YUKLANGAN KASR TARTIBLI INTEGRO-DIFFERENSIAL TENGLAMALAR UCHUN MASALALAR

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Annotatsiya. *Ushbu maqolada Kaputo kasr tartibli operator qatnashgan yuklangan tenglama uchun masala o`rganilgan. Bu masalalar yechimlari Koshi masalasi yechimdan foydalanib topilgan.*

Kalit so`zlar: *yuklangan integro-differensial tenglama, kasr tartibli operator, Koshi masalasi.*

ЗАДАЧИ ДЛЯ ЗАГРУЖЕННЫХ ИНТЕГРО- ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ДРОБНОГО ПОРЯДКА

Аннотация: *В данной статье изучались две задачи для нагруженного уравнения с дробным оператором по Капуто. Найдены решения этих задач с использованием решения задачи Коши.*

Ключевые слова: *нагруженное интегро-дифференциальное уравнение, оператор дробного порядка, задача Коши.*

PROBLEMS FOR LOADED INTEGRO-DIFFERENTIAL EQUATIONS OF FRACTIONAL ORDER

Abstract: *In this article,the problem for the loaded equation involving the Caputo fractional operator is studied.The solution of these problems are found using the solution of the Cauchy problem.*

Keywords: *loaded integro-differential equation, fractional order operator, Cauchy problem.*

I. Kirish. So‘ngi vaqtlarda noma`lum funksiyani biror qiymati qatnashgan differensial tengalamalar bilan shug‘ullanishga bo`lgan qiziqish ortib bormoqda. Bunga sabab ko`plab issiqlik tarqalish va diffuziya jarayonlarini matematik modelini tuzish funksiyani biror qiymati qatnashgan differensial tenglama uchun qo‘yiladigan masalalarga keltiriladi. Odatda, bunday turdagi tenglamalar yuklangan differensial tenglama deb yuritiladi. Yuklangan xususiy hosilali va oddiy differensial tenglamalar yuklangan differensial tenglama ko`plab tadqiqotchilar tomonidan o‘rganilgan (masalan, ushbu [1]–[3] ishlarga qaralsin).

II. Masalaning qo‘yilishi va tadqiqoti.

(0,1) oraliqda ushbu

$$ {}_cD_{0x}^{\alpha}y(x)-\lambda I_{0x}^{\gamma}y(x)=f(x) \qquad (1)$$

kasr tartibli integro - differensial tenglamani qaraylik, bu yerda $y(x)$ - noma’lum funksiya; α,γ,λ - o‘zgarmas haqiqiy sonlar bo'lib, $\gamma>0$; ${}_cD_{0x}^{\alpha}y(x)$ - Kaputo ma'nosida α (kasr) tartibli hosila operatori, $I_{0x}^{\gamma}y(x)$ - Riman-Liuvill ma'nosida γ (kasr) tartibli integral operatori:

$$ {}_cD_{0x}^{\alpha}y(x)=\frac{1}{\Gamma(1-\alpha)}\int_0^x (x-t)^{-\alpha}\,y'(t)\,dt,\, x>0, $$

$$ I_{0x}^{\gamma}y(x)=\frac{1}{\Gamma(\gamma)}\int_0^x (x-t)^{\gamma-1}\,y(t)\,dt,\, x>0. $$

A masala. Shunday $y(x)$ funksiya topilsinki, u quyidagi xossalarga ega bo'lsin:

1) (0,1) oraliqda (1) tenglamani qanoatlantirsin;
2) $x=0$ nuqtada esa

$$ y(0)=A, \qquad (2) $$

shartni qanoatlantirsin, bu yerda A - berilgan o'zgarmas haqiqiy son.

(1) tenglamaga $I_{0x}^{\alpha}y(x)$ ni ta'sir ettirib,

$$ I_{0x}^{\alpha}\{I_{0x}^{\gamma}y(x)\}=I_{0x}^{\alpha+\gamma}y(x),\; I_{0x}^{\alpha}\{{}_cD_{0x}^{\alpha}y(x)\}=y(x)-y(0) \qquad (3) $$

(3) xossalardan va y(0)=A shartdan foydalanib,uni quyidagicha yozib olamiz:

$$ y(x)=\lambda I_{0x}^{\alpha+\gamma}y(x)+I_{0x}^{\alpha}f(x)+A \qquad (4) $$

ko'rinishdagi integral tenglamani hosil qilamiz.

(4) Volterra integral tenglamasi bo‘lib,

$$ y(x)-\frac{\lambda}{\Gamma(\alpha+\gamma)}\int_0^x (x-z)^{\alpha+\gamma-1}y(z)dz=A+\frac{1}{\Gamma(\alpha)}\int_0^x (x-z)^{\alpha-1}f(z)dz \qquad (5) $$

uni yechish uchun ba’zi belgilashlarni kiritamiz:

$$ g(x)=\frac{1}{\Gamma(\alpha)}\int_0^x (x-z)^{\alpha-1}f(z)dz+A,\; K(x,z)=\frac{(x-z)^{\alpha+\gamma-1}}{\Gamma(\alpha+\gamma)} \qquad (6) $$

(5) tenglamani ketma-ket yaqinlashish usuli orqali yechamiz.
Buning uchun

$$ K_1(x,z)=\frac{(x-z)^{\alpha+\gamma-1}}{\Gamma(\alpha+\gamma)} \quad \text{va} \quad K_i(x,y)=\int_y^x K_1(x,t)K_{i-1}(t,y)dt $$

formulalardan foydalanib,ba’zi hisoblashlarni amalga oshirib,

$$ K_n(x,z)=\frac{(x-z)^{n(\alpha+\gamma)-1}}{\Gamma(n(\alpha+\gamma))} $$

ko‘rinishda topamiz. $K_n(x,z)$ yadrolarning rezolventasi

$$ R(x,z,\lambda)=\sum_{n=1}^{+\infty}\frac{\lambda^{n-1}(x-z)^{n(\alpha+\gamma)-1}}{\Gamma(m(\alpha+\gamma))} $$

ko‘rinishda bo‘ladi.

Integral tenglamalar nazariyasiga ko‘ra (5) tenglamani yechimini,

$$y(x)=g(x)-\lambda\int_0^x R(x,z,\lambda)g(z)dz$$

ko‘rinishda topamiz, bu yerda $R(x,z,\lambda)=\sum_{n=1}^{+\infty}\frac{\lambda^{n-1}(x-z)^{n(\alpha+\gamma)-1}}{\Gamma(m(\alpha+\gamma))}$.

(6) belgilashlarni e’tiborga olib, ba'zi hisoblashlarni amalga oshirib, A masalaning yechimini

$$y(x)=AE_{\alpha+\gamma,1}\left(\lambda x^{\alpha+\gamma}\right)+\int_0^x (x-z)^{\alpha-1} f(z)E_{\alpha+\gamma,\alpha}\lambda(x-z)^{\alpha+\gamma}dz \qquad (7)$$

ko'rinishda topamiz, bu yerda $E_{p,q}(z)=\sum_{n=0}^{+\infty}\frac{z^n}{(pn+q)}$ - Mittag-Leffer funksiyasi.

Endi

$${}_cD_{0x}^{\alpha}y(x)-\lambda I_{0x}^{\gamma}y(x)=y(x_0) \qquad (8)$$

yuklangan tartibli integro-differensial tenglamani qaraylik

B masala. Shunday $y(x)$ funksiya topilsinki, u (8) tenglamani va (2) shartni qanoatlantirsin.

Bu masalaning yechimini A masalaning yechimidan foydalanib,

$$y(x)=AE_{\alpha+\gamma,1}\left(\lambda x^{\alpha+\gamma}\right)+y(x_0)x^{\alpha}E_{\alpha+\gamma,\alpha+1}\left(\lambda x^{\alpha+\gamma}\right) \qquad (9)$$

ko'rinishda aniqlanadi.

(9) dan $x=x_0$ ni o'rniga qo'yib, $y(x_0)$ ni

$$y(x_0)=\frac{AE_{\alpha+\gamma,1}\left(\lambda x_0^{\alpha+\gamma}\right)}{1-x_0^{\alpha}E\left(\lambda x_0^{\alpha+\gamma}\right)} \qquad (10)$$

ko`rinishda topamiz.

(10) ni (9) ga olib borib qo'yilib yechim B masalani yechimini

$$y(x)=AE_{\alpha+\gamma,1}\left(\lambda x^{\alpha+\gamma}\right)+\frac{AE_{\alpha+\gamma,1}\left(\lambda x_0^{\alpha+\gamma}\right)}{1-x_0^{\alpha}E_{\alpha+\gamma,\alpha+1}\left(\lambda x_0^{\alpha+\gamma}\right)}x_0^{\alpha}E_{\alpha+\gamma,\alpha+1}\left(\lambda x_0^{\alpha+\gamma}\right) \qquad (11)$$

ko'rinishda topamiz.

1-teorema. Agar $x_0^{\alpha}E_{\alpha+\gamma,\alpha+1}\left(\lambda x_0^{\alpha+\gamma}\right)\neq 1$ bo‘lsa, u holda B masala yagona yechimga ega bo‘lib, u (11) formula bilan aniqlanadi.

Ta’kidlash joizki B masalaga o`xshash masala [4] ishda ko`rilgan.

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