Shirinboy Bo‘ron o‘g‘li
Samarqand iqtisodiyot va servis instituti o‘qituvchisi
Rustamov Sarvar Jasurovich
Samarqand iqtisodiyot va servis instituti talabasi
Jozilova Zuhra O‘tkir qizi
TATU Samarqand filiali talabasi

Annotatsiya: *Koshi tipidagi integralning ko‘p kompleks o‘zgaruvchili sohada mavjudlik sharti haqida.*

Аннотация: *Об условии существования интеграла типа Коши в области многих комплексных переменных*.

Annotation: *On the existence condition of the Koshi type integral in the domain of many complex variables.*

$\boldsymbol{\varphi(t) = \varphi(t_1, t_2, \dots, t_n)}$ funksiya $\Delta -$ ostovda aniqlangan bo‘lsin.

$\boldsymbol{t = (t_1, t_2, \dots, t_n)} \in \Delta$ nuqtani markaz qilib istalgancha kichik $\boldsymbol{r_k}$ radiusli $\boldsymbol{C(t_k, r_k)}$ polisilindir chizaylikki, har bir $\boldsymbol{D_k^{\cdot}}$ kontur polisilindirlar bilan faqat ikki nuqtada kesishsin. $\boldsymbol{D_k^{\cdot}}$ konturning $\boldsymbol{C(t_k, r_k)}$ polisilindirning ichkarisidagi qismini $\boldsymbol{d_k^{\cdot}}$ qolgan qismini esa $\boldsymbol{D_k^{\cdot} - d_k^{\cdot}}$ bilan belgilaymiz. $\boldsymbol{D_\varepsilon = (D_1^{\cdot} - d_k^{\cdot}) \times (D_2^{\cdot} - d_k^{\cdot}) \times \dots \times (D_n^{\cdot} - d_k^{\cdot})}$ deb belgilaymiz. Ravshanki, $\boldsymbol{\varepsilon \to 0}$ da $\boldsymbol{D_\varepsilon} \to \Delta$.

1-ta’rif. Agar $\lim_{\varepsilon\to 0} \Phi_\varepsilon(t)$ (bunda

$$\Phi_\varepsilon(t) = \frac{1}{(2\pi i)^n} \int_{D_\varepsilon} \frac{\varphi(\tau)}{\prod_{k=1}^{n}(\tau_k - z_k)} d\tau) \qquad (1)$$

limit mavjud bo‘lsa, u holda

$$\Phi(t) = \frac{1}{(2\pi i)^n} \int_{\Delta} \frac{\varphi(\tau)}{\prod_{k=1}^{n}(\tau_k - t_k)} d\tau \qquad (2)$$

maxsus integral Koshining bosh qiymat manosida mavjud deyiladi va u

$$\lim_{\varepsilon\to 0} \Phi_\varepsilon(t) = V \cdot P \frac{1}{(2\pi i)^n} \int_{\Delta} \frac{\varphi(\tau)}{\prod_{k=1}^{n}(\tau_k - t_k)} d\tau = V \cdot P\, \Phi(t)$$

kabi belgilanadi.

Bundan keyin (2) maxsus integralning bosh qismini oddiy integral deb ataymiz va uni qisqacha $\Phi(t) = \frac{1}{2^n} S\varphi(\tau)$ deb ham belgilaymiz.

(2) maxsus integralning bosh qiymati har doim ham mavjud bo‘lavermaydi. Shuning uchun u qanday funksiyalar sinfi uchun o‘rinli bo‘ladi degan masala bilan shug‘ullanamiz.

1-teorema. Agar (2) maxsus integralning zichligi Δ ostovda Gyolder shartini qanoatlantirsa, u holda (2) maxsus integral bosh qiymat ma’nosida mavjud bo‘ladi.

Isbot. Yozishni qisqartirish maqsadida teoremani $n = 3$ bo‘lganda isbotlaymiz. Teoremani isbot qilish jarayonida Gyolder shartini qanoatlantiruvchi tengsizliklardan foydalanamiz.

(2) integralning zichligi $\varphi(\tau_1, \tau_2, \tau_3)$ ni $\varphi(\tau_1, \tau_2, \tau_3) = \varphi_3(\tau; t) + \varphi_3(\tau_{t_1}; t) + \varphi_3(\tau_{t_2}; t) + \varphi_3(\tau_{t_3}; t) + +\varphi_3(t_{\tau_1}; t) +$ $+\varphi_3(t_{\tau_2}; t) + \varphi_3(t_{\tau_3}; t) + \varphi(t_1, t_2, t_3)$ ayniyatning o‘ng tomoni bilan almashtirib topamiz.

$$\Phi_\varepsilon(t) = \frac{1}{(2\pi i)^3} \int_{D_\varepsilon} \frac{\varphi_3(\tau, t)}{\prod_{k=1}^{3}(\tau_k - t_k)} d\tau + \frac{1}{(2\pi i)^3} \sum_{k=1}^{3} \int_{D_\varepsilon} \frac{\varphi_3(\tau_{t_k}, t)}{\prod_{k=1}^{3}(\tau_k - t_k)} d\tau +$$

$$+ \frac{1}{(2\pi i)^3} \sum_{k=1}^{3} \int_{D_\varepsilon} \frac{\varphi_3(t_{\tau_k}, t)}{\prod_{k=1}^{3}(\tau_k - t_k)} d\tau + \frac{\varphi(t)}{(2\pi i)^3} \int_{D_\varepsilon} \frac{1}{\prod_{k=1}^{3}(\tau_k - t_k)} d\tau \qquad (3)$$

(3) ning o‘ng tomonidagi integralni mos ravishda $\overset{3}{S}_\varepsilon \varphi_3(\tau, t)$; $\overset{2}{S}_\varepsilon \varphi_3(\tau_{t_k}, t)$ $(k = 1, 2, 3)$; $\overset{1}{S}_\varepsilon \varphi_3(t_{\tau_k}, t)$ $(k = 1, 2, 3)$ va $\overset{0}{S}_\varepsilon$.

$\overset{3}{S}_\varepsilon \varphi_3(\tau, t)$ ni baholash uchun $|\varphi_3(\tau; t)| \le 4 \prod_{k=1}^{3} A_k^{\frac{1}{3}} |\tau_k - t_k|^{\frac{\alpha_k}{3}}$ dan foydalanib topamiz.

$$\left| \overset{3}{S}_\varepsilon \varphi_3(\tau, t) \right| \le \frac{4\sqrt[3]{A_1 A_2 A_3}}{(2\pi)^3} \int_{D_\varepsilon} \prod_{k=1}^{3} |\tau_k - t_k|^{\frac{\alpha_k}{3} - 1} |d\tau_k| =$$

$$= \frac{1}{2\pi^3} \prod_{k=1}^{3} \sqrt[3]{A_k} \int_{D_k^{\cdot} - d_k^{\cdot}} |\tau_k - t_k|^{\frac{\alpha_k}{3} - 1} |d\tau_k| \qquad (4)$$

(4) ning o‘ng tomonidagi ko‘paytmaning tagida turgan integrallarning har biri ε→0 da oddiy Riman ma’nosida mavjud. Qolgan integrallarni baholashda quydagi tengsizliklardan foydalanamiz.

$$\left| \frac{1}{2\pi i} \int_{D_k^{\cdot} - d_k^{\cdot}} \frac{d\tau_k}{\tau_k - t_k} \right| < 1, \qquad k = 1, 2, 3 \qquad (5)$$

$\overset{3}{S}_\varepsilon \varphi_3(\tau_{t_3}, t)$ ni baholashda (5) va $\left| \varphi_3\left(t_{\tau_p}; t\right) \right| \le 2 \prod_{k=1}^{3} A_k^{\frac{1}{2}} |\tau_k - t_k|^{\frac{\alpha_k}{2}}$; $p = \overline{1, 3}$; $k \ne p$, dan $p = 3$ da foydalanib topamiz.

$$\left| \overset{3}{S}_\varepsilon \varphi_3(\tau_{t_3}, t) \right| = \left| \frac{1}{2\pi i} \int_{D_k^{\cdot} - d_3^{\cdot}} \frac{d\tau_3}{\tau_3 - t_3} \right| \left| \frac{1}{(2\pi i)^2} \int_{D_k^{\cdot} - d_1^{\cdot}} \int_{D_k^{\cdot} + d_2^{\cdot}} \frac{\varphi_3(\tau_{t_3}, t) d\tau_1 d\tau_2}{(\tau_1 - t_1)(\tau_2 - t_2)} \right|$$

$$\le \frac{1}{2\pi^2} \prod_{k=1}^{2} \sqrt{A_k} \int_{D_k^{\cdot} - d_k^{\cdot}} |\tau_k - t_k|^{\frac{\alpha_k}{2} - 1} |d\tau_k| \qquad (6)$$

(6) ning o‘ng tomonidagi ko‘paytmaning tagida turgan har biri ε→0 da oddiy Riman ma’nosida mavjud bo‘ladi. Xuddi shunday $\overset{2}{S}_\varepsilon \varphi_3(\tau_{t_2}, t)$ va $\overset{2}{S}_\varepsilon \varphi_3(\tau_{t_1}, t)$ integrallar uchun ham baholar o‘rinli bo‘ladi.

$\overset{1}{S}_{\varepsilon}\varphi_3(t_{\tau_3},t)$ uchun (5) ni va $\left|\varphi_3\left(t_{\tau_p},t\right)\right| \le A_p|\tau_p - t_p|^{\alpha_p}$, $p=\overline{1,3}$ ni $p=3$ da e’tiborga olgan holda topamiz.

$$\left|\overset{1}{S}_{\varepsilon}\varphi_3(t_{\tau_3},t)\right| = \left|\frac{1}{2\pi i}\int_{\dot{D_1}-\dot{d_1}}\frac{d\tau_1}{\tau_1-t_1}\right|\cdot\left|\frac{1}{2\pi i}\int_{\dot{D_2}-\dot{d_2}}\frac{d\tau_2}{\tau_2-t_2}\right|\cdot$$

$$\cdot\left|\frac{1}{2\pi i}\int_{\dot{D_3}-\dot{d_3}}\frac{\varphi_3(t_{\tau_3},t)d\tau_3}{\tau_3-t_3}\right| \le \frac{A_3}{2\pi}\int_{\dot{D_3}-\dot{d_3}}|\tau_3-t_3|^{\alpha_3-1}|d\tau_3|.$$

Bundan ε→0 da $\overset{1}{S}_{\varepsilon}\varphi_3(t_{\tau_3},t)$ ning yaqinlashuvchiligi kelib chiqadi.

Xuddi shunday $\overset{1}{S}_{\varepsilon}\varphi_3(t_{\tau_2},t)$va $\overset{1}{S}_{\varepsilon}\varphi_3(t_{\tau_1},t)$ larning yaqinlashuvchiligini ko‘rish qiyin emas. Ma’lumki,

$$\frac{1}{2\pi i}\int_{\dot{D_k}}\frac{d\tau_k}{\tau_k-z_k} = \begin{cases}1, & z_k\in D_k^+\\ \frac{1}{2}, & z_k\in \dot{D_k}\\ 0, & z_k\in D_k^-\end{cases} \qquad (7)$$

(7) e’tiborga olgan holda ε→0 da $\overset{0}{S}_{\varepsilon}\to\frac{1}{8}$ ga intilishi kelib chiqadi.

$\overset{3}{S}_{\varepsilon}\varphi_3(\tau,t)$; $\overset{2}{S}_{\varepsilon}\varphi_3(\tau_{t_k},t)$ $(k=1,2,3)$; $\overset{1}{S}_{\varepsilon}\varphi_3(t_{\tau_k},t)$ $(k=1,2,3)$ larni ε→0 da mos ravishda

$\overset{3}{\psi}(t)$, $\frac{1}{2}\overset{2}{\psi}_1(t)$, $\frac{1}{2}\overset{2}{\psi}_2(t)$, $\frac{1}{2}\overset{2}{\psi}_3(t)$, $\frac{1}{4}\overset{1}{\psi}_1(t)$, $\frac{1}{4}\overset{1}{\psi}_2(t)$, $\frac{1}{4}\overset{1}{\psi}_3(t)$,
deb belgilaymiz, bunda misol uchun

$$\overset{3}{\psi}(t_1,t_2,t_3) = \frac{1}{(2\pi i)^3}\int_{\Delta}\frac{\varphi_3(\tau,t)}{\prod_{k=1}^{3}(\tau_k-t_k)}d\tau \qquad (8)$$

$$\overset{2}{\psi}_1(t_1,t_2,t_3) = \frac{1}{(2\pi i)^2}\int_{\dot{D_2}}\int_{\dot{D_3}}\frac{\varphi_3(\tau_{t_1},t)}{(\tau_2-t_2)(\tau_3-t_3)}d\tau_2 d\tau_3 \qquad (9)$$

$$\overset{1}{\psi}_1(t_1,t_2,t_3) = \frac{1}{2\pi i}\int_{\dot{D_1}}\frac{\varphi_3(t_{\tau_1},t)}{\tau_1-t_1}d\tau_1 \qquad (10).$$

Shunday qilib $n=3$ bo‘lganda (2) integralning bosh qiymati:

$$\Phi(t_1,t_2,t_3) = \overset{3}{\psi}(t_1,t_2,t_3) + \frac{1}{2}\overset{2}{\psi}_1(t_1,t_2,t_3) + \frac{1}{2}\overset{2}{\psi}_2(t_1,t_2,t_3) +$$

$$+\frac{1}{2}\overset{2}{\psi}_3(t_1,t_2,t_3) + \frac{1}{4}\overset{1}{\psi}_1(t_1,t_2,t_3) + \frac{1}{4}\overset{1}{\psi}_2(t_1,t_2,t_3) + \frac{1}{4}\overset{1}{\psi}_3(t_1,t_2,t_3) +$$

$$+\frac{1}{8}\varphi(t_1,t_2,t_3) \qquad (11)$$

bo‘ladi.

FOYDALANILGAN ADABIYOTLAR:

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