MATHEMATICAL MODEL USED TO IMPROVE IMAGE QUALITY

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Annotation: *The study examined the use of piecewise polynomial methods in digital image processing. A Hermitian spline function of piecewise polynomials is selected as a mathematical model for digital signal processing, and the construction of a third-order Hermitian spline function with two variables is presented. Based on the constructed mathematical model, an image restoration algorithm has been developed.*

Keywords: *image, piecewise polynomial methods, cubic Hermite spline, local spline functions.*

For the rapidly developing information and communication technologies in the world, it is important to develop high-performance, efficient computational methods for image restoration, the use of digital processing algorithms and the search for optimal solutions. Local spline is an important mathematical apparatus for creating digital signal processing algorithms due to the small number of calculations when constructing functions, high accuracy, flexibility of digital processing algorithms, optimal differential and extreme properties, and ease of calculation of parameters. Advanced scientific research is being conducted in this direction in developed countries of the world, such as the USA, Germany, Great Britain, India, China, South Korea, the Russian Federation, and Japan [4], [11]. This article describes in detail the construction of a two-dimensional third-order Hermitian spline function and numerical image processing [1].

Based on the cubic spline functions with one variable $S_3(x_i, y)$ and $S_3(x_{i+1}, y)$, constructed above, after some simplifications, the following type of Hermitian spline function with two variables is formed:

$$ES_{3,3}(x,y) = (1-t)^2(1+2t)S_3(x_i,y) +$$
$$+t^2(3-2t)S_3(x_{i+1},y)+h_i t(1-t)^2 S_3(x_i,y) - h_i t^2(1-t)S_3(x_{i+1},y)$$

где, $j=\overline{0, M-1}$, $0 \le u \le 1$, $t=\frac{x-x_i}{h}$, $u=\frac{y-y_j}{l}$, $h_i = x_{i+1}-x_i$,

$l_i = y_{j+1} - y_j$.

Substituting the values of the one-dimensional cubic spline functions $SS_3(x_i, y) - S_3(x_{i+1}, y)$, we obtain the following model (1).

$$ES_{3,3}(x,y) = (1-t)^2(1+2t)[(1-u)^2(1+2u)f_{i,j} + u^2(3-2u)f_{i,j+1} + l_j u(1-u)^2 f_{i,j}$$
$$-l_j u^2(1-u)f_{i,j+1}] + t^2(3-2t)[(1-u)^2(1+u)f_{i+1,j} + u^2(3-2u)f_{i+1,j+1} +$$
$$+l_j u(1-u)^2 f_{i+1,j} - l_j u^2(1-u)f_{i+1,j+1}] + h_i t(1-t)^2[(1-u)^2(1+u)f_{i,j}$$
$$+ +u^2(3-2u)f_{i,j+1} +$$
$$+l_j u(1-u)^2 f_{i+1,j} - l_j u^2(1-u)f_{i+1,j+1}] - h_i t^2(1-t)[(1-u)^2(1+u)f_{i+1,j} +$$

$$+u^2(3-2u)f_{i+1,j+1} + l_j u(1-u)^2 f_{i+1,j} - l_j u^2(1-u) f_{i+1,j+1}], \quad (1)$$

где, $i = \overline{0, N-1}$, $0 \le t \le 1$, $t = \frac{x-x_i}{h}$, $h_i = x_{i+1} - x_i$,

$j = \overline{0, M-1}$, $0 \le u \le 1$, $u = \frac{x-x_i}{l}$, $l_j = y_{i+1} - y_i$.

After some simplifications, a two-dimensional local Hermite spline function was formed:

$$ES_{3,3}(x,y) = \varphi_1(t)[\varphi_1(u)f_{i,j} + \varphi_2(u)f_{i,j+1} + l_j\varphi_3(u)f_{i,j} + l_j\varphi_4(u)f_{i,j+1}] +$$
$$+\varphi_2(t)[\varphi_1(u)f_{i+1,j} + \varphi_2(u)f_{i+1,j+1} + l_j\varphi_3(u)f_{i,j} + l_j\varphi_4(u)f_{i,j+1}] +$$
$$+h_i\varphi_3(t)\left[\varphi_1(u)f_{i,j} + \varphi_2(u)f_{i,j+1} + l_j\varphi_3(u)f_{i,j} + l_j\varphi_4(u)f_{i,j+1}\right] +$$
$$+h_i\varphi_4(t)\left[\varphi_1(u)f_{i+1,j} + \varphi_2(u)f_{i+1,j+1} + l_j\varphi_3(u)f_{i,j} + l_j\varphi_4(u)f_{i,j+1}\right]. \quad (2)$$

где, $j = \overline{0, M-1}$, $0 \le u \le 1$, $t = \frac{x-x_i}{h}$, $u = \frac{y-y_j}{l}$, $h_i = x_{i+1} - x_i$, $l_j = y_{j+1} - y_j$,

$$\varphi_1(t) = (1-t)^2(1+2t), \qquad \varphi_1(u) = (1-u)^2(1+2u),$$
$$\varphi_2(t) = t^2(3-2t), \qquad \text{и} \qquad \varphi_2(u) = u^2(3-2u),$$
$$\varphi_3(t) = t(1-t)^2, \qquad \varphi_3(u) = u(1-u)^2,$$
$$\varphi_4(t) = -t^2(1-t). \qquad \varphi_4(u) = -u^2(1-u).$$

Эту функцию (2) можно назвать двумерной локальной кубической сплайн-функцией Эрмита.

This function (2) can be called a two-dimensional local cubic Hermite spline function.

Let's consider the use of the above two-dimensional mathematical model in digital image processing. For this we use a JPEG image file. For digital processing of images in this format, the process of converting them into digital form is relatively simple. Below is a 50x50 pixel JPEG image (Figure 1).

Figure 1. Initial view of the image

To digitally process a given image, we can convert the image into digital form and place it in a table (Table 1).

Table 1.

Digital image representation

№	x_1	x_2	x_3	x_4	x_5	x_5	x_6	x_7	...	x_n
y_1	96	78	77	77	80	79	77	77	...	101
y_2	71	56	61	60	60	60	61	59	...	80

y_3	79	59	60	60	60	60	60	59	...	94
y_4	80	59	59	57	59	59	57	57	...	97
y_5	78	59	59	59	60	60	60	59	...	97
y_6	78	54	56	59	60	60	60	60	...	97
y_7	79	57	60	61	60	60	60	61	...	96
y_8	80	61	62	60	60	60	59	60	...	97
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
y_n	78	68	158	205	201	205	206	206	78	0

Using the considered models of cubic splines, we perform restoration to the digital representation of the image presented in Table 4. Using the above sequence, a program for constructing two-dimensional third-order Hermitian splines was developed in the

MATLAB software environment, which was used for digital processing of the presented image (Fig. 2).

Figure 2. Result of the image recovery process

Из процесса восстановления мы видим, что за счет цифровой обработки изображений с помощью двумерной эрмитовой сплайн-функции количество пикселей от The original state of the image has increased several times, and we have achieved improved image quality. image. During the recovery process, the original image size increased from 54 KB to 572 KB. This shows that the mathematical model used has good accuracy in digital image processing.

Conclusion.

JPEG image data was interpolated using a two-variable Hermite spline function. The results show that the number of pixels during image restoration increased by 10 times. This results in improved image quality. When digitizing images using the 2D Hermitian spline function, the size of the original image increased from 54 KB to 572 KB. It is shown that the use of a mathematical model of a Hermite spline with two variables is effective in digital processing of medical images and detection of ambiguous areas during examinations.

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