TA'RIF YORDAMIDA 4-TARTIBLI DETERMINANTLARNI HISOBLASH

Eshtemirov Eshtemir Salim o'g'li
eshtemireshtemirov577@gmail.com
[1] Denov tadbirkorlik va pedagogika instituti Aniq va tabiiy fanlar fakulteti Oliy matematika kafedrasi o'qituvchisi
Sayitniyozov Adhan Annamuhammad o'g'li
[2] Denov tadbirkorlik va pedagogika instituti Matematika va Informatika yo'nalishi 1- bosqich talabasi
Bakirova Mo'tabar Norsafar qizi
[3] Denov tadbirkorlik va pedagogika instituti Matematika yo'nalishi 3- bosqich talabasi

Anatatsiya: *$n-$ Tartibli determinantning ixtiyoriy satr va ustunidan bittadan olingan n ta elementlarining ko'paytmasini tuzish, o'ringa qo'yishlarni tuzish, yig'indini hisoblash. Ushbu maqolada determinantlarni ta'rif yordamida hisoblash ko'rib o'tilgan.*

Kalit so'zlar: *$n-$ Tartibli matritsa, $n-$ tartibli o'ringa qo'yishlar, o'ringa qo'yishning inversiyasi va signaturasi, $n-$ tartibli determinantning ta'rifi.*

Bizga $(1,2,\dots,i,\dots,j,\dots,n)$ o'rin almashtirish berilgan bo'lsin.

1-ta'rif. Agar berilgan o'rin almashtirishda i>j bo'lib, o'rin almashtirishda i soni j dan oldin turgan bo'lsa, i va j sonlar inversiya tashkil etadi deyiladi va $inv(i,j)$ shaklda belgilanadi.

O'rin almashtirishdagi inversiya tashkil etuvchi juftliklar soniga o'rin almashtirishning inversiyasi deyiladi va $inv(i_{1,}i_{2,\dots,}i_n)$ kabi belgilanadi. Inversiyasi toq va juft son bo'lgan o'rin almashtirishlar mos ravishda toq va juft o'rin almashtirishlar deb ataladi. Berilgan $(i_{1,}i_{2,\dots,}i_n)$ o'rin almashtirishning signaturasi deb,

$$sign(i_{1,}i_{2,\dots,}i_n) = (-1)^{inv(i_{1,}i_{2,\dots,}i_n)}$$

miqdorga aytiladi. Ma'lumki, o'rin almashtirishning signaturasi uning toq va juftligiga qarab, -1 yoki 1 ga teng bo'ladi.

Bizga $A \in M_n(K)$ kvadrat matritsa berilgan bo'lsin: bu yerda $K = R$ yoki C

$$\begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,,n} \\ \dots & \dots & \dots & \dots \\ a_{n,1} & a_{n,2} & \dots & a_{n,n} \end{pmatrix}$$

Bu matritsaning ixtiyoriy satr va ustunidan bittadan olingan n ta elementlarining ko'paytmasini qaraymiz:

$$a_{1,\alpha_1} \cdot a_{2,\alpha_2} \cdot \dots \cdot a_{n,\alpha_n}$$

Ko'paytmaning ko'paytuvchilaridagi indekslaridan

$\alpha = \begin{pmatrix} 1 & 2 & \dots & n \\ \alpha_1 & \alpha_2 & \dots & \alpha_n \end{pmatrix}$ o'rniga qo'yishni tuzib olamiz.

Demak, har bir ko'paytuvchiga bitta o'rniga qo'yishni mos qo'yish mumkin. Aksincha, har bir n − tartibli o'rniga qo'yishga matritsadan yuqoridagi kabi olingan ko'paytmani mos qilib qo'yishimiz mumkin.

Ko'paytmaning ishorasini o'rniga qo'yishni signaturasi bilan aniqlaymiz, ya'ni

$$sgn(\alpha) = (-1)^{inv\alpha}$$

Quyidagi ko'paytmani hosil qilamiz:

$sgn(\alpha) \cdot a_{1,\alpha_1} \cdot a_{2,\alpha_2} \cdot \ldots \cdot a_{n,\alpha_n}$

Hamma o'rniga qo'yishlar soni $n!$ bo'lganligi uchun, tuzilgan ko'paytmalar soni ham $n!$ ta bo'ladi. Bu elementlarning

$\sum_{\alpha \in S_n} sgn(\alpha) \cdot a_{1,\alpha_1} \cdot a_{2,\alpha_2} \cdot \ldots \cdot a_{n,\alpha_n}$ (1)

yig'indisini qaraymiz.

2-ta'rif. Yuqorida hosil bo'lgan (1) yig'indiga berilgan n − tartibli A kvadrat matritsaning determinanti deyiladi. Determinant odatda $detA$ yoki $|A|$ kabi belgilanadi.

Shunday qilib, determinantni quyidagicha yozib olishimiz mumkin:

$$|A| = \begin{vmatrix} a_{1,1} & a_{1,2} & \ldots & a_{1,n} \\ a_{2,1} & a_{2,2} & \ldots & a_{2,,n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n,1} & a_{n,2} & \ldots & a_{n,n} \end{vmatrix} = \sum_{\alpha \in S_n} sgn(\alpha) \cdot a_{1,\alpha_1} \cdot a_{2,\alpha_2} \cdot \ldots \cdot a_{n,\alpha_n} \quad (1)$$

Misol: Determinantning qiymatini ta'rif yordamida hisoblang:

$$|A| = \begin{vmatrix} 1 & 0 & 2 & 3 \\ 2 & 2 & 1 & 7 \\ 0 & 1 & 4 & 0 \\ 4 & 3 & 2 & 0 \end{vmatrix}$$

Bu determinantda barcha qo'yishlari quyidagilar bo'ladi.

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

Har bir o'rniga qo'yishga bitta ko'paytma mos keladi, bu ko'paytmalarning ishorasini o'rniga qo'yishning signaturasi orqali aniqlaymiz.

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} ==$$

$sign(\alpha_1)a_{11}a_{22}a_{33}a_{44} + sign(\alpha_2)a_{11}a_{22}a_{34}a_{43} + sign(\alpha_3)a_{11}a_{24}a_{32}a_{43} +$
$sign(\alpha_4)a_{11}a_{24}a_{33}a_{42} + sign(\alpha_5)a_{11}a_{23}a_{34}a_{42} + sign(\alpha_6)a_{11}a_{23}a_{32}a_{44} +$
$sign(\alpha_7)a_{12}a_{21}a_{33}a_{44} + sign(\alpha_8)a_{12}a_{21}a_{34}a_{43} + sign(\alpha_9)a_{12}a_{24}a_{31}a_{43} +$
$sign(\alpha_{10})a_{12}a_{24}a_{33}a_{41} + sign(\alpha_{11})a_{12}a_{23}a_{31}a_{44} + sign(\alpha_{12})a_{12}a_{23}a_{34}a_{41} +$

$sign(\alpha_{13})a_{13}a_{21}a_{32}a_{44} + sign(\alpha_{14})a_{13}a_{21}a_{34}a_{42} + sign(\alpha_{15})a_{13}a_{24}a_{31}a_{42} + sign(\alpha_{16})a_{13}a_{24}a_{32}a_{41} + sign(\alpha_{17})a_{13}a_{22}a_{31}a_{44} + sign(\alpha_{18})a_{13}a_{22}a_{34}a_{41} + sign(\alpha_{19})a_{14}a_{21}a_{32}a_{43} + sign(\alpha_{20})a_{14}a_{21}a_{33}a_{42} + sign(\alpha_{21})a_{14}a_{22}a_{33}a_{41} + sign(\alpha_{22})a_{14}a_{22}a_{31}a_{43} + sign(\alpha_{23})a_{14}a_{23}a_{31}a_{42} + sign(\alpha_{24})a_{14}a_{23}a_{32}a_{41}.$

$$|A| = \begin{vmatrix} 1 & 0 & 2 & 3 \\ 2 & 2 & 1 & 7 \\ 0 & 1 & 4 & 0 \\ 4 & 3 & 2 & 0 \end{vmatrix} =$$

$$= (-1)^0 \cdot 1 \cdot 2 \cdot 4 \cdot 0 + (-1)^1 \cdot 1 \cdot 2 \cdot 0 \cdot 2 + (-1)^2 \cdot 1 \cdot 7 \cdot 1 \cdot 2 + (-1)^3 \cdot 1 \cdot 7 \cdot 4 \cdot 3 + (-1)^2 \cdot 1 \cdot 1 \cdot 4 \cdot 3 + (-1)^1 \cdot 1 \cdot 7 \cdot 1 \cdot 0 + (-1)^1 \cdot 0 \cdot 2 \cdot 4 \cdot 0 + (-1)^2 \cdot 0 \cdot 2 \cdot 4 \cdot 2 + (-1)^3 \cdot 0 \cdot 7 \cdot 0 \cdot 2 + (-1)^3 \cdot 0 \cdot 7 \cdot 4 \cdot 0 + (-1)^2 \cdot 0 \cdot 1 \cdot 0 \cdot 4 + (-1)^2 \cdot 0 \cdot 1 \cdot 4 \cdot 4 + (-1)^2 \cdot 0 \cdot 2 \cdot 1 \cdot 0 + (-1)^3 \cdot 2 \cdot 2 \cdot 4 \cdot 3 + (-1)^4 \cdot 2 \cdot 7 \cdot 0 \cdot 3 + (-1)^4 \cdot 2 \cdot 7 \cdot 1 \cdot 4 + (-1)^2 \cdot 2 \cdot 2 \cdot 0 \cdot 0 + (-1)^3 \cdot 2 \cdot 2 \cdot 4 \cdot 4 + (-1)^3 \cdot 3 \cdot 2 \cdot 1 \cdot 2 + (-1)^4 \cdot 3 \cdot 2 \cdot 4 \cdot 3 + (-1)^5 \cdot 3 \cdot 2 \cdot 4 \cdot 4 + (-1)^4 \cdot 3 \cdot 2 \cdot 0 \cdot 2 + (-1)^5 \cdot 3 \cdot 2 \cdot 0 \cdot 3 + (-1)^6 \cdot 3 \cdot 1 \cdot 1 \cdot 4 = -150$$

FOYDALANILGAN ADABIYOTLAR:

1. Sh.A.Ayupov, B.A.Omirov,A.X.Xudoyberdiyev, F.H.Haydarov. Algebra va sonlar nazariyasi. Toshkent-2019 (62-67)

2. R.N.Nazarov, B.T.Toshpo'latov, A.D.Do'sumbetov. Algebra va sonlar nazariyasi. I qism. Toshkent-1993.(216-219)

3. Malik D.S., Mordeson J.N., Sen M.K., Fundamentals of abstract algebra(o'quv qo'llanma) WCB McGrew-Hill, 1997, p.636.

4. Allakov. Sonlar nazariyasidan misol va masalalar. (o'quv qo'llanma) – T. Inovatsion rivojlanish nashriyot matbaa uyi. 2020.348b.

5. Abdurashidov N.G'.,Simmetrik Li va Leybnits algebralari va ularning xossalari. "O'ZBEKISTONDA FANLARARO INNOVATSIYALAR VA ILMIY TADQIQOTLAR" APREL 2022. (7), 62-63.

6. Eshtemirov Eshtemir Salim o'g'li, Abdurashidov Nuriddin G'iyoziddin o'g'li. VEYL-TITCHMARSH FUNKSIYASI VA SPEKTRAL FUNKSIYA ORASIDAGI MUNOSABAT. "O'ZBEKISTONDA FANLARARO INNOVATSIYALAR VA ILMIY TADQIQOTLAR" 20-iyun 2023-yil 20-son (870-875).

7. *Abdurashidov N. G'., Eshtemirov E. S* .SIMMETRIK LEYBNITS ALGEBRALARI VA ULARNING XOSSALARI. *<< Matematik modellashtirish va axborot texnologiyalarining dolzarb masalalari>> xalqaro ilmiy-amaliy anjuman. Nukus 2-3-may 2023-yil 1-Tom.*

8. *Abdurashidov Nuriddin , Toshtemirova Sarvara,Yo'ldoshev Husan.* Laplas teoremasi yordamida 4-tartibli determinantni hisoblash. "O'ZBEKISTONDA FANLARARO INNOVATSIYALAR VA ILMIY TADQIQOTLAR" 20-fevral 2024-yil 27-son (163-166).

9. www.lib.math.msu.ru internet sahifasi (Rossiya).

10. www.mathlinks.ro internet sahifasi (Ruminiya).
11. www.zaba.ru internet sahifasi (Rossiya).