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IKKINCHI TARTIBLI INTEGRO-DIFFERENSIAL TENGLAMA UCHUN IKKI TESKARI MASALALAR HAQIDA

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Annotatsiya: *Ushbu ishda ikkinchi tartibli integro-differensial tenglamalar uchun bir integral shartli teskari masalalarni bir qiymatli yechilishi isbotlangan.*

Аннотация: *В работе доказано однозначное решение обратных задач с одним интегральным условием для интегро-дифференциальных уравнений второго порядка.*

Abstract: *In this work, the one-value solution of inverse problems with one integral condition for second-order integro-differential equations is proved.*

Kalit so`zlar: *ikkinchi tartibli integro-differensial tenglama, teskari masala.*

Ключевые слова: *интегро-дифференциальное уравнение второго порядка, обратная задача.*

Key words: *second-order integro-differential equation, inverse problem.*

KIRISH

So‘nggi vaqtlarda noma’lum manbali differensial tengalamalar bilan shug‘illanishga bo‘lgan qiziqish ortib bormoqda. Bunga sabab ko‘plab issiqlik taqalish va diffuziya jarayonlarini matematik modelini tuzish noma’lum manbali differensial tenglama uchun qo‘yiladigan masalalarga keltiriladi. Shu sababdan biz ushbu ishda ikkinchi tartibli integro-differensial tenglama uchun bir teskari masalani tadqiq etamiz.

(0, 1) oraliqda ushbu

$$y''(x)-\lambda I_{0x}^{\gamma}y(x)=kf(x) \qquad (1)$$

ikkinchi tartibli integro-differensial tenglamani qaraylik, bu yerda $y(x)$-noma’lum funksiya, λ,γ-o‘zgarmas haqiqiy sonlar, $f(x)$-berilgan funksiya, k-noma’lum sonlar bo‘lib, $I_{0x}^{\gamma}y(x)$ - Riman-Liuvill ma’nosida kasr tartibli integral [1]

$$I_{0x}^{\gamma}y(x)=\frac{1}{\Gamma(\gamma)}\int_0^x (x-t)^{\gamma-1}y(t)dt .$$

T_1 masala Shunday $y(x)$-funksiya va k sonni topilsinki u quyidagi xossalarga ega bo‘lsin:

1) (0, 1) oraliqda (1) tenglamani qanoatlantirsin;

2) $C^1[0,1]$ I $C^2(0,1)$ sinfga tegishli bo‘lsin;

3) $x=0$ nuqtada esa

$$y(0)=A_1,\ y'(0)=A_2 \qquad (2)$$

$$B=\int_0^1 y(x)\,dx \qquad (3)$$

shartlarni qanoatlantirsin, bu yerda $A_1, A_2, B-$berilgan o‘zgarmas haqiqiy sonlar.

(1) tenglamani (2) shartlarni qanoatlantiruvchi yechimini

$$y(x)=A_1E_{\beta,1}\left(\lambda x^{\beta}\right)+A_2xE_{\beta,2}\left(\lambda x^{\beta}\right)+k\int_0^x (x-z)E_{\beta,4}\left[\lambda(x-z)^{\beta}\right]f(z)\,dz \qquad (4)$$

yozib olamiz [2], bu yerda $E_{\xi,\eta}(z)=\sum_{n=0}^{+\infty}\frac{z^n}{\Gamma(\xi n+\eta)}$ - Mittag –Leffler funksiyasi [1], $\beta=\gamma+1.$

(4) ni (3) shartga qo`yib, ba’zi hisoblashlarni amalga oshirib k sonini

$$k=\frac{B-A_1E_{\beta,2}(\lambda)-A_2E_{\beta,3}(\lambda)}{\int_0^1 (1-z)^2 E_{\beta,3}\left[\lambda(1-z)^{\beta}\right]f(z)\,dz} \qquad (5)$$

ko‘rinishda topamiz.

(5) formulani (4) formulga qo`yib (1), (2) masalaning yechimini

$$y(x)=A_1E_{\beta,1}\left(\lambda x^{\beta}\right)+A_2xE_{\beta,2}\left(\lambda x^{\beta}\right)+$$

$$+\left[\int_0^1 (1-z)^2 E_{\beta,3}\left[\lambda(1-z)^{\beta}\right]f(z)\,dz\right]^{-1}\left[B-A_1E_{\beta,2}(\lambda)-A_2E_{\beta,3}(\lambda)\right]\times$$

$$\times\int_0^x (x-z)E_{\beta,4}\left[\lambda(x-z)^{\beta}\right]f(z)\,dz \qquad (6)$$

ko`rinishda topamiz.

1-teorema. Agar $\int_0^1 (1-z)^2 E_{\beta,3}\left[\lambda(1-z)^{\beta}\right]f(z)\,dz\neq 0$ bo‘lsa, u holda T_1 masala yagona yechimga ega bo‘ladi va u (5), (6) formulalar bilan aniqlanadi.

T_2 masala Shunday $y(x)$-funksiya va k sonni topilsinki u quyidagi xossalarga ega bo‘lsin:

1) (0, 1) oraliqda (1) tenglamani qanoatlantirsin;

2) $C^1[0,1]$ I $C^2(0,1)$ sinfga tegishli bo‘lsin;

3) $x=0$ nuqtada esa

$$y(1)-a\int_0^1 y(x)\,dx=D \qquad (7)$$

(2) va (7) shartlarni qanoatlantirsin, bu yerda $A_1, A_2, a, D-$berilgan o‘zgarmas haqiqiy sonlar.

(1) tenglamani (2) shartlarni qanoatlantiruvchi yechimini

$$y(x)=A_1E_{\beta,1}\left(\lambda x^{\beta}\right)+A_2xE_{\beta,2}\left(\lambda x^{\beta}\right)+k\int_0^x (x-z)E_{\beta,4}\left[\lambda(x-z)^{\beta}\right]f(z)\,dz \qquad (4)$$

yozib olamiz [2], bu yerda $E_{\xi,\eta}(z)=\sum_{n=0}^{+\infty}\frac{z^n}{\Gamma(\xi n+\eta)}$ - Mittag –Leffler funksiyasi [1], $\beta=\gamma+1$.

(4) ni (3) shartga qo`yib, ba'zi hisoblashlarni amalga oshirib k sonini

$$k=\frac{D-A_1\left[E_{\beta,1}(\lambda)-aE_{\beta,2}(\lambda)\right]-A_2\left[E_{\beta,2}(\lambda)-aE_{\beta,3}(\lambda)\right]}{\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz-a\int_0^1(1-z)^2E_{\beta,3}\left[\lambda(1-z)^\beta\right]f(z)dz} \tag{8}$$

ko'rinishda topamiz.

(8) formulani (4) formulga qo`yib (1), (2) masalaning yechimini

$$y(x)=A_1E_{\beta,1}\left(\lambda x^\beta\right)+A_2xE_{\beta,2}\left(\lambda x^\beta\right)+$$

$$+\frac{D-A_1\left[E_{\beta,1}(\lambda)-aE_{\beta,2}(\lambda)\right]-A_2\left[E_{\beta,2}(\lambda)-aE_{\beta,3}(\lambda)\right]}{\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz-a\int_0^1(1-z)^2E_{\beta,3}\left[\lambda(1-z)^\beta\right]f(z)dz}\times$$

$$\times\int_0^x(x-z)E_{\beta,4}\left[\lambda(x-z)^\beta\right]f(z)dz \tag{9}$$

ko`rinishda topamiz.

1-teorema. Agar

$$\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^\beta\right]f(z)dz\neq a\int_0^1(1-z)^2E_{\beta,3}\left[\lambda(1-z)^\beta\right]f(z)dz$$

bo'lsa, u holda T_1 masala yagona yechimga ega bo'ladi va u (8), (9) formulalar bilan aniqlanadi.

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