KVAZICHIZIQLI XUSUSIY HOSILALI DIFFERENSIAL TENGLAMALAR HAQIDA

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Annotatsiya: *Ushbu ishda qo'shimcha shart asosida kvazichiziqli tenglamani chiziqli tenglamaga keltirish usuli o'rganilgan.*

Kalit so'zlar: *kvazichiziqli tenglama, chiziqli tenglama, Kaputo integro-differensial operatori, sub-diffuziya tenglamasi, kasr tartibli to'lqin tenglamasi, teskari masalalar.*

ON A QUASILINEAR PARTIAL DIFFERENTIAL EQUATIONS

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Abstract: *In this work, a method of reducing a quasi-linear equation to a linear equation based on an additional condition is studied.*

Keywords: *quasi-linear equation, linear equation, Caputo integro-differential operator, sub-diffusion equation, fractional order wave equation, inverse problems.*

Biz ushbu ishda ikki xil holatni ko'rib chiqamiz. Dastlab, kasr tartibli so'ngra yuqori tartibli xususiy hosilali differensial tenglamalarni.

So'ngi yillarda nochiziqli differensial tenglamalarni o'rganishga bo'lgan qiziqish ortib bormoqda. Bunga sabab gazlar va suyuqliklar dinamikasi, aylanma sirtlarning cheksiz kichik bukilish nazariyasi, momentsiz qobiqlar nazariyasi, kompyuter tomografiyasi, qonning arteriyadagi harakati va boshqa ko'plab sohalar masalalarining matematik modeli sifatida qo'llaniladi. Shu sababdan ushbu ishda biz nochiziqli bir nechta xususiy hosilali differensial tenglamalarni qo'shimcha shart asosida chiziqli tenglamaga keltirish usuli bayon qilamiz.

Quyidagi kvazichiziqli

$$ {}_cD_{ot}^{\alpha}u(t,x)-k(t)u_{xx}(t,x)+u(t,x)\left[u_x(t,\xi_1)-u_x(t,\xi_2)\right]=f(t,x) $$

tenglamani $\Omega=\{(t,x):t>0,-\infty<x<\infty\}$ sohada tadqiq etamiz, bu yerda $k(t)\neq 0$, $f(t,x)$ - berilgan funksiyalar, $-\infty<\xi_1<\xi_2<\infty$, $n-1<\alpha\leq n$, $n\in N$.

$$ {}_cD_{ot}^{\alpha}g(t)=\frac{1}{\Gamma(n-\alpha)}\int_0^t (t-z)^{n-\alpha-1}g^{(n)}(z)dz $$

kasr tartibli Kaputo integro-differensial operatori $[1]$.

Quyidagi tatbiq o'rinli:

1- lemma. Agar (1) tenglama

$$\int_{\xi_1}^{\xi_2} u(t,x)dx = 0$$

shartni qanoatlantirsa, uni quyidagi chiziqli tenglamaga keltirish mumkin:

$${}_cD_{ot}^{\alpha}u(t,x) - k(t)u_{xx}(t,x) + p(t)u(t,x) = f(t,x)$$

bu yerda $p(t) = \frac{1}{k(t)}\int_{\xi}^{\xi_2} f(t,x)dx$.

Isbot. (2) shartning har ikki tomoniga ${}_cD_{ot}^{\alpha}$ operatorni qo'llaymiz:

$$\int_{\xi_1}^{\xi_2} {}_cD_{ot}^{\alpha}u(t,x)dx = 0.$$

So'ngra (1) tenglamadan foydalanib quyidagini hosil qilamiz:

$$k(t)\int_{\xi_1}^{\xi_2} u_{xx}(t,x)dx - \left[u_x(t,\xi_1) - u_x(t,\xi_2)\right]\int_{\xi_1}^{\xi_2} u(t,x)dx = -\int_{\xi_1}^{\xi_2} f(t,x)dx.$$

Oxirgi tenglikda (2) shartni hisobga olsak,

$$k(t)\left[u_x(t,\xi_2) - u_x(t,\xi_1)\right] = -\int_{\xi_1}^{\xi_2} f(t,x)dx$$

hosil bo'ladi. Ushbu ifodadan

$$u_x(t,\xi_1) - u_x(t,\xi_2) = \frac{1}{k(t)}\int_{\xi_1}^{\xi_2} f(t,x)dx$$

ni topib (1) tenglamaga qo'ysak (3) chiziqli tenglama hosil bo'ladi.

Izoh. α kasr tartibli hosila 0 va 1 orasida bo'lsa (1) tenglama sub-diffuziya tenglamasiga, 1 bilan 2 ning orasida bo'lsa kasr tartibli to'lqin tenglamasiga o'tadi.

Endi ushbu

$$L(u) - k(t)u_{xx}(t,x) + u(t,x)\left[u_x(t,\xi_1) - u_x(t,\xi_2)\right] = f(t,x)$$

yuqori tartibli xususiy hosilali differensial tenglamani $\Omega = \{(t,x): t > 0, -\infty < x < \infty\}$ sohada tadqiq etamiz, bu yerda $k(t) \neq 0$, $f(t,x)$ - berilgan funksiyalar, $-\infty < \xi_1 < \xi_2 < \infty$.

$$L(u) = \sum_{i=1}^{n} \alpha_i \frac{\partial^i u(t,x)}{\partial t^i},\ n \in N,$$

α_i lar bir vaqtda nolga aylanmaydigan o'zgarmaslar haqiqiy sonlar.

Quyidagi tasdiq o'rinli.

2- lemma. Agar (4) tenglama (2)

shartni qanoatlantirsa, uni quyidagi chiziqli tenglamaga keltirish mumkin:

$$L(u) - k(t)u_{xx}(t,x) + p(t)u(t,x) = f(t,x)$$

bu yerda $p(t)=\frac{1}{k(t)}\int_{\xi}^{\xi_2} f(t,x)dx$.

Isbot. (2) shartning har ikki tomoniga $L(u)$ operatorni qo‘llaymiz:

$$\int_{\xi_1}^{\xi_2} L\left[u(t,x)\right]dx=0$$

So‘ngra (4) tenglamadan foydalanib, quyidagini hosil qilamiz:

$$k(t)\int_{\xi_1}^{\xi_2} u_{xx}(t,x)dx-\left[u_x(t,\xi_1)-u_x(t,\xi_2)\right]\int_{\xi_1}^{\xi_2} u(t,x)dx=-\int_{\xi_1}^{\xi_2} f(t,x)dx.$$

Bu yerdan (2) shartni xisobga olsak

$$k(t)\left[u_x(t,\xi_2)-u_x(t,\xi_1)\right]=-\int_{\xi_1}^{\xi_2} f(t,x)dx$$

hosil bo‘ladi. Ushbu ifodadan

$$u_x(t,\xi_1)-u_x(t,\xi_2)=\frac{1}{k(t)}\int_{\xi_1}^{\xi_2} f(t,x)dx$$

ni topib (4) tenglamaga qo‘ysak (5) chiziqli tenglama hosil bo‘ladi.

Shuni ta’kidlash kerakki, (2) ko‘rinishdagi shart ko‘plab teskari masalalarni tadqiq etishda ishlatiladi $[2,3]$.

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