GYOLDER SHARTINI QANOATLANTIRUVCHI FUNKSIYALAR SINFI

O‘razaliyev Shirinboy Bo‘ron o‘g‘li
Samarqand iqtisodiyot va servis instituti o’qituvchisi
Rustamov Sarvar Jasurovich
Samarqand iqtisodiyot va servis instituti talabasi
Jozilova Zuhra O‘tkir qizi
TATU Samarqand filiali talabasi

Annotatsiya: $\varphi(t_1, t_2, \dots, t_n)$ *funksiyaning ko‘p o‘zgaruvchili kompleks tekislikda Gyolder shartini qanoatlantirishi haqida.*

Аннотация: $\varphi(t_1, t_2, \dots, t_n)$ *относится к удовлетворению условия Голдера в комплексной плоскости с несколькими переменными.*

Annotation: $\varphi(t_1, t_2, \dots, t_n)$ *is about the satisfaction of the Golder condition in the multivariable complex plane.*

$\varphi(t)$ funksiya Δ-ostovda aniqlangan bo‘lsin.

1-ta’rif. Agar $\varphi(t) = \varphi(t_1, t_2, \dots, t_n)$ funksiya $t = (t_1, t_2, \dots, t_n), \tau = (\tau_1, \tau_2, \dots, \tau_n) \in \Delta$ nuqtalar uchun

$$|\varphi(t) - \varphi(\tau)| \le \sum_{k=1}^{n} A_k\, |t_k - \tau_k|^{\alpha_k} \qquad (1)$$

tengsizlikni qanoatlantirsa, u holda $\varphi(t)$ funksiya Δ da Gyolder shartini qanoatlantiradi deyiladi, bunda A_k $(k = \overline{1, n})$ musbat sonlar Gyolder o‘zgarmasi, α_k esa $0 < \alpha_k \le 1$ $(k = \overline{1, n})$ tengsizlikni qanoatlantiruvchi o‘zgarmas sonlar bo‘lib, u Gyolder ko‘rsatkichi deyiladi.

Gyolder shartini qanoatlantiruvchi funksiyalar sinfini $H_{\alpha_k}(\Delta)$ deb belgilaymiz.

Agar $\varphi(t) \in H_{\alpha_k}(\Delta)$ bo‘lsa, u holda $\varphi(t) \in H_{\beta_k}(\Delta)$ $(\beta_k < \alpha_k,\ k = \overline{1, n})$ bo‘ladi. Haqiqatdan ham shartga ko‘ra $\varphi(t) \in H_{\alpha_k}(\Delta)$;

$$|\varphi(t) - \varphi(\tau)| \le \sum_{k=1}^{n} A_k\, |t_k - \tau_k|^{\alpha_k}$$

bundan $\sum_{k=1}^{n} A_k\, |t_k - \tau_k|^{\alpha_k} = \sum_{k=1}^{n} A_k\, |t_k - \tau_k|^{\alpha_k - \beta_k} \cdot |t_k - \tau_k|^{\beta_k} \le$
$\le \sum_{k=1}^{n} \breve{A}_k\, |t_k - \tau_k|^{\beta_k} \quad \Rightarrow \ \varphi(t) \in H_{\beta_k}(\Delta)$ ekanligi kelib chiqadi.

1-teorema. $\varphi(t)$ funksiya Δ da (1) Gyolder shartini qanoatlantirishi uchun har bir t_p argumenti bo‘yicha boshqa t_k $(p \ne k, k = \overline{1, n})$ argumentlariga nisbatan tekis Gyolder shartini qanoatlantirishi zarur va yetarli.

Isbot. Haqiqatdan ham $\tau_k = t_k, k = \overline{1, n}, p \ne k$ bo‘lganda (1) dan

$$\left|\varphi(t) - \varphi(t_{\tau_p})\right| \le A_p \left|t_p - \tau_p\right|^{\alpha_p}, \quad p = \overline{1, n} \qquad (2)$$

o‘rinli bo‘ladi. Xuddi shunday quydagi tengsizliklarni isbotlash qiyin emas:

$$\begin{cases} \left|\varphi\left(t_{\tau_p}\right)-\varphi(t_{\tau_{pq}})\right| \le A_q\left|t_q-\tau_q\right|^{\alpha_q}, \quad p,q=\overline{1,n}\,, p<q \\ \left|\varphi\left(t_{\tau_{pq}}\right)-\varphi(t_{\tau_{pqr}})\right| \le A_r|t_r-\tau_r|^{\alpha_r}, \quad p,q,r=\overline{1,n}\,, p<q<r \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ \left|\varphi\left(\tau_{t_p}\right)-\varphi(\tau_{t_{pq}})\right| \le A_q\left|t_q-\tau_q\right|^{\alpha_q}, \quad p,q=\overline{1,n}\,, p<q \\ \left|\varphi(\tau)-\varphi(\tau_{t_p})\right| \le A_p\left|t_p-\tau_p\right|^{\alpha_p}, \quad p=\overline{1,n} \end{cases} \qquad (3)$$

(3) tengsizliklar sistemasini e'tiborga olgan holda ushbu

$$|\varphi(t)-\varphi(\tau)| \le \left|\varphi(t)-\varphi\left(t_{\tau_1}\right)\right| + \left|\varphi\left(t_{\tau_1}\right)-\varphi\left(t_{\tau_{12}}\right)\right| + \cdots +$$
$$+\left|\varphi\left(\tau_{t_{n-1,n}}\right)-\varphi\left(\tau_{t_n}\right)\right| + \left|\varphi\left(\tau_{t_n}\right)-\varphi(\tau)\right|,$$

tengsizlikdan teoremaning tasdiqi kelib chiqadi.

Keyinchalik ishlatiladigan ba'zi yig'indilar.

$$\varphi_n(\tau;t)=\varphi_n(\tau_1,\tau_2,\dots,\tau_n;t_1,t_2,\dots,t_n)$$

orqali quydagi yig'indini belgilaymiz

$$\varphi_n(\tau;t)=\varphi(\tau)-\sum_{p=1}^{n}\varphi\left(\tau_{t_p}\right)+\sum_{p=1}^{n}\sum_{q=1}^{n}\varphi\left(\tau_{t_{pq}}\right)-\cdots+$$

$$+(-1)^{n-2}\sum_{p=1}^{n}\sum_{q=1}^{n}\varphi\left(t_{\tau_{pq}}\right)+(-1)^{n-1}\sum_{p=1}^{n}\varphi\left(t_{\tau_p}\right)+(-1)^{n}\varphi(t)\,, p<q \qquad (4)$$

(4) dan

$$\varphi_n(\tau;t)=(-1)^n\varphi_n(t;\tau) \qquad (5)$$

tenglikning to'g'riligini tekshirib ko'rish qiyin emas.

$\varphi_n(\tau;t)$ yig'indi 2^n ta qo'shiluvchiga ega.

Masalan, $n=3$ bo'lganda

$$\varphi_3(\tau;t)=\varphi(\tau_1,\tau_2,\tau_3)-\varphi(\tau_1,\tau_2,t_3)-\varphi(\tau_1,t_2,\tau_3)-\varphi(t_1,\tau_2,\tau_3)+$$
$$+\varphi(\tau_1,t_2,t_3)+\varphi(t_1,\tau_2,t_3)+\varphi(t_1,t_2,\tau_3)-\varphi(t_1,t_2,t_3) \qquad (6)$$

$$\varphi_n\left(\tau_{t_p};t\right)=\varphi_n(\tau_1,\tau_2,\dots,\tau_{p-1},t_p,\tau_{p+1},\dots,\tau_n;t_1,t_2,\dots,t_n)$$

orqali ushbu

$$\varphi_n\left(\tau_{t_p};t\right)=\varphi\left(\tau_{t_p}\right)-\sum_{\substack{q=1\\q\ne p}}^{n}\varphi\left(\tau_{t_{pq}}\right)+\sum_{\substack{q=1\\q,r\ne p}}^{n}\sum_{\substack{r=1\\q<r}}^{n}\varphi\left(\tau_{t_{pqr}}\right)-\cdots+$$

$$+(-1)^{n-3}\sum_{\substack{q=1\\q,r\ne p}}^{n}\sum_{\substack{r=1\\q<r}}^{n}\varphi\left(t_{\tau_{qr}}\right)+(-1)^{n-2}\sum_{\substack{q=1\\q\ne p}}^{n}\varphi\left(t_{\tau_q}\right)+(-1)^{n+1}\varphi(t) \qquad 7)$$

(7) yig'indi 2^{n-1} ta qo'shiluvchiga ega va u (6) ga o'xshash tuziladi, lekin undan bitta qo'shiluvchi kam.

$n=3$ bo'lganda (7) ning ko'rinishi quydagi ko'rinishga keladi:

$$\begin{cases} \varphi_3\left(\tau_{t_3};t\right)=\varphi_3\left(t_{\tau_{12}};t\right)=\varphi(\tau_1,\tau_2,t_3)-\varphi(\tau_1,t_2,t_3)-\varphi(t_1,\tau_2,\tau_3)+\varphi(\tau_1,\tau_2,t_3) \\ \varphi_3\left(\tau_{t_2};t\right)=\varphi_3\left(t_{\tau_{13}};t\right)=\varphi(\tau_1,t_2,\tau_3)-\varphi(\tau_1,t_2,t_3)-\varphi(t_1,t_2,\tau_3)+\varphi(t_1,t_2,t_3) \\ \varphi_3\left(\tau_{t_1};t\right)=\varphi_3\left(t_{\tau_{23}};t\right)=\varphi(t_1,\tau_2,\tau_3)-\varphi(t_1,t_2,\tau_3)-\varphi(t_1,\tau_2,t_3)+\varphi(t_1,t_2,t_3) \end{cases} \qquad (8)$$

Xuddi shunday $\varphi_n\left(\tau_{t_{pq}};t\right)$, $\varphi_n\left(\tau_{t_{pqr}};t\right),\dots,\varphi_n\left(t_{\tau_{pq}};t\right)$ va $\varphi_n\left(t_{\tau_p};t\right)$ va hokazo yig'indilar uchun ham tuziladi.

$n=3$ bo'lganda bu yig'indilarning ko'rinish quydagicha bo'ladi:

$$\begin{cases}\varphi_3(\tau_{t_{12}};t)=\varphi_3(t_{\tau_3};t)=\varphi(t_1,t_2,\tau_3)-\varphi(t_1,t_2,t_3)\\ \varphi_3(\tau_{t_{13}};t)=\varphi_3(t_{\tau_2};t)=\varphi(t_1,\tau_2,t_3)-\varphi(t_1,t_2,t_3)\\ \varphi_3(\tau_{t_{23}};t)=\varphi_3(t_{\tau_1};t)=\varphi(\tau_1,t_2,t_3)-\varphi(t_1,t_2,t_3)\end{cases}\qquad(9)$$

(6), yuqoridagi ikkita sistema va $\varphi(t_1,t_2,t_3)$ lar qo'shish natijasida ushbu

$$\varphi(\tau_1,\tau_2,\tau_3)=\varphi_3(\tau;t)+\varphi_3(\tau_{t_1};t)+\varphi_3(\tau_{t_2};t)+\varphi_3(\tau_{t_3};t)++\varphi_3(t_{\tau_1};t)+$$
$$+\varphi_3(t_{\tau_2};t)+\varphi_3(t_{\tau_3};t)+\varphi(t_1,t_2,t_3)\qquad(10)$$

ayniyatni hosil qilamiz. Umumiy holda bu ayniyatning ko'rinishi quydagicha bo'ladi.

$$\varphi(\tau)=\varphi_n(\tau;t)+\sum_{p=1}^{n}\varphi_n(\tau_{t_p};t)+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}\varphi_n\left(\tau_{t_{pq}};t\right)+\cdots+$$
$$+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}\varphi_n\left(t_{\tau_{pq}};t\right)+\sum_{p=1}^{n}\varphi_n(t_{\tau_p};t)+\varphi(t)\qquad(11)$$

bu ayniyatning to'g'riligini yuqoridagi singari tekshirib ko'rish qiyin emas.

Qaralayotgan (4) yig'indining hadlarini ikkitadan shunday gruppalarga ajrataylikі, har bir gruppa faqat bitta aniq argument bilan farq qiladigan funksiyalar ayirmasi shaklida tasvirlansin. (4) yig'indida shunday gruppalar soni 2^{n-1} ta bo'ladi. Gruppalashlar soni esa aniq n ga teng bo'ladi.

Masalan $n=3$ bo'lganda (4) yig'indi uchun:

$$\begin{cases}\varphi_3(\tau;t)=[\varphi(\tau_1,\tau_2,\tau_3)-\varphi(\tau_1,\tau_2,t_3)]+[\varphi(\tau_1,t_2,t_3)-\varphi(\tau_1,t_2,\tau_3)]\\ \quad+[\varphi(t_1,\tau_2,t_3)-\varphi(t_1,\tau_2,\tau_3)]+[\varphi(t_1,t_2,\tau_3)-\varphi(t_1,t_2,t_3)],\\ \varphi_3(\tau;t)=[\varphi(\tau_1,\tau_2,\tau_3)-\varphi(\tau_1,t_2,\tau_3)]+[\varphi(\tau_1,t_2,t_3)-\varphi(\tau_1,\tau_2,t_3)]\\ \quad+[\varphi(t_1,\tau_2,t_3)-\varphi(t_1,t_2,t_3)]+[\varphi(t_1,t_2,\tau_3)-\varphi(t_1,\tau_2,\tau_3)],\\ \varphi_3(\tau;t)=[\varphi(\tau_1,\tau_2,\tau_3)-\varphi(t_1,\tau_2,\tau_3)]+[\varphi(\tau_1,t_2,t_3)-\varphi(t_1,t_2,t_3)]\\ \quad+[\varphi(t_1,\tau_2,t_3)-\varphi(\tau_1,\tau_2,t_3)]+[\varphi(t_1,t_2,\tau_3)-\varphi(\tau_1,t_2,\tau_3)].\end{cases}$$

Xuddi shunday mulohazalarni $\varphi_n\left(\tau_{t_p};t\right)$, $\varphi_n\left(\tau_{t_{pq}};t\right),\dots,\varphi_n\left(t_{\tau_{pq}};t\right)$ va $\varphi_n\left(t_{\tau_p};t\right)$ yig'indilarga ham qo'llash mumkin.

Bu punktda (4) da qaralgan yig'indilarni baholaymiz; buning uchun avvalo $n=3$ uchun qaraymiz. Yuqoridagi sistemadan yig'indining moduli, modullar yig'indisidan kichik yoki tengligini e'tiborga olgan holda (4) dan topamiz:

$$|\varphi_3(\tau;t)|\le 4A_k|\tau_k-t_k|^{\alpha_k},\quad k=1,2,3.$$

osonlik bilan ko'rish mumkinki, umumiy holda:

$$\begin{cases} |\varphi_n(\tau;t)| \le 2^{n-1}A_k|\tau_k - t_k|^{\alpha_k}, \quad k=\overline{1,n} \\ \left|\varphi_n\left(\tau_{t_p};t\right)\right| \le 2^{n-2}A_k|\tau_k - t_k|^{\alpha_k}, \quad k\ne p; \ k,p=\overline{1,n} \\ \left|\varphi_n\left(\tau_{t_{pq}};t\right)\right| \le 2^{n-3}A_k|\tau_k - t_k|^{\alpha_k}, \quad k\ne p,q; \ p<q; \ k,p,q=\overline{1,n} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \left|\varphi_n\left(t_{\tau_{pqr}};t\right)\right| \le 2^{2}A_k|\tau_k - t_k|^{\alpha_k}, \quad p<q<r; \ k\ne p,q,r; \ k,p,q,r=\overline{1,n} \\ \left|\varphi_n\left(t_{\tau_{pq}};t\right)\right| \le 2A_k|\tau_k - t_k|^{\alpha_k}, \quad k\ne p,q; \ p<q; \ k,p,q=\overline{1,n} \end{cases} \qquad (12)$$

bu birinchisidan foydalanib topamiz:

$$|\varphi_n(\tau;t)| \le [|\varphi_n(\tau;t)|^n]^{\frac{1}{n}} \le \left[\prod_{k=1}^{n} 2^{n-1}A_k\,|\tau_k - t_k|^{\alpha_k}\right]^{\frac{1}{n}} =$$

$$= 2^{n-1}\prod_{k=1}^{n} A_k^{\frac{1}{n}}|\tau_k - t_k|^{\frac{\alpha_k}{n}}$$

Xuddi shunday mulohazalardan keyin yuqoridagi sistemaning qolgan tensizliklari uchun ham quydagi tengsizliklarni olish mumkin:

$$\left|\varphi_n\left(t_{\tau_{pq}};t\right)\right| \le 2\sqrt{A_p\cdot A_q}\,|\tau_p - t_p|^{\frac{\alpha_p}{2}}\cdot|\tau_q - t_q|^{\frac{\alpha_q}{2}};$$

$$p,q=\overline{1,n}; \ p<q, \qquad (13)$$

$$\left|\varphi_n\left(t_{\tau_{pqr}};t\right)\right| \le 4\sqrt[3]{A_p\cdot A_q\cdot A_r}\,|\tau_p - t_p|^{\frac{\alpha_p}{3}}\cdot|\tau_q - t_q|^{\frac{\alpha_q}{3}}\cdot|\tau_r - t_r|^{\frac{\alpha_r}{3}};$$

$$p<q<r; \ p,q,r=\overline{1,n}; \qquad (14)$$

… … … … … … … … … … … … … … …

$$\left|\varphi_n\left(\tau_{t_{pq}};t\right)\right| \le 2^{n-3}\prod_{k=1}^{n} A_k^{\frac{1}{n-2}}\,|\tau_p - t_p|^{\frac{\alpha_k}{n-2}};$$

$$p,q=\overline{1,n}; \quad p<q \quad k\ne p,q \qquad (15)$$

$$\left|\varphi_n\left(\tau_{t_p};t\right)\right| \le 2^{n-2}\prod_{k=1}^{n} A_k^{\frac{1}{n-1}}\,|\tau_k - t_k|^{\frac{\alpha_k}{n-1}}; \quad p=\overline{1,n}; \quad k\ne p, \qquad (16)$$

$$|\varphi_n(\tau;t)| \le 2^{n-1}\prod_{k=1}^{n} A_k^{\frac{1}{n}}\,|\tau_k - t_k|^{\frac{\alpha_k}{n}}; \qquad (17)$$

(3) ning birinchisini $\varphi_n\left(\tau_{t_p};t\right)$ yig'indi orqali quydagicha yozamiz:

$$\left|\varphi_n\left(t_{\tau_p},t\right)\right| \le A_p|\tau_p - t_p|^{\alpha_p}, \quad p=\overline{1,n} \qquad (18)$$

takidlaymizki, $n=2,3$ hollar uchun ham xuddi shunday yig'indilar uchun baholarni olamiz:

$$\left|\varphi_2\left(t_{\tau_p},t\right)\right| \le A_p|\tau_p - t_p|^{\alpha_p}, \quad p=\overline{1,2} \qquad (19)$$

$$|\varphi_2(\tau;t)| \le 2\sqrt{A_1\cdot A_2}\,|\tau_1 - t_1|^{\frac{\alpha_1}{2}}\cdot|\tau_2 - t_2|^{\frac{\alpha_2}{2}}, \qquad (20)$$

$$\left|\varphi_3\left(t_{\tau_p},t\right)\right| \le A_p|\tau_p - t_p|^{\alpha_p}, \quad p=\overline{1,3} \qquad (21)$$

$$\left|\varphi_3\left(t_{\tau_p};t\right)\right| \leq 2\prod_{k=1}^{3} A_k^{\frac{1}{2}}|\tau_k - t_k|^{\frac{\alpha_k}{2}};\quad p=\overline{1,3};\quad k\neq p, \quad (22)$$

$$|\varphi_3(\tau;t)| \leq 4\prod_{k=1}^{3} A_k^{\frac{1}{3}}|\tau_k - t_k|^{\frac{\alpha_k}{3}}; \quad (23)$$

FOYDALANILGAN ADABIYOTLAR:

1. В. А. Какичев. «Граничные свойства интеграла типа Коши многих переменных». Уч. зап. Шахт. пед. ин-та. 2,вып. 6,1959-г,25-90 с.
2. A. G‘oziyev. R. Mardiyev «Analitik funksiyaning chegaraviy masalalari va singuliyar integral tenglamalar». Samarqand-2014.
3. Н. И. Мусхелишвили. «Сингулярные интегральные уравнения». Москва. Наука. 1968-г.
4. С. Г. Михлин «Сингулярные интегральные уравнения» Усп.мат. наук. вып.3, т. 3. 1948-г.
5. O‘razaliyev. Sh.B. «Ko‘p kompleks o‘zgaruvchili Koshi tipidagi integralning mavjudlik sharti ». GOSPODARKA I INNOWACJE. Volume 34 (2023).