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KO'P O'ZGARUVCHILI FUNKSIYA EKSTREMUMI, EKSTREMUM BO'LISHINING ZARURIY VA YETARLI SHARTI, SHARTLI EKSTREMUM.

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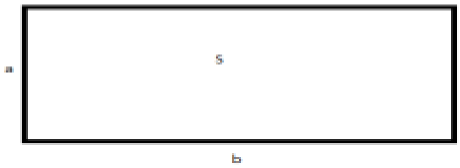
Iishlab chiqarishning turli sohalarida 2 va undan ko'p o'zgaruvchili funksiyalar bilan ishlashga to'g'ri keladi, masalan

1)

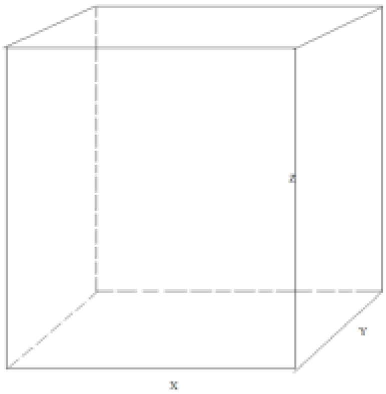


S – to'g'ri to'rtburchakning yuzi 2 o'zgaruvchiga bog'liq: S = a*b , S=f(a,b)

2) Gazning hajmi V uning temperaturasi t ga va bosimi p ga bog'liq:

$V=\frac{T}{P}$, V = f(p,t)

3) To'g'ri parallellelipipedning hajmi 3 ta o'zgaruvchiga bog'liq:

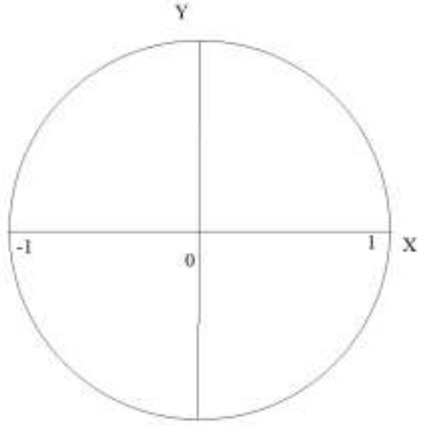


V=x*y*z, V=f(x,y,z)

Biz bundan buyon faqat 2 o'zgaruvchili funksiyalarni o'rganamiz, 2 o'zgaruvchili funksiyalar uchun bildirilgan mulohazalar ko'p o'zgaruvchili funksiyalar uchun ham o'rinli bo'ladi.

Ta'rif-1 Agar bir-biriga bog'liq bo'lmagan ikki o'zgaruvchi x va y ning har bir juft (x,y) qiymatiga z miqdorning yagona qiymati mos kelsa, z = f(x,y) ikki o'zgaruvchili funksiya berilgan deyiladi.

Ta'rif-2 z = f(x,y) ikki o'zgaruvchili funksiya aniqlangan (x,y) juftliklar to'plami funksiyaning aniqlanish sohasi deyiladi.



Ta'rif-3 z=f(x,y) funksiyaning x bo'yicha xususiy hosilasi deb xususiy orttirma $\Delta_x z$ ning Δx orttirmaga nisbatining Δx nolga intilgandagi limitga aytiladi:

$$\frac{\partial z}{\partial x} = \lim_{\Delta x \to 0} \frac{\Delta_x z}{\Delta x} = \lim_{Vx \to 0} \frac{f(x + Vx, y) - f(x, y)}{Vx} \quad \text{va} \quad z'_y,\ f'_x(x,y), \frac{\partial z}{\partial x}$$ kabi belgilanadi.

Demak, z=f(x,y) funksiyaning x bo'yicha xususiy hosilasi deb, y ni o'zgarmas faraz qilib , x bo'yicha topilgan; y bo'yicha xususiy hosila deb, x ni o'zgarmas faraz qilib, y bo'yicha topilgan hosilaga aytiladi.

Misol-1 z=x²siny funksiyaning $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ xususiy hosilalari topilsin.

$$\frac{\partial z}{\partial x} = \frac{\partial (x^2 \sin y)}{\partial x} = 2x \sin y$$

$$\frac{\partial z}{\partial y} = \frac{\partial (x^2 \sin y)}{\partial y} = x^2 \cos y \qquad \frac{\partial z}{\partial y} = \frac{\partial (x^2 \sin y)}{\partial y} = x^2 \cos y$$

Endi to'liq differensialning ta'rifini keltiramiz. Z=f(x,y) funksiya berilgan bo'lsin.

Ta'rif-4 z=f(x,y) funksiyaning to'liq differensiali deb $f'_x(x,y)Vx + f'_y(x,y)Vy$ ifodaga aytiladi.

Bu yerda $f'_x(x,y) = \frac{\partial z}{\partial x}$ va $f'_y(x,y) = \frac{\partial z}{\partial y}$, hamda $Vx = dx, Vy = dy$ ni hisobga olib :

dz= $\frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$ ni hosil qilamiz.

Misol-2

$z = x^3 y + xtgy$, dz=?

(1) formulaga ko'ra :

$$\frac{\partial z}{\partial x} = 3x^2 y + tgy; \qquad \frac{\partial z}{\partial y} = x^3 + x\frac{1}{\cos^2 y}$$

$$dz = (3x^2 y + tgy)dx + (x^3 + x\frac{1}{\cos^2 y})dy$$

Har xil tartibli xususiy hosilalarni hisoblashni ko'rib chiqamiz:

Z=f(x,y) berilgan va u $\frac{\partial z}{\partial x}$ va $\frac{\partial z}{\partial y}$ xususiy hosilalarga ega bo'lsin. Ulardan yana xususiy hosilalar olish mumkin. Ikki o'zgaruvchili funksiyaning 2-tartibli xususiy hosilalari 4 ta bo'ladi:

$$\frac{\partial^2 z}{\partial x \partial x} = f''_{xx}(x, y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = f''_{xy}(x, y)$$

$$\frac{\partial^2 z}{\partial y \partial x} = f''_{yx}(x, y)$$

$$\frac{\partial^2 z}{\partial y \partial y} = f''_{yy}(x, y)$$

Ulardan yana x va y bo'yicha hosila olinsa, 3-tartibli xususiy hosilalarga ega bo'lamiz –ular 8 ta. Umuman, (n)-tartibli xususiy hosila – bu (n-1) – tartibli xususiy hosilaning birinchi tartibli hosilasidir.

Misol-3 $z = y^2e^{2x} + x^4y^3 + 5$ bo'lsa, $\frac{\partial^3 z}{\partial^2 x \partial y}$ ni toping.

$$\frac{\partial z}{\partial x} = 2y^2e^{2x} + 4x^3y^3$$

$$\frac{\partial^2 z}{\partial x^2} = 4y^2e^{2x} + 12x^2y^3$$

$$\frac{\partial^3 z}{\partial^2 x \partial y} = 8ye^{2x} + 36y^2x^2$$

Funksiya ekstremumi

Z=f(x,y) funksiya biror G sohada berilgan bo'lsin. $M_0(x_0,y_0) \in G$.

Ta'rif-5 G sohada barcha (x_0,y_0) nuqtalar uchun

$f(x_0, y_0) > f(x, y)$ $\quad (f(x_0,y_0) < f(x,y))$

bo'lsa, u holda Z=f(x,y) funksiya $M_0(x_0,y_0)$ nuqtada

maksimum (minimum) ga ega deyiladi. Funksiyaning maksimumlari va minimumlari birgalikda funksiyaning ekstremumlari deyiladi.

Teorema-1 (ekstremum mavjudligining yetarlilik sharti)

Z=f(x,y) funksiya $M_0(x_0,y_0)$ nuqtada va uning biror atrofida 3-tartibgacha xususiy hosilalarga ega hamda $M_0(x_0,y_0)$ nuqta Z=f(x,y) funksiyaning kritik nuqtasi bo'lsin, ya'ni $z'_x(x_0, y_0) = 0$ va $z'_y(x_0, y_0) = 0$ bo'lsin. U holda $\Delta(M_0) = z''_{xx}(M_0) * z''_{yy}(M_0) - \left[z''_{xy}(M_0)\right]^2 > 0$ shartda

$M_0(x_0,y_0)$ nuqta $\Delta(M_0) < 0$ da maksimum, $\Delta(M_0) > 0$ da minimum nuqtasi bo'ladi.

Misol-4 $z = x^2 - 2xy + 3y^2 + x - 2y + 5$ funksiyani ekstremumga tekshiring.

Dastlab, berilgan funksiyaning kritik nuqtalarini topamiz:

$z'_x = 2x - 2y + 1 \qquad z'_y = 6y - 2x - 2$

$\begin{cases} 2x - 2y + 1 = 0 \\ -2x + 6y - 2 = 0 \end{cases} \Rightarrow \quad X = -\frac{1}{4}; \quad y = \frac{1}{4};$ Kritik nuqta $\left(-\frac{1}{4};\frac{1}{4}\right)$

Endi 2-tartibli hosilalarni topamiz: $z''_{xx} = 2$; $z''_{yy} = 6$; $z''_{xy} = -2$

$\Delta(M_0)$ ifodani tuzamiz:

$\Delta(M_0) = z''_{xx}(M_0) * z''_{yy}(M_0) - \left[z''_{xy}(M_0)\right]^2 = 2*6 - (-2)2 = 12 - 4 = 8 > 0,$ *demak*

Z=f(x,y) funksiya $\left(-\frac{1}{4};\frac{1}{4}\right)$ nuqtada minimumga erishadi.

Shartli ekstremum_ Z=f(x,y) funksiyaning x va y ga nisbatan biror shart asosida ekstremumini topish – shartli ekstremum deyiladi. Quyidagi misol yordamida ko'rib chiqamiz.

Misol-5 $z = x^2 + y^2$ funksiyaning $x - y + 2 = 0$ shart asosida ekstremumini toping.

x-y+2=0 $\Rightarrow$ *y=x+2* $\Rightarrow$ *z=x²+(x+2)²*

$z' = 2x + 2(x+2) = 4x + 4 = 0 \Rightarrow$ *x=-1 -kritik nuqta*

$z'' = 4 > 0 \Rightarrow$ *x=-1 da minimumgz erishadi.*

y ni topamiz: *y=x+2* $\Rightarrow$ *x=-1 da y=1*

Shartli ekstremumni topamiz: $z = x^2 + y^2 \Rightarrow z = (-1)^2 + 1^2 = 2$

Demak, javob: $z_{\min} = z(-1;1) = 2$ ***(x-y=2=0 shart asosida!)***

Shartli ekstremumni Lagranj ko'paytuvchilari usulida topish_ ni quyidagi misol asosida ko'rib chiqamiz:

Misol-6 *z=f(x,y)=5-3x-4y funksiyani $x^2+y^2=25$ shart asosida ekstremumini toping.*

Dastlab, z ni φ(x,y)=0 ko'rinishga keltirib Lagranj funksiyasini tuzib olamiz:

L=f(x,y) + λ φ(x,y) , λ – Lagranj ko'paytuvchisi.

L=5-3x-4y + λ(x^2+y^2-25)

$L'_x = -3 + \lambda(2x + 0 - 0) = -3 + 2\lambda x$

$L'_y = -4 + \lambda(2y + 0 - 0) = -4 + 2\lambda y$

$$\begin{cases} L'_x = 0 \\ L'_y = 0 \\ \varphi(x,y) = 0 \end{cases} \Rightarrow \begin{cases} -3 + 2\lambda x = 0 \\ -4 + 2\lambda y = 0 \\ x^2 + y^2 - 25 = 0 \end{cases} \Rightarrow \begin{cases} 2\lambda x = 3 \\ 2\lambda y = 4 \end{cases} \Rightarrow \begin{cases} x = \frac{3}{2\lambda} \\ y = \frac{2}{\lambda} \end{cases}$$

$$\Rightarrow \left(\frac{3}{2\lambda}\right)^2 + \left(\frac{2}{\lambda}\right)^2 - 25 = 0$$

$$\frac{9}{4\lambda^2} + \frac{4}{\lambda^2} - 25 = 0 \Rightarrow \frac{25}{4\lambda^2} = 25 \Rightarrow \lambda^2 = \frac{1}{4} \Rightarrow \lambda = \pm\frac{1}{2}$$

$\lambda = -\frac{1}{2}$ da $x = \frac{3}{2(-\frac{1}{2})} = -3$; $y = \frac{2}{-\frac{1}{2}} = -4$; $\Rightarrow M_1(-3;-4)$

$\lambda = \frac{1}{2}$ da $x = \frac{3}{2*\frac{1}{2}} = 3;$ $y = \frac{2}{-\frac{1}{2}} = -4;$ $\Rightarrow$ $M_2(3;4)$

$z(-3;-4) = 5 - 3*(-3) - 4*(-4) = 30$ -maksimum;

$z(3;4) = 5 - 3*3 - 4*4 = -20$ -minimum;

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