UMUMLASHGAN HILFER MA'NOSIDAGI HOSILA VA RIMAN-LIUVILL MA'NOSIDAGI INTEGRALNI O'Z ICHIGA OLUVCHI YUKLANGAN ODDIY DIFFERENSIAL TENGLAMA BA'ZI MASALALARI

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Annotatsiya: *Ushbu maqolada umumlashgan Hilfer ma'nosidagi hosila va Riman-Liuvill ma'nosidagi integralni o'z ichiga oluvchi yuklangan oddiy differensial tenglama uchun ba'zi masalalar o'rganilgan. Masalaning yechimi Mittag-Leffler funksiyasi yordamida topilgan.Berilgan funksiya yetarli shart topilgan va teorema shaklida bayon qilingan.*

Kalit so`zlar: *yuklangan oddiy differensial tenglama, kasr tartibli operator, Mittag-Leffer funksiyasi, yuklangan had.*

НЕКОТОРЫЕ ЗАДАЧИ НАГРУЖЕННОГО ОБЫКНОВЕННОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ, ВКЛЮЧАЯ ПРОИЗВОДНУЮ В ОБОБЩЕННОМ СМЫСЛЕ ГИЛЬФЕРА И ИНТЕГРАЛ В СМЫСЛЕ РИМАНА-ЛИУВИЛЯ

Аннотация: *В данной работе изучаются некоторые задачи для нагруженного обыкновенного дифференциального уравнения, содержащего производную в обобщенном смысле Гильфера и интеграл в смысле Римана-Лиувилля. Решение задачи было найдено с помощью функции Миттаг-Леффлера, для данной функции найдено достаточное условие и сформулировано в виде теоремы.*

Ключевые слова: *нагруженное обыкновенное дифференциальное уравнение, оператор дробного порядка, функция Миттаг-Леффера, нагруженный член.*

SOME PROBLEMS OF THE LOADED ORDINARY DIFFERENTIAL EQUATION INCLUDING THE DERIVATIVE IN THE GENERALIZED HILFER SENSE AND THE INTEGRAL IN THE RIMAN-LIOUVILLE SENSE

Abstract: *In this paper, some problems for the loaded ordinary differential equation involving the derivative in the generalized Hilfer sense and the integral in the Riemann-Liouville sense are studied. The solution to the problem was found using the Mittag-Leffler function. A sufficient condition was found for the given function and it was stated in the form of a theorem.*

Keywords: *loaded ordinary differential equation, fractional order operator, Mittag-Leffer function, loaded term.*

I.Kirish. So‘ngi vaqtlarda noma`lum funksiyani biror qiymati qatnashgan differensial tengalamalar bilan shug‘ullanishga bo`lgan qiziqish ortib bormoqda. Bunga sabab ko`plab issiqlik tarqalish va diffuziya jarayonlarini matematik modelini tuzish funksiyani biror qiymati qatnashgan differensial tenglama uchun qo‘yiladigan masalalarga keltiriladi. Odatda, bunday turdagi tenglamalar yuklangan differensial tenglama deb yuritiladi. Yuklangan xususiy hosilali va oddiy differensial tenglamalar yuklangan differensial tenglama ko`plab tadqiqotchilar tomonidan o‘rganilgan (masalan, ushbu [1]–[3] ishlarga qaralsin).

II. Masalaning qo‘yilishi.

(0,1) oraliqda ushbu

$$D_{0x}^{(\alpha,\varphi),\beta} y(x) - \lambda I_{0x}^{\gamma} y(x) = y(x_0) \qquad (1)$$

kasr tartibli yuklangan oddiy differensial tenglamani qaraylik, bu yerda $y(x)$ - noma’lum funksiya, $\alpha, \varphi, \beta, \lambda$ -o‘zgarmas haqiqiy sonlar bo‘lib, $0<\alpha<1, 0<\varphi<1, 0\le\beta\le1$; $D_{0x}^{(\alpha,\varphi),\beta} y(x)$ -Hilfer ma’nosidagi kasr tartibli hosilasi[4].

$$D_{0x}^{(\alpha,\varphi),\beta} y(x) = I_{0x}^{\beta(1-\alpha)} \frac{d}{dx} I_{0x}^{(1-\beta)(1-\varphi)} y(x) \ , \qquad (2)$$

$I_{0x}^{\gamma} y(x)$ – Riman-Liuvill ma’nosida γ (kasr) tartibli integral [5]

$$I_{0x}^{\gamma} y(x) = \frac{1}{\Gamma(\gamma)} \int_0^x (x-t)^{\gamma-1} y(t)dt, \ \gamma > 0.$$

A_1 **masala.** Shunday $y(x)$ funksiya topilsinki, u quyidagi xossalarga ega bo‘lsin:

1) $x^{(1-\beta)(1-\varphi)} y(x) \in a\,C[0,1]$, $D_{0x}^{(\alpha,\varphi),\beta} y(x) \in C(0,1)$ sinfga tegishli

2) (1) tenlamani qanoatlantirsin;

3) $x=0$ nuqtada esa

$$\lim_{x\to0} I_{0x}^{(1-\beta)(1-\varphi)} y(x) = A \qquad (3)$$

shartni qanoatlantirsin, bu yerda, A - berilgan o‘zgarmas haqiqiy son.

Ma’lumki [6],

$$D_{0x}^{(\alpha,\varphi),\beta} y(x) - \lambda I_{0x}^{\gamma} y(x) = f(x)$$

tenglamaning (3) shartni bajaruvchi yechimi

$$y(x) = Ax^{-(1-\varphi)(1-\beta)} E_{\varphi+\gamma+\beta(\alpha-\varphi),1-(1-\beta)(1-\varphi)}[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}] +$$

$$+\int_0^x (x-z)^{\varphi+\beta(\alpha-\varphi)-1} E_{\varphi+\gamma+\beta(\alpha-\varphi),\varphi+\beta(\alpha-\varphi)}[\lambda(x-z)^{\varphi+\gamma+\beta(\alpha-\varphi)}] f(z)dz \qquad (4)$$

ko‘rinishda bo‘ladi, bu yerda $E_{\gamma,\sigma}(z)=\sum_{k=0}^{+\infty}\frac{z^k}{\Gamma(\gamma k+\sigma)}$ - Mittag-Leffler funksiyasi.

Bizning masalada $f(x)=y(x_0)$ bo‘lgani uchun (1) tenglamaning (3) shartni qanoatlantiruvchi yechimi

$$y(x)=Ax^{-(1-\varphi)(1-\beta)}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-(1-\beta)(1-\varphi)}[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}]+$$
$$+y(x_0)\int_0^x(x-z)^{\varphi+\beta(\alpha-\varphi)-1}E_{\varphi+\gamma+\beta(\alpha-\varphi),\varphi+\beta(\alpha-\varphi)}[\lambda(x-z)^{\varphi+\gamma+\beta(\alpha-\varphi)}]dz \qquad (5)$$

(5) ko‘rinishda bo‘ladi. Uni

$$y(x)=Ax^{-(1-\varphi)(1-\beta)}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-(1-\beta)(1-\varphi)}[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}]+$$
$$+y(x_0)x^{\varphi+\beta(\alpha-\varphi)}E_{\varphi+\gamma+\beta(\alpha-\varphi)+1,\varphi+\beta(\alpha-\varphi)}\left[\lambda(x-z)^{\varphi+\gamma+\beta(\alpha-\varphi)}\right] \qquad (6)$$

ko‘rinishda yozib olamiz.

(6) formulada $x=x_0$ deb, $y(x_0)$ ni

$$y(x_0)=\frac{Ax_0^{-(1-\varphi)(1-\beta)}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-(1-\beta)(1-\varphi)}\left[\lambda x_0^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]}{1-x_0^{\varphi+\beta(\alpha-\varphi)}E_{\varphi+\gamma+\beta(\alpha-\varphi)+1,\varphi+\beta(\alpha-\varphi)}\left[\lambda(x_0-z)^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]} \qquad (7)$$

ko‘rinishda topamiz.

(7) ni (6) ga qo‘yib, A_1 masalaning yechimini

$$y(x)=Ax^{-(1-\varphi)(1-\beta)}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-(1-\beta)(1-\varphi)}[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}]+$$
$$+\frac{Ax_0^{-(1-\varphi)(1-\beta)}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-(1-\beta)(1-\varphi)}\left[\lambda x_0^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]}{1-x_0^{\varphi+\beta(\alpha-\varphi)}E_{\varphi+\gamma+\beta(\alpha-\varphi)+1,\varphi+\beta(\alpha-\varphi)}\left[\lambda x_0^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]}\times$$
$$\times x^{\varphi+\beta(\alpha-\varphi)}E_{\varphi+\gamma+\beta(\alpha-\varphi)+1,\varphi+\beta(\alpha-\varphi)}\left[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}\right] \qquad (8)$$

ko‘rinishda topamiz.

1-teorema. Agar $x_0^{\varphi+\beta(\alpha-\varphi)}E_{\varphi+\gamma+\beta(\alpha-\varphi)+1,\varphi+\beta(\alpha-\varphi)}\left[\lambda x_0^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]\neq 1$ bo‘lsa,u holda A_1 masala yagona yechimga ega bo‘lib, u (8) formula bilan aniqlanadi.

Endi (1) tenglamaning o‘rniga

$$D_{0x}^{\alpha,\beta}y(x)-\lambda I_{0x}^{\gamma}y(x)=D_{0x_0}^{\alpha}y(x_0) \qquad (9)$$

tenglamani (0,1) oraliqda qaraylik, bu yerda $D_{0x}^{\alpha}y(x)$ -Riman-Liuvill ma’nosidagi kasr tartibli operatori,

$$D_{0x}^{\alpha}y(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_0^x(x-t)^{-\alpha}y(t)dt.$$

B_1 **masala.** (1) tenglamani oʻrniga (9) tenglamani A_1 masalaning shartlarini bajaruvchi $y(x)$ funksiya topilsin.

(4) formuladan foydalanib, B_1 masalaning (3) shartni qanoatlantiruvchi yechimini

$$y(x) = Ax^{-(1-\varphi)(1-\beta)}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-(1-\beta)(1-\varphi)}[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}]+$$

$$+x^{\varphi+\beta(\alpha-\varphi)}E_{\varphi+\gamma+\beta(\alpha-\varphi)+1,\varphi+\beta(\alpha-\varphi)}\left[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]D^{\alpha}_{0x_0}y(x_0) \qquad (10)$$

koʻrinishda yozib olamiz.

(10) formulaga D^{α}_{0x} ni ta’sir ettirib,

$$D^{\alpha}_{0x}y(x) = AD^{\alpha}_{0x}\left[x^{-(1-\varphi)(1-\beta)}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-(1-\beta)(1-\varphi))}\left[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]\right]+$$

$$+D^{\alpha}_{0x}\left[x^{\varphi+\beta(\alpha-\varphi)}E_{\varphi+\gamma+\beta(\alpha-\varphi)+1,\varphi+\beta(\alpha-\varphi)}\left[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]\right]D^{\alpha}_{0x_0}y(x_0) \qquad (11)$$

tenglikni hosil qilamiz

Ba’zi hisoblashlarni amalga oshirib,

$$D^{\alpha}_{0x}y(x) = Ax^{-\alpha-(1-\varphi)(1-\beta)}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-\alpha-(1-\beta)(1-\varphi))}\left[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]+$$

$$+x^{\varphi+\beta(\alpha-\varphi)-\alpha}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-\alpha+\varphi+\beta(\alpha-\varphi)}\left[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]D^{\alpha}_{0x_0}y(x_0)$$

(12)

(12) formulada $x = x_0$ deb, $D^{\alpha}_{0x_0}y(x_0)$ ni

$$D^{\alpha}_{0x_0}y(x_0) = \frac{Ax_0^{-\alpha-(1-\varphi)(1-\beta)}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-\alpha-(1-\beta)(1-\varphi)}\left[\lambda x_0^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]}{1-x_0^{\varphi+\beta(\alpha-\varphi)-\alpha}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-\alpha+\varphi+\beta(\alpha-\varphi)}\left[\lambda x_0^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]}$$

(13)

koʻrinishda topamiz.

(13) ni (10) ga qoʻyib, B_1 masalaning yechimini

$$y(x) = Ax^{-(1-\varphi)(1-\beta)}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-(1-\beta)(1-\varphi)}[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}]+$$

$$+x^{\varphi+\beta(\alpha-\varphi)}E_{\varphi+\gamma+\beta(\alpha-\varphi)+1,\varphi+\beta(\alpha-\varphi)}\left[\lambda x^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]\times$$

$$\times\frac{Ax_0^{-\alpha-(1-\varphi)(1-\beta)}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-\alpha-(1-\beta)(1-\varphi)}\left[\lambda x_0^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]}{1-x_0^{\varphi+\beta(\alpha-\varphi)-\alpha}E_{\varphi+\gamma+\beta(\alpha-\varphi),1-\alpha+\varphi+\beta(\alpha-\varphi)}\left[\lambda x_0^{\varphi+\gamma+\beta(\alpha-\varphi)}\right]} \qquad (14)$$

ko‘rinishda topamiz.

2-teorema. Agar $x_0^{\varphi+\beta(\alpha-\varphi)-\alpha} E_{\varphi+\gamma+\beta(\alpha-\varphi),1-\alpha+\varphi+\beta(\alpha-\varphi)}\left[\lambda x_0^{\varphi+\gamma+\beta(\alpha-\varphi)}\right] \neq 1$ bo‘lsa, u holda B_1 masala yagona yechimga ega bo‘lib, u (14) formula bilan aniqlanadi.

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