G'OVAK MUHITLARDA MODDA KO'CHIRILISHI TENGLAMASI MANBA HADINI TIKLASH TESKARI MASALASINI SONLI YECHISH.

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Dusnazarova Roksana Kaxramonovna
Termiz davlat universiteti Amaliy matematika va axborot texnologiyari mutaxassisligi 2-kurs magistranti

Annotatsiya: *Ushbu maqolani yozishimdan maqsad axborot texnologiyalarida* ***x****ususiy hosilali differensial tenglama uchun chegaraviy masalalarda aniqlovchi tenglama, hisoblash sohasi, chegaraviy va boshlang'ich shartlarni berish bilan tavsiflanadi. Shu sababli teskari masalalarni koeffitsientli, geometrik, chegaraviy va evolyutsion teskari masalalarga bo'ladi. Bu dasturning afzalligi shundaki, simvollar soni cheklanmangan, hamda qaysi tilga o'girishni dasturni o'zi aniqlaydi.*

Kalit so'z: *Identifikatsiya, approksimatsiya, parabolik tenglama, differensial tenglama , chegaraviy masala, aktiv g'ovaklik, joriy konsentratsiya, sizish tezligi, diffuziya koeffitsiyenti, manba hadi, koordinata, – vaqt, g'ovak qatlam uzunligi, jarayonning davomiyligi .*

Xususiy hosilali differensial tenglama uchun chegaraviy masalalarda aniqlovchi tenglama, hisoblash sohasi, chegaraviy va boshlang'ich shartlarni berish bilan tavsiflanadi. Shu sababli teskari masalalarni koeffitsientli, geometrik, chegaraviy va evolyutsion teskari masalalarga bo'lish mumkin.

Umumiy metodologik nuqtai nazardan, to'g'ri masalalar deb sabablari berilgan va izlanayotgan kattaliklar oqibat bo'lgan masalalarni aytish mumkin. Bunday farazga ko'ra oqibatlari ma'lum bo'lgan va sabablari noma'lum bo'lgan masalalarni teskari masalalar desak bo'ladi. Teskari masalalar nokorrekt masalalar hisoblanadi. Teskari masalalar bo'yicha nazariy va amaliy masalalarni [1-6] adabiyotlarda batafsil yoritilgan. Ko'pincha bizda kirill yoki lotin alifbosidagi matnlarni kirilldan-lotinga, lotindan-kirillga o'girishga ehtiyoj bo'ladi.Ushbu kiril lotin, lotin kiril onlayn translit dasturi o'zbek tilidagi har qanday matnlarni hech qanday muammosiz hal qilishingizda sizga yordam beradi.Bu dasturning afzalligi shundaki, simvollar soni cheklanmangan, hamda qaysi tilga o'girishni dasturni o'zi aniqlaydi. Butun bir maqolani yoki matnni birinchi bo'sh katakka qo'ysangiz, bir necha soniyadan so'ng tayyor natijani qo'lga kiritasiz.

Hozircha sinov tartibida ishlayotganligi sababli, natijani tekshirib oling.

Ba'zi bir kamchiliklar bo'lsa administratorga murojat qiling yoki izoh yozib qoldiring.

$$\frac{\partial c}{\partial t}+\frac{v}{m}\frac{\partial c}{\partial x}=\frac{D}{m}\frac{\partial^2 c}{\partial x^2}+\frac{1}{m}f(x,t),\quad 0<x<l,\quad 0<t\le T, \qquad (1)$$

bu yerda m – aktiv g'ovaklik, $c(x,t)$ – joriy konsentratsiya, v – sizish tezligi, D – diffuziya koeffitsiyenti, $f(x,t)$ – manba hadi, x – koordinata, m, t – vaqt, s, l – g'ovak qatlam uzunligi, m, T – jarayonning davomiyligi, s.

(1) tenglamani quyidagi boshlang'ich va chegaraviy shartlarda qaraymiz:

$$c(x,0)=0, \quad 0 \le x \le l, \tag{2}$$

$$c(0,t)=c_0=\text{const}, \quad c(l,t)=0, \quad 0 \le t \le T. \tag{3}$$

To'g'ri masala (1)-(3) ko'rinishda ifodalanadi.

(1) tenglama o'ng tarafi $f(x,t)$ ni tiklash teskari masalasini qaraymiz. $f(x,t)$ quyidagi ko'rinishda tasvirlansin deb hisoblaymiz

$$\frac{1}{m} f(x,t)=\frac{1}{m} f_0 \eta(t)\psi(x), \tag{4}$$

bu yerda $\psi(x)$ berilgan funksiya, manbaning vaqtdan bog'liq finksiyasi $\eta(t)$ esa noma'lum bo'lsin. Ushbu bog'lanish qandaydir $0 < x^* < l$ ichki nuqtada $c(x,t)$ funksiya bo'yicha qo'shimcha kuzatish orqali tiklansin:

$$c(x^*,t)=\varphi(t). \tag{5}$$

(1)-(5) parabolik tenglamaning o'ng tarafini tiklash bo'yicha sodda masalaga kelamiz.

2. Teskari masalani yechish algoritmi. Tiklash masalasi quyidagi cheklashlarda qaraladi [6, 19]:

1. $\psi(x^*) \ne 0$,
2. $\psi(x)$ – yetarlicha sillqi funksiya ($\psi \in C[0,1]$),
3. Soha chegarasida $\psi(x)=0$.

Bu yerda asosiy farazlardan biri birinchi farazdir, chunki x^* kuzatish nuqtasida tiklanayotgan manba ta'sir qiladi. Aynan shu narsa qaralayotgan tiklash masalasining korrektligini ta'minlaydi.

Teskari masala yechimini quyidagi ko'rinishda izlaymiz

$$c(x,t)=\theta(t)\psi(x)+w(x,t), \tag{6}$$

bu yerda

$$\theta(t)=\frac{f_0}{m}\int_0^t \eta(s)ds. \tag{7}$$

(4), (6), (7) ni (1) ga qo'yib, $w(x,t)$ uchun quyidagi tenglamaga kelamiz:

$$\frac{f_0}{m}\eta(t)\psi(x)+\frac{\partial w}{\partial t}+\frac{v}{m}\theta(t)\frac{\partial \psi}{\partial x}+\frac{v}{m}\frac{\partial w}{\partial x}=\frac{D}{m}\theta(t)\frac{\partial^2 \psi}{\partial x^2}+\frac{D}{m}\frac{\partial^2 w}{\partial x^2}+\frac{f_0}{m}\eta(t)\psi(x),$$

tenglamani soddalashtirib quyidagiga kelamiz

$$\frac{\partial w}{\partial t}+\frac{v}{m}\frac{\partial w}{\partial x}=\frac{D}{m}\frac{\partial^2 w}{\partial x^2}+\frac{\theta(t)}{m}\left(D\frac{\partial^2 \psi}{\partial x^2}-v\frac{\partial \psi}{\partial x}\right). \tag{8}$$

(6) ko'rinishni hisobga olib (5) shartdan $\theta(t)$ noma'lum uchun quyidagi ko'rinishni hosil qilamiz:

$$\theta(t) = \frac{1}{\psi(x^*)}\left(\varphi(t) - w(x^*,t)\right). \qquad (9)$$

(9) ni (8) ga qo‘yib quyidagi parabolik tenglamani hosil qilamiz

$$\frac{\partial w}{\partial t} + \frac{v}{m}\frac{\partial w}{\partial x} = \frac{D}{m}\frac{\partial^2 w}{\partial x^2} + \frac{1}{m\psi(x^*)}\left(\varphi(t) - w(x^*,t)\right)\left(D\frac{\partial^2 \psi}{\partial x^2} - v\frac{\partial \psi}{\partial x}\right). \qquad (10)$$

$w(x,t)$ uchun chegaraviy shartlar ushbu ko‘rinishga ega bo‘ladi:

$$w(0,t) = c_0, \quad w(l,t) = 0, \ 0 \le t \le T. \quad (11)$$

(7) dan $\theta(t)$ yordamchi funksiya uchun quyidagi kelib chiqadi

$$\theta(0) = 0. \qquad (12)$$

Bu esa quyidagi boshlang‘ich shartni qo‘llash imkonini beradi

$$w(x,0) = 0, \ \ 0 < x < l. \qquad (13)$$

Shunday qilib (1)-(5) teskari masala noma’lum manbaning (7), (9) ko‘rinishdagi vaqtdan bog‘lanishi bo‘yicha (10)-(13) tenglama uchun chegaraviy masalaga keltiriladi.

3. Ayirmali masala. Identifikatsiya masalasini sonli yechishning asosiy nuqtalariga to‘xtalib o‘tamiz. Hisoblash algoritmi (10)-(13) masalani taqribiy yechish algoritmiga tayanadi. Matematik fizikaning bunday noklassik masalalarini sonli yechish algortimlari hozircha yetarlicha ishlab chiqilmagan.

$E = \{0 \le x \le l, \ 0 \le t \le T\}$ sohada x koordinata va t vaqt bo‘yicha tekis to‘r kiritamiz [22]:

$$\overline{\Omega} = \Omega_h \times \Omega_\tau = \left\{(x_i, t_j), \ x_i = ih, \ t_j = j\tau, \ i = 0,1,\ldots,N, \ \ j = 0,1,\ldots,M, \ h = \frac{l}{N}, \ \tau = \frac{T}{M}\right\}.$$

Soddalik uchun $x = x^*$ kuzatish nuqtasi $i = k$ ichki tugun bilan ustma-ust tushadi deb hisoblaymiz.

(10) tenglamani yechish uchun sof oshkormas ayirmali sxemani qo‘llaymiz. (10) ni approksimatsiyalab quyidagiga ega bo‘lamiz

$$\frac{w_i^{j+1} - w_i^j}{\tau} + \frac{v}{m}\frac{w_{i+1}^{j+1} - w_{i-1}^{j+1}}{2h} = \frac{D}{m}\frac{w_{i+1}^{j+1} - 2w_i^{j+1} + w_{i-1}^{j+1}}{h^2} +$$
$$+ \frac{1}{m\psi_k}\left(\varphi^{j+1} - w_k^{j+1}\right)\left(D(\psi''_{xx})_i - v(\psi'_x)_i\right), \ i = 1,2,\ldots,N-1, \ \ j = 0,1,\ldots,M-1. \qquad (14)$$

(11), (13) ni approksimatsiyalab quyidagini hosil qilamiz

$$w_0^{j+1} = c_0, \quad w_N^{j+1} = 0, \quad j = 0,1,\ldots,N-1, \ (15)$$

$$w_i^0 = 0, \ \ i = 0,1,\ldots,N. \qquad (16)$$

(14)-(16) ayirmali masala yechimidan (9) ga mos ravishda quyidagini aniqlaymiz

$$\theta^{j+1} = \frac{1}{\psi_k}\left(\varphi^{j+1} - w_k^{j+1}\right), \quad j = 0,1,\ldots,M-1, \qquad (17)$$

bu munosabatni $\theta^0 = 0$ shart bilan to'ldiramiz. (7) munosabatni inobatga olib, izlanayotgan vaqtdan bog'liq bo'lgan o'ng taraf uchun quyidagi oddiy sonli differensiallash protsedurasini qo'llashimiz mumkin:

$$\eta^{j+1} = \frac{m}{f_0}\frac{\theta^{j+1} - \theta^j}{\tau}, \quad j = 0,1,\ldots,M-1. \quad (18)$$

Qaralayotgan oshkormas sxemani realizatsiya qilish uchun to'rli masalani yechish muammosiga alohida to'xtalish zarur.

4. To'rli nolokal masala va uni realizatsiya qilish

(14)-(16) sxema bo'yicha hisoblashni realizatsiya qilish hech qanday maxsus muammo tug'dirmaydi, to'rli masalaning nostandartligiga (nolokalligiga) qaramasdan yangi vaqt qatlamida hech qanday muammo paydo bo'lmaydi. (14) tenglama ichki tugunlarda quyidagi ko'rinishda bo'ladi

$$\frac{w_i^{j+1}}{\tau} + \frac{v}{m}\frac{w_{i+1}^{j+1} - w_{i-1}^{j+1}}{2h} - \frac{D}{m}\frac{w_{i+1}^{j+1} - 2w_i^{j+1} + w_{i-1}^{j+1}}{h^2} +$$
$$+ \frac{1}{m\psi_k} w_k^{j+1}\left(D(\psi''_{xx})_i - v(\psi'_x)_i\right) = g_i^j, \quad i = 1,2,\ldots,N-1, \quad j = 0,1,\ldots,M-1. \quad (19)$$

bu yerda $g_i^j = \frac{w_i^j}{\tau} + \frac{1}{m\psi_k}\varphi^{j+1}\left(D(\psi''_{xx})_i - v(\psi'_x)_i\right)$. (19) tenglama uchun chegaraviy shart (15) bo'ladi. (15), (19) sistema yechimi quyidagi ko'rinishda izlanadi

$$w_i^{j+1} = y_i + w_k^{j+1} z_i, \quad i = 0,1,\ldots,N. \quad (20)$$

(20) ni (19) ga qo'yib y_i, z_i yordamchi funksiyalar uchun quyidagi to'rli masalani hosil qilamiz

$$\frac{y_i}{\tau} + \frac{v}{m}\frac{y_{i+1} - y_{i-1}}{2h} - \frac{D}{m}\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = g_i^j, \quad i = 1,2,\ldots,N-1, \quad (21)$$

$$y_0 = c_0, \quad y_N = 0, \quad (22)$$

$$\frac{z_i}{\tau} + \frac{v}{m}\frac{z_{i+1} - z_{i-1}}{2h} - \frac{D}{m}\frac{z_{i+1} - 2z_i + z_{i-1}}{h^2} + \frac{1}{m\psi_k}\left(D(\psi''_{xx})_i - v(\psi'_x)_i\right) = 0,$$
$$i = 1,2,\ldots,N-1, \quad (23)$$

$$z_0 = 0, \quad z_N = 0. \quad (24)$$

Shundan keyin (20) ni hisobga olib w_k^{j+1} ni topamiz:

$$w_k^{j+1} = \frac{y_k}{1 - z_k}. \quad (25)$$

Algoritmning korrektligini (25) ning maxraji nolga aylanmasligi ta'minlaydi.

(21), (22) va (23), (24) to'rli masalalar standart masalalar hisoblanadi va ularni sonli yechish hech qanday qiyinchlik tug'dirmaydi.

(21), (23) tenglamalar quyidagi ko'rinishga keladi:

$$Ay_{i-1} - Cy_i + By_{i+1} = -F_i^j, \quad (26)$$

$$Az_{i-1} - Cz_i + Bz_{i+1} = -\overline{F}_i^{\,j}, \qquad (27)$$

bu yerda

$$A = \frac{D\tau}{mh^2} + \frac{\nu\tau}{2mh}, \quad B = \frac{D\tau}{mh^2} - \frac{\nu\tau}{2mh}, \quad C = \frac{2D\tau}{mh^2} + 1,$$

$$F_i^{\,j} = \tau g_i^{\,j}, \quad \overline{F}_i^{\,j} = \frac{\tau}{m\psi_k}\left(\nu(\psi'_x)_i - D(\psi''_{xx})_i\right).$$

(22), (26) chegaraviy masalani progonka usuli yordamida yechish algoritmi quyida ko‘rinishda bo‘ladi:

$$y_i = \alpha_{i+1} y_{i+1} + \beta_{i+1}, \; i = N-1, N-2, \ldots, 1, 0, \qquad (28)$$

$$\alpha_{i+1} = \frac{B}{C - A\alpha_i}, \; \beta_{i+1} = \frac{A\beta_i + F_i^{\,j}}{C - A\alpha_i}, \; i = 1, 2, \ldots, N-1, \qquad (29)$$

$$y_0 = \alpha_1 y_1 + \beta_1 = c_0, \quad \alpha_1 = 0, \; \beta_1 = c_0, \; (30)$$

$$y_N = 0. \qquad (31)$$

Endi (24), (27) chegaraviy masalaning ham progonka usuli yordamida yechish algoritmini keltiramiz:

$$z_i = \overline{\alpha}_{i+1} z_{i+1} + \overline{\beta}_{i+1}, \; i = N-1, N-2, \ldots, 1, 0, \qquad (32)$$

$$\overline{\alpha}_{i+1} = \frac{B}{C - A\overline{\alpha}_i}, \; \overline{\beta}_{i+1} = \frac{A\overline{\beta}_i + \overline{F}_i^{\,j}}{C - A\overline{\alpha}_i}, \; i = 1, 2, \ldots, N-1, \qquad (33)$$

$$z_0 = \overline{\alpha}_1 z_1 + \overline{\beta}_1 = 0, \quad \overline{\alpha}_1 = 0, \quad \overline{\beta}_1 = 0, \; (34)$$

$$z_N = 0. \qquad (35)$$

Kvazireal eksperiment o‘tkazish qandaydir o‘ng taraf bilan berilgan (1)-(3) to‘g‘ri masala qaraladi. O‘ng taraf (4) munosabat beriladi, bu yerda

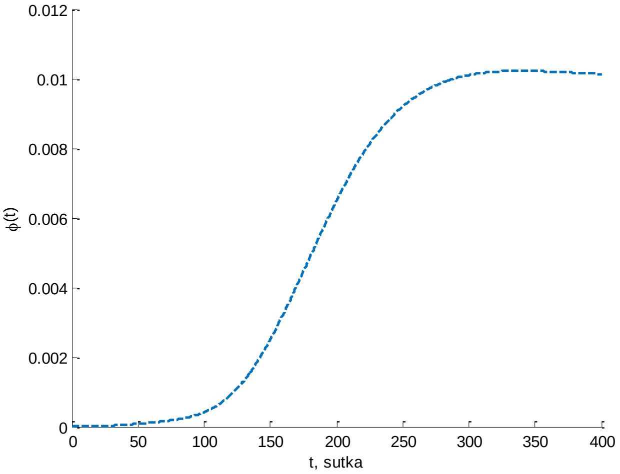
$$\psi(x) = \sin\frac{\pi x}{l}, \quad 0 \le 0 \le l,$$

$$\eta(t) = \begin{cases} t, & 0 < t < 0{,}6T, \\ 0, & 0{,}6T < t < T. \end{cases}$$

Teskari masalani yechishda $\varphi(t)$ to`r funksiya tasodifiy xatoliklar bilan quyidagicha modellashtiriladi [6, 19]

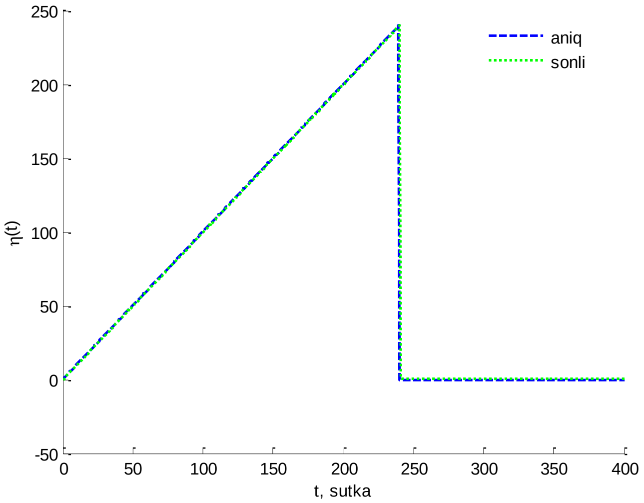
$$\varphi_\delta^{\,j} = \varphi^{\,j} + 2\delta(\sigma^{\,j} - 0{,}5), \; j = 0, 1, \ldots, M,$$

bu yerda $\sigma^{\,j}$ – tasodifiy funksiya bo’lib $[0,1]$ kesmada tekis taqsimlangan sonlarni beradi. δ kattalik xatolik darajasini beradi. 1-rasmda $\varphi(t)$ funksiyaning grafigi $\delta = 0$ da tasvirlangan.

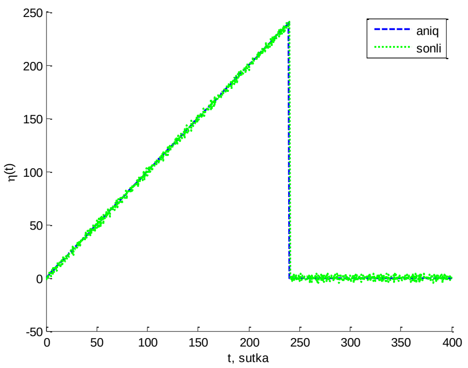


1-rasm. $\varphi(t)$ funksiyaning grafigi ($\delta = 0$ da)

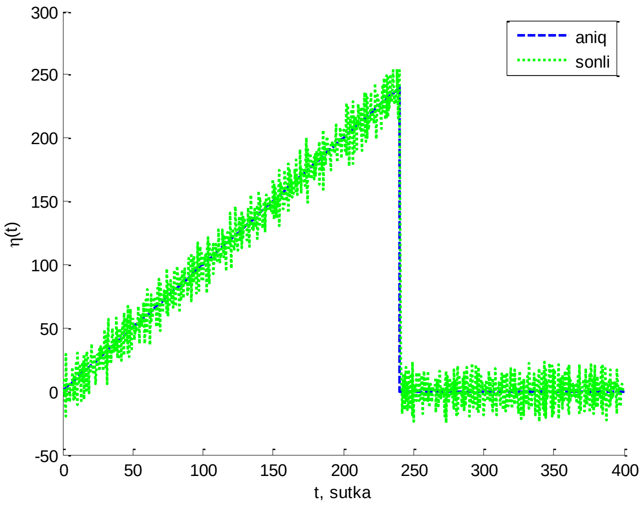
2-7-rasmlarda $\eta(t)$ tiklanishi bo'yicha sonli natijalari tasvirlangan. 2-rasmda $\delta = 0$ da tiklangan $\eta(t)$ funksiya aniq $\eta(t)$ funksiya bilan ustma-ust tushganligini ko'rishimiz mumkin. 3-7 rasmlarda $\varphi(t)$ funksiya mos ravishda $\delta = 0{,}00001$; 0,00005; 0,0001; 0,00025; 0,0005 xatoliklar bilan berilgandagi $\eta(t)$ funksiyaning tiklanish grafiklari berilgan.



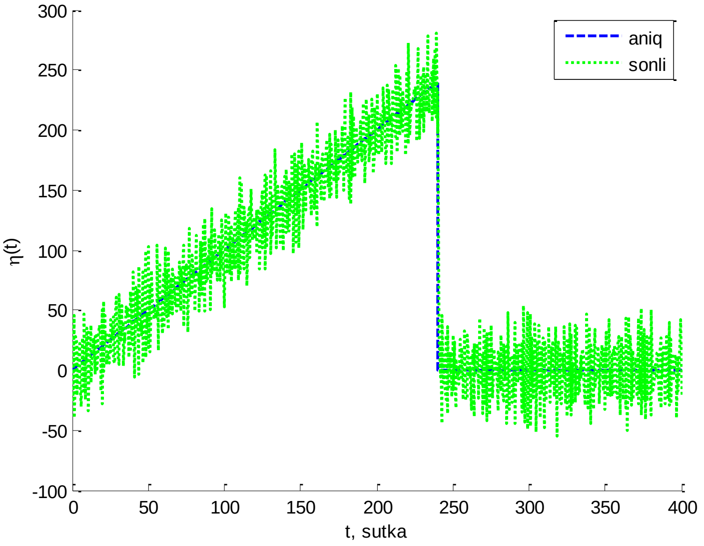
2-rasm. $\delta = 0$ da $\eta(t)$ funksiyaning tiklanishi



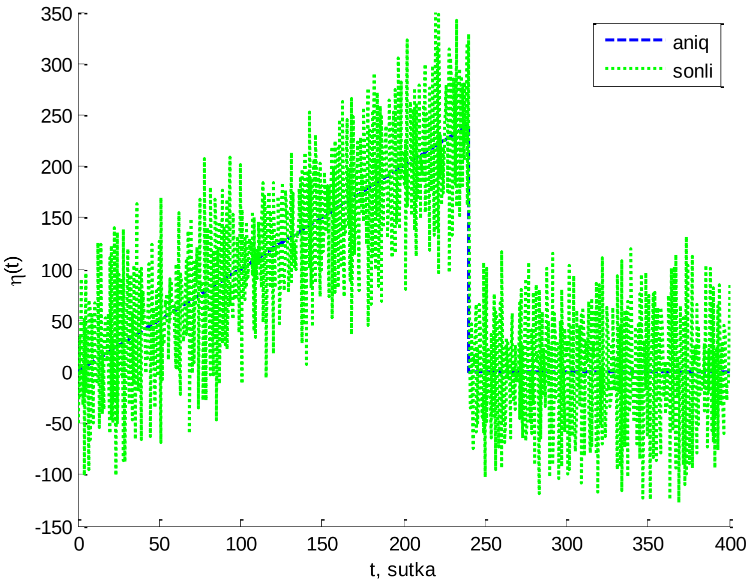
3-rasm. $\delta = 0{,}00001$ da $\eta(t)$ funksiyaning tiklanishi



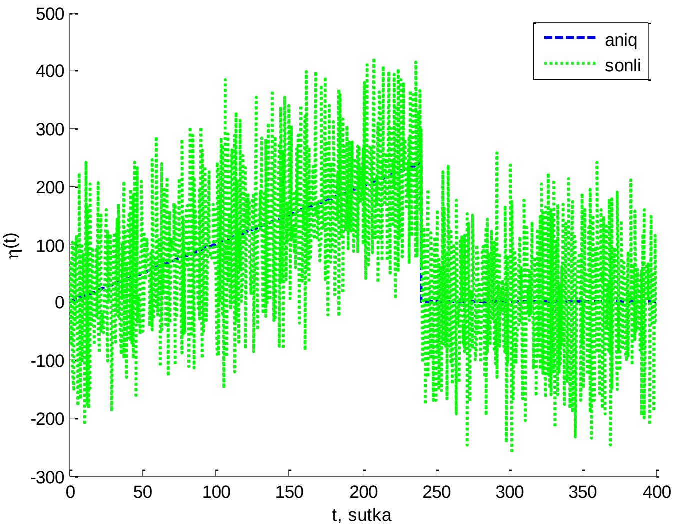
4-rasm. $\delta = 0{,}00005$ da $\eta(t)$ funksiyaning tiklanishi



5-rasm. $\delta = 0{,}0001$ da $\eta(t)$ funksiyaning tiklanishi



6-rasm. $\delta = 0{,}00025$ da $\eta(t)$ funksiyaning tiklanishi



7-rasm. $\delta = 0{,}0005$ da $\eta(t)$ funksiyaning tiklanishi

3-4 rasmlardagi sonli natijalardan ko'rinib turibdiki, δ xatolik darajasining kichik qiymatlarida $\eta(t)$ tiklanishini amalda qo'llash mumkin. Ammo δ ning kattaroq qiymatlarida $\eta(t)$ o`ng taraf funksiyasining tiklanishi turg'unmas xarakterga ega (5-7 rasmlar). Bundan ko'rinadiki o'ng tarafni tiklash masalasida yechim haqidagi qo'shimcha ma'lumotlar ($\varphi(t)$ funksiya) xatoligi ma'lum darajadan oshib ketmasligi kerak ekan.

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