KAPUTO VA RIMAN-LIUVILL MA’NOSIDAGI HOSILA VA RIMAN-LIUVILL MA’NOSIDAGI INTEGRALNI O‘Z ICHIGA OLUVCHI YUKLANGAN ODDIY DIFFERENSIAL TENGLAMA UCHUN IKKI MASALA HAQIDA

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Annotatsiya: *Ushbu ishda Kaputo va Riman-Liuvill ma’nosidagi hosila va Riman-Liuvill integralni o‘z ichiga oluvchi yuklangan oddiy differensial tenglama uchun ikki masala bayon qilingan va uning bir qiymatli yechilishi isbotlangan.*

Kalit so`zlar: *Kaputo va Riman-Liuvill ma’nosidagi hosila, Riman-Liuvill ma’nosidagi integral.*

Аннотация: *В работе представлены две задачи для нагруженного обыкновенного дифференциального уравнения, содержащие производную и интеграл Римана-Лиувилля в смысле Капуто и Римана-Лиувилля, и доказано ее однозначное решение.*

Ключевые слова: *производная по Капуто и Риману-Лиувиллю, интеграл по Риману-Лиувиллю.*

Abstract: *In this work, two problems for the loaded ordinary differential equation containing the derivative and the Riemann-Liouville integral in the sense of Caputo and Riemann-Liouville are stated and its one-valued solution is proved.*

Keywords: *derivative in the sense of Caputo and Riemann-Liouville, integral in the sense of Riemann-Liouville.*

So‘ngi vaqtlarda noma`lum funksiyani biror qiymati qatnashgan differensial tengalamalar bilan shug‘ullanishga bo`lgan qiziqish ortib bormoqda. Bunga sabab ko`plab issiqlik tarqalish va diffuziya jarayonlarini matematik modelini tuzish funksiyani biror qiymati qatnashgan differensial tenglama uchun qo‘yiladigan masalalarga keltiriladi. Odatda, bunday turdagi tenglamalar yuklangan differensial tenglama deb yuritiladi. Shu sababdan biz ushbu ishda Kaputo va Riman-Liuvill ma’nosidagi hosila va Riman-Liuvill ma’nosidagi integralni o‘z ichiga oluvchi yuklangan oddiy differensial tenglama uchun bir masalani tadqiq etamiz.

$(0,1)$ oraliqda ushbu

$$ {}_C D_{0x}^{\alpha} y(x) - \lambda I_{0x}^{\gamma} y(x) = I_{0x_0}^{\delta} y(x_0) \qquad (1)$$

kasr tartibli yuklangan oddiy differensial tenglamani qaraylik, bu yerda $y(x)$-noma’lum funksiya, $\alpha, \lambda, \delta, x_0$-o‘zgarmas haqiqiy sonlar bo‘lib, $0<\alpha<1, \gamma>0$, $0<x_0<1$, ${}_C D_{0x}^{\alpha} y(x)$-Kaputo ma’nosidagi kasr tartibli hosilasi [1]

$${}_C D_{0x}^{\alpha} y(x) = I_{0x}^{1-\alpha} y'(x),$$

$I_{0x}^{\alpha} y(x)$ - Riman-Liuvill ma'nosida kasr tartibli integral [1]

$$I_{0x}^{\alpha} y(x) = \frac{1}{\Gamma(\gamma)} \int_0^x (x-t)^{\gamma-1} y(t)\,dt.$$

Masala. (1) tenglamani va

$$y(0) = A \qquad (2)$$

boshlang`ich shartni qanoatlantiruvchi $y(x)$ funksiya topilsin, bu yerda, A - berilgan o'zgarmas haqiqiy son.

Ma'lumki [2],

$${}_C D_{0x}^{\alpha} y(x) - \lambda I_{0x}^{\gamma} y(x) = f(x)$$

tenglamaning (2) shartni qanoatlantiruvchi yechimi

$$y(x) = AE_{\alpha+\gamma,1}\left[\lambda x^{\alpha+\gamma}\right] + \int_0^x (x-z)^{\alpha-1} E_{\alpha+\gamma,\alpha}\left[\lambda (x-z)^{\alpha+\gamma}\right] f(z)\,dz \qquad (3)$$

ko'rinishda bo'ladi, bu yerda $E_{\gamma,\sigma}(z) = \sum_{k=0}^{+\infty} \frac{z^k}{\Gamma(\gamma k + \sigma)}$ - Mittag-Leffler funksiyasi [1].

Bizning masalada $f(x) = I_{0x_0}^{\delta} y(x_0)$ bo'lgani uchun (1) tenglamaning (2) shartni qanoatlantiruvchi yechimi

$$y(x) = AE_{\alpha+\gamma,1}\left[\lambda x^{\alpha+\gamma}\right] + x^{\alpha-1} E_{\alpha+\gamma,\alpha+1}\left[\lambda x^{\alpha+\gamma}\right] I_{0x_0}^{\delta} y(x_0) \qquad (4)$$

ko'rinishda bo'ladi.

(4) ni har ikkala tarafini I_{0x}^{δ} ni ta'sir ettirib yuboramiz va ba'zi hisoblashlarni amalga oshirib,

$$I_{0x}^{\delta} y(x) = Ax^{\delta} E_{\alpha+\gamma,\delta+1}\left[\lambda x^{\alpha+\gamma}\right] + x^{\delta-1} E_{\alpha+\gamma,\delta+1}\left[\lambda x^{\alpha+\gamma}\right] I_{0x_0}^{\delta} y(x_0) \qquad (5)$$

tenglikni hosil qilamiz.

(5) tenglikda $x = x_0$ deb, $I_{0x_0}^{\delta} y(x_0)$ ni

$$I_{0x_0}^{\delta} y(x_0) = \frac{Ax_0^{\delta} E_{\alpha+\gamma,\delta+1}\left[\lambda x_0^{\alpha+\delta}\right]}{1 - x_0^{\delta-1} E_{\alpha+\gamma,\delta+1}\left[\lambda x_0^{\alpha+\gamma}\right]} \qquad (6)$$

ko'rinishda topamiz.

(6) ni (4) ga qo'yib, masalaning yechimini

$$y(x) = AE_{\alpha+\gamma,1}\left[\lambda x^{\alpha+\gamma}\right] + \frac{Ax_0^{\delta} E_{\alpha+\gamma,\delta+1}\left[\lambda x_0^{\alpha+\delta}\right]}{1 - x_0^{\delta-1} E_{\alpha+\gamma,\delta+1}\left[\lambda x_0^{\alpha+\gamma}\right]} x^{\alpha-1} E_{\alpha+\gamma,\alpha+1}\left[\lambda x^{\alpha+\gamma}\right] \qquad (7)$$

ko'rinishda topamiz.

Teorema. Agar $x_0^{\delta-1} E_{\alpha+\gamma,\delta+1}\left[\lambda x_0^{\alpha+\gamma}\right] \neq 1$ bo'lsa,u holda masala yagona yechimga ega bo'lib, u (7) formula bilan aniqlanadi.

Masala. (1) tenglamani va

$$\lim_{x\to 0} I_{0x}^{1-\varphi} y(x) = A \qquad (3)$$

boshlang`ich shartni qanoatlantiruvchi $y(x)$ funksiya topilsin, bu yerda, A - berilgan o'zgarmas haqiqiy son.

Ma'lumki [3],

$$D_{0x}^{\varphi} y(x) - \lambda I_{0x}^{\gamma} y(x) = f(x)$$

tenglamaning (3) shartni qanoatlantiruvchi yechimi

$$y(x) = Ax^{\varphi-1} E_{\varphi+\gamma,\varphi}\left[\lambda x^{\varphi+\gamma}\right] + \int_0^x (x-z)^{\varphi-1} E_{\varphi+\gamma,\varphi}\left[\lambda (x-z)^{\varphi+\gamma}\right] f(z)dz \qquad (4)$$

ko'rinishda bo'ladi, bu yerda $E_{\gamma,\sigma}(z) = \sum_{k=0}^{+\infty} \frac{z^k}{\Gamma(\gamma k + \sigma)}$ - Mittag-Leffler funksiyasi [1].

Bizning masalada $f(x) = I_{0x_0}^{\delta} y(x_0)$ bo'lgani uchun (1) tenglamaning (3) shartni qanoatlantiruvchi yechimi

$$y(x) = Ax^{\varphi-1} E_{\varphi+\gamma,\varphi}\left[\lambda x^{\varphi+\gamma}\right] + x^{\varphi} E_{\varphi+\gamma,\varphi+1}\left[\lambda x^{\varphi+\gamma}\right] I_{0x_0}^{\delta} y(x_0) \qquad (5)$$

ko'rinishda bo'ladi.

(5) ni har ikkala tarafini I_{0x}^{δ} ni ta'sir ettirib yuboramiz va ba'zi hisoblashlarni amalga oshirib,

$$I_{0x}^{\delta} y(x) = Ax^{\varphi+\delta-1} E_{\varphi+\gamma,\varphi+\delta}\left[\lambda x^{\varphi+\delta}\right] + x^{\varphi+\delta} E_{\varphi+\gamma,\delta+\varphi+1}\left[\lambda x^{\varphi+\gamma}\right] I_{0x_0}^{\delta} y(x_0) \qquad (6)$$

tenglikni hosil qilamiz.

(6) tenglikda $x = x_0$ deb, $I_{0x_0}^{\delta} y(x_0)$ ni

$$I_{0x_0}^{\delta} y(x_0) = \frac{Ax_0^{\varphi+\delta-1} E_{\varphi+\gamma,\varphi+\delta}\left[\lambda x_0^{\varphi+\delta}\right]}{1 - x_0^{\varphi+\delta} E_{\varphi+\delta,\varphi+\delta+1}\left[\lambda x_0^{\varphi+\gamma}\right]} \qquad (7)$$

ko'rinishda topamiz.

(7) ni (5) ga qo'yib, masalaning yechimini

$$y(x) = Ax^{\varphi-1} E_{\varphi+\gamma,\varphi}\left[\lambda x^{\varphi+\gamma}\right] + \frac{Ax_0^{\varphi+\delta-1} E_{\varphi+\gamma,\varphi+\delta}\left[\lambda x_0^{\varphi+\delta}\right]}{1 - x_0^{\varphi+\delta} E_{\varphi+\delta,\varphi+\delta+1}\left[\lambda x_0^{\varphi+\gamma}\right]} x^{\varphi} E_{\varphi+\gamma,\varphi+1}\left[\lambda x^{\varphi+\gamma}\right] \qquad (8)$$

ko'rinishda topamiz.

Teorema. Agar $x_0^{\varphi+\delta} E_{\varphi+\delta,\varphi+\delta+1}\left[\lambda x_0^{\varphi+\gamma}\right] \neq 1$ bo'lsa,u holda masala yagona yechimga ega bo'lib, u (8) formula bilan aniqlanadi.

FOYDALANILGAN ADABIYOTLAR:

1. Kilbas A.A., Srivastava H.M., Trujillo J.J. *T*heory and applications of fractional differential equations **(North-Holland Mathematics Studies, 204). Amsterdam: Elsevier, 2006.-523p.**

2. Omonova D. D. Umumlashgan Hilfer ma’nosidagi hosila va Riman-Liuvill ma’nosidagi integralni o‘z ichiga oluvchi oddiy differensial tenglama uchun Koshi masalasi. O‘zbekistonda fanlararo innovatsiyalar va ilmiy tadqiqotlar jurnali. 18 - son. 2023. 371-376 betlar.