IKKINCHI TARTIBLI XUSUSIY HOSILALI BUZILADIGAN DIFFERENSIAL TENGLAMA UCHUN TESKARI MASALALAR

Yigitaliyeva Muazzasxon Mo'sajon qizi

Annatotsiya. *Ushbu maqolada buziladigan ikkinchi tartibli tenglama uchun boshlang`ich-chegaraviy malasa bayon qilingan va tadqiq etilgan. Malasaning yechimining yagonali energiya integrallari usulidan foydalanib isbotlangan. Malasa yechimining mavjud ekanligi esa o`zgaruvchilarni ajratish usuli yordamida ko`rsatilgan.*

Kalit so'zlar: *buziladigan differensial tenglamalar, chegaraviy masala, energiya integrallari usuli, o`zgaruvchilarni ajratish usuli.*

Annotation. *In this article, the initial-boundary problem for the degenerative second-order equation is described and researched. The solution of the problem is proved using the method of energy integrals. The existence of a solution to the problem is shown using the method of separation of variables.*

Key words: *Degenerative differential equations, boundary value problem, method of energy integrals, method of separation of variables.*

Аннотация. *В данной статье описана и исследована начально-краевая задача для вырождающегося уравнения второго порядка. Решение задачи доказывается методом интегралов энергии. Существование решения задачи показано с помощью метода разделения переменных.*

Ключевые слова: *Вырождающиеся дифференциальные уравнения, краевая задача, метод интегралов энергии, метод разделения переменных.*

KIRISH

I. Masalaning qo'yilishi.

Biz ushbu ishda ikkinchi tartibli xususiy hosilali buziladigan differensial tenglamalar uchun chegaraviy masalani ko'rib chiqamiz.

$\Omega=\{(x,t):0<x<1,0<t<T\}$ sohada

$$D_{0t}^{a}u(x,t)=\left[x^{\beta}u_x(x,t)\right]_x+f(x) \quad (1)$$

tenglamani qaraylik, bu yerda ${}_cD_{0t}^{a}$-Kaputo ma'nosidagi kasr tartibli operator $[7]$

$$D_{0t}^{\alpha}u(x,t)=\frac{1}{\Gamma(1-\alpha)}\int_0^1\frac{u_z(x,z)}{(t-z)^{\alpha}}dz\ ,$$

α,β,T-o'zgarmas sonlar bo'lib, $0\le\beta<1$, $0<\alpha<1$, $T>0$, $u=u(x,t)$ va $f(x)$-noma'lum funksiyalar.

A masala. Shunday $\{u(x,t),f(x)\}$ funksiyalar juftligi topilsinki ular quyidagi xossalarga ega bo'lsin:

1) $u(x,t),x^{\beta}u_x(x,t)\in C\left(\bar{\Omega}\right)$; ${}_cD_{0t}^{\alpha}u(x,t)$, $\left[x^{\beta}u_x\right]_x\in C(\Omega)$; $f(x)\in C(0,1)\cap L(0,1)$;

2) Ω sohada (1) tenglamani qanoatlantiradi;

3) Ω soha chegarasida ushbu

$$u(0,t)=0,\ u(1,t)=0,\ t\in[0,T]; \tag{2}$$

$$u(x,0)=\varphi_1(x),\ u(x,T)=\varphi_2(x), x\in[0,1]; \tag{3}$$

chegaraviy shartlarni qanoatlantiradi, bu yerda $\varphi_1(x)$ va $\varphi_2(x)$-berilgan funksiyalar.

II. Yechimning mavjudligi.

Masalaning yechimini ko'rsatish uchun (1) tenglamaning yechimini

$$u(x,t)=X(x)T(t) \tag{4}$$

ko'rinishda qidiramiz. Uni (1) tenglamaga qo'yib, ba'zi almashtirishlarni bajarib,

$$\left[x^{\beta}X'(x)\right]'+\lambda X(x)=0,\quad 0<x<1 \tag{5}$$

ko'rinishidagi tenglamani hosil qilamiz. (3) shartlardan $X(x)$ funksiya uchun

$$X(0)=0,\ X(1)=0 \tag{6}$$

shartlarni olamiz.

$\{(5),(6)\}$ spektral masalaning xos sonlarini toppish uchun (5) tenglamani $X(x)$ ga ko'paytirib $(0,1)$ oraliqda integrallaymiz:

$$\int_0^1\left[x^{\beta}X'(x)\right]_x X(x)dx+\lambda\int_0^1 X^2(x)dx=0. \tag{7}$$

(7) ni bir marta bo'laklab integrallab, (6) shartdan foydalanib,

$$\int_0^1 x^{\beta}\left[X'(x)\right]^2 X(x)dx=\lambda\int_0^1 X^2(x)dx \tag{8}$$

tenglik hosil qilamiz. Undan $\lambda\geq 0$ ekanligi kelib chiqadi.

Dastlab, $\lambda=0$ bo'lsin, (7) ga ko'ra $\left[x^{\beta}X'(x)\right]'=0$ bo'lishidan $X(x)=C_1\dfrac{x^{1-\beta}}{1-\beta}+C_2$ ekanligi kelib chiqadi, uni (7) shatga bo'ysundirsak, $X_n(x)\equiv 0$ kelib chiqadi. Demak, $\lambda=0$ xos son emas.

Endi $\lambda>0$ bo'lsin

$$X(x)=V(z),\qquad z=\frac{x^{2-\beta}}{(2-\beta)^2} \tag{9}$$

belgilash kiritamiz. Uni (5) tenglamaga qo'yib quyidagi tenglamani hosil qilamiz

$$zV''(z)+\frac{V'(z)}{2-\beta}+\lambda V(z)=0\ . \tag{10}$$

Bu tenglama Bessel tenglamasiga keltirilgan tenglama bo'lib, uning yechimi

$$V(z)=\left(2\sqrt{z}\right)^{\frac{1-\beta}{2-\beta}}\left[C_1J_{\nu}\left(2\sqrt{z\lambda}\right)+C_2J_{-\nu}\left(2\sqrt{z\lambda}\right)\right]$$

ko'rinishga keladi.

(10) belgilashga ko'ra (5) tenglamaning umumiy yechimi

$$X(x)=\left(\frac{2}{1-\beta}\right)^{\frac{1-\beta}{2-\beta}} x^{\frac{1-\beta}{2}}\left[C_1 J_\nu\left(2\sqrt{\lambda}\,\frac{x^{\frac{2-\beta}{2}}}{2-\beta}\right)+C_2 J_{-\nu}\left(2\sqrt{\lambda}\,\frac{x^{\frac{2-\beta}{2}}}{2-\beta}\right)\right]$$

(11)

ko‘rinishda bo‘ladi, bu yerda C_1, C_2, ν -o‘zgarmas sonlar, $\nu=\frac{1-\beta}{2-\beta}$.

Endi (12) yechimni X(0)=0 chegaraviy shartga bo‘ysundirib, $C_2=0$ ekanligini topamiz. X(1)=0 dan $C_1 J_\nu\left(\frac{2\sqrt{\lambda}}{2-\beta}\right)=0$ tenglama hosil bo‘ladi. $C_1\neq 0$ deb, $J_\nu\left(\frac{2\sqrt{\lambda}}{2-\beta}\right)=0$ tenglamani yechamiz. $\nu>0$ bo‘lganligi uchun $J_\nu\left(\frac{2\sqrt{\lambda}}{2-\beta}\right)=0$ tenglama absolyut qiymati bo‘yicha cheksiz kattalashib boruvchi sanoqli sondagi haqiqiy yechimlarga ega. Uning n-musbat yechimini θ_n bilan belgilasak $\{(5),(6)\}$ masalaning sanoqli sondagi

$$\lambda_n=\left[\frac{2-\beta}{2}\theta_n\right]^2,\quad n=1,2,...$$

xos qiymatlari kelib chiqadi, unga mos keluvchi xos funksiyalar esa

$$X_n(x)=x^{\frac{1-\beta}{2}} J_\upsilon\left[\theta_n x^{\frac{2-\beta}{2}}\right], n=1,2,... \qquad (12)$$

ko‘rinishda bo‘ladi.

Bu yerda,

$$\nu=\frac{1-\beta}{2-\beta},\ \theta_n=\frac{2}{1-\beta}\sqrt{\lambda_n}.$$

1-lemma. (12) formula bilan aniqlanuvchi $X_n(x)$, $n\in N$ funksiyalar (0,1) kesmada ortonormal va to‘la sistema tashkil etadi [ ].

Endi masalaning yechimini

$$u(x,t)=\sum_{n=1}^{+\infty} X_n(x)T_n(t)$$

(13)

ko‘rinishda qidiramiz, bu yerda $T_n(t)$ noma’lum funksiya. (13) ni (1) tenglamaga va (3) shartlarga qo‘yib,

$$_cD_{0t}^{a}T_n(t)+\lambda T_n(t)=f_n \quad 0<t<T \qquad (14)$$

$$T_n(0)=\varphi_{1n},\ T_n(T)=\varphi_{2n} \qquad (15)$$

chegaraviy masalani hosil qilamiz, bu yerda $\varphi_{jn}=\frac{1}{\mu_n}\int_0^1\varphi(x)X_n(x)dx$,

$f_n=\frac{1}{\mu_n}\int_0^1 f(x)X_n(x)dx$, $\mu_n=\int_0^1 X^2{}_n(x)dx=\frac{1}{2-\beta}J_{\upsilon+1}^2(\theta_n)$.

(14) tenglamaning umumiy yechimi

$$T_n(t)=C_nE_{\alpha,1}\left(-\lambda t^{\alpha}\right)+\frac{f_n}{\lambda_n}$$

(16)

ko‘rinishda bo‘ladi[ ]. (16) tenglamani (15) shartlarga bo‘ysundirib,

$$C_n=\frac{\varphi_{1n}-\varphi_{2n}}{1-E_{\alpha,1}\left(-\lambda_nT^{\alpha}\right)},\qquad f_n=\frac{\lambda_n}{1-E_{\alpha,1}\left(-\lambda_nT^{\alpha}\right)}\left(\varphi_{2n}-\varphi_{1n}\right)+\lambda_n\varphi_{1n} \qquad (17)$$

tengliklarni topamiz. Topilgan tengliklarni (16) ga qo’yib, ba’zi soddalashtirishlarni amalga oshirib

$$T_n(t)=\frac{\left(\varphi_{2n}-\varphi_{1n}\right)}{E_{\alpha,1}\left(-\lambda_nT^{\alpha}\right)}\left(E_{\alpha,1}\left(-\lambda_nt^{\alpha}\right)-1\right)+\varphi_{1n}$$

tenglikka ega bo‘lamiz.

Yuqoridagilarga asosan, A masalaning formal yechimini

$$u\left(x,t\right)=\sum_{n=1}^{\infty}\left[\frac{\left(\varphi_{2n}-\varphi_{1n}\right)}{E_{\alpha,1}\left(-\lambda_nT^{\alpha}\right)}\left(E_{\alpha,1}\left(-\lambda_nt^{\alpha}\right)-1\right)+\varphi_{1n}\right]x^{\frac{1-\beta}{2}}J_{\upsilon}\left[\theta_nx^{\frac{2-\beta}{2}}\right] \qquad (18)$$

$$f\left(x\right)=\sum_{n=1}^{\infty}\left[\frac{\lambda_n}{1-E_{\alpha,1}\left(-\lambda_nT^{\alpha}\right)}\left(\varphi_{2n}-\varphi_{1n}\right)+\lambda_n\varphi_{1n}\right]x^{\frac{1-\beta}{2}}J_{\upsilon}\left[\theta_nx^{\frac{2-\beta}{2}}\right] \qquad (19)$$

ko‘rinishida yozish mumkin bo‘ladi.

1-teorema. Agar $\varphi_j\left(x\right)$, $x^{\beta}\varphi_j'\left(x\right)$, $\left[x^{\beta}\varphi_j'\left(x\right)\right]'$, $\varphi_j\left(0\right)=\varphi_j\left(1\right)=0$ va $\left[x^{\beta}\varphi_j'\left(x\right)\right]'_{x=0}=0$, $\left[x^{\beta}\varphi_j'\left(x\right)\right]'_{x=1}=0$, $j=1,2$ bo‘lsa, (18) va (19) qator bilan aniqlangan $u\left(x,t\right)$ funksiya A masalaning yechimi bo‘ladi.

Isbot. Teoremani isbotlash uchun (16) va $x^{\beta}u_x\left(x,t\right)$, $\left[x^{\beta}u_x\left(x,t\right)\right]_x$, ${}_cD_{0t}^{\alpha}u(x,t)$ ga mos keluvchi qatorlarni tekis yaqinlashuvchi ekanligini ko‘rsatish yetarli.

(16) qatorning tekis yaqinlashuvchi ekanligini ko‘rsatish maqsadida ushbu qatoni baholaymiz:

$$\left|u(x,t)\right|\le\sum_{n=0}^{\infty}=\left|X_n(x)\right|\left|T_n(t)\right|=\left|x^{\frac{1-\beta}{2}}J_{\upsilon}\left(\theta_nx^{\frac{2-\beta}{2}}\right)\right|\left|\frac{\left(\varphi_{2n}-\varphi_{1n}\right)}{E_{\alpha,1}\left(-\lambda_nT^{\alpha}\right)}\left(E_{\alpha,1}\left(-\lambda_nt^{\alpha}\right)-1\right)+\varphi_{1n}\right|.$$

(20)

Bessel-Klifford funksiyasidan foydalanib,

$$X_n(x)=C_3(\sqrt{\lambda_n})^{\frac{1-\beta}{2-\beta}}x^{1-\beta}\bar{J}_{\nu}\left(\frac{2\sqrt{\lambda_n}}{2-\beta}x^{\frac{2-\beta}{2}}\right) \qquad (21)$$

ko‘rinishda yozib olish mumkin. Bu yerda

$$\bar{J}_{\upsilon}\left(\theta_nx^{\frac{2-\beta}{2}}\right)=\left(\frac{\theta_n}{2}\right)^{-\nu}\Gamma(\nu+1)x^{\frac{\beta-1}{2}}J_{\nu}\left(\theta_nx^{\frac{2-\beta}{2}}\right)$$

$\bar{J}_{\upsilon}\left(\theta_nx^{\frac{2-\beta}{2}}\right)\le1$ ekanligini inobatga olsak, $X_n(x)=(\sqrt{\lambda_n})^{\frac{1-\beta}{2-\beta}}$ hosil bo‘ladi.

$T_n(t)$ qatorni $\left|E_{\alpha,1}\left(-\lambda_n t^\alpha\right)\right|<1$ekanligini inobatga olsak, uni quyidagicha yozishimiz mumkin:

$$\left|T_n(t)\right| \le \frac{\left|\varphi_{1n}\right|+\left|\varphi_{2n}\right|}{\left|E_{\alpha,1}\left(-\lambda_n T^\alpha\right)\right|}C+\left|\varphi_{1n}\right| \le \frac{\left|\varphi_{2n}\right|}{\left|\delta_n\right|}+\left|\varphi_{1n}\right|\left(1+\frac{C}{\delta_n}\right) \le \left|\varphi_{1n}\right|+\left|\varphi_{2n}\right|$$

demak, yechimni

$$\left|u(x,t)\right| \le C_4\sum_{n=0}^{\infty}\left[\left|\varphi_{1n}\right|+\left|\varphi_{2n}\right|\right]\left|\sqrt{\lambda_n}\right|^{\frac{1-\beta}{2-\beta}} = C_4\sum_{n=0}^{\infty}\left|\varphi_{1n}\right|\left|\sqrt{\lambda_n}\right|^{\frac{1-\beta}{2-\beta}}+C_4\sum_{n=0}^{\infty}\left|\varphi_{2n}\right|\left|\sqrt{\lambda_n}\right|^{\frac{1-\beta}{2-\beta}} \quad (22)$$

ko‘rinishida yozish mumkin.

(22) tengsizlikning har ikkala qo‘shiluvchisiga Koshi-Bunyakovskiy tengsizligini qo‘llab,

$$\left|u(x,t)\right| \le C_4\left(\sum_{n=0}^{+\infty}\lambda_n\varphi_{1n}^2\sum_{n=0}^{+\infty}\lambda_n^{(-2)/(2-\beta)}\right)^{\frac{1}{2}}+C_4\left(\sum_{n=0}^{+\infty}\lambda_n\varphi_{2n}^2\sum_{n=0}^{+\infty}\lambda_n^{(-2)/(2-\beta)}\right)^{\frac{1}{2}} \quad (23)$$

tengsizlikni hosil qilamiz.

Endi

$$\varphi_{1n}=\frac{2-\beta}{\bar{J}_{\nu+1}^2(\theta_n)}\int_0^1\varphi(x)X_n(x)dx \quad (24)$$

tenglikni bo‘laklab integrallab,

$$\sqrt{\lambda_n}\varphi_{1n}=\frac{2-\beta}{\bar{J}_{\nu+1}^2(\theta_n)}\int_0^1\varphi'(x)x^{\frac{\beta}{2}}dx$$

tenglikka ega bo‘lamiz. $\varphi'(x)\in L_2(0,1)$ ekanligidan oxirgi ifoda Furye koeffitsienti bo‘ladi, Bessel tengsizligiga ko‘ra:

$$\sum_{n=0}^{+\infty}\left|\lambda_n\varphi_{1n}^2\right| \le C_4\int_0^1\left(\varphi'(x)\right)^2x^\beta dx \le M \quad (25)$$

ifoda kelib chiqadi, bu yerda $C_4=\left(\frac{\beta-2}{J_{\nu+1}^2(\theta_n)}\right)^2$.

(25) ga ko‘ra (23) tengsizlikdagi birinchi qator yaqinlashuvchi, $\beta\in(0,1)$ bo‘lgani uchun $\sum_{n=0}^{+\infty}\lambda_n^{(-2)/(2-\beta)}$ qator umumlashgan garmonik qator hisoblanadi. Undan ma’lumki, bu qator ham yaqinlashuvchi. Xuddi shu ishlarni (23) tengsizlikning ikkinchi qo‘shiluvchi uchun ham bajarsak, (23) qator tekis yaqinlashuvchi ekanligi kelib chiqadi.

Endi $\left[x^\beta u_x(x,t)\right]_x$ funksiya mos qatorni yaqinlashuvchiligini ko‘rsatish maqsadida

$$\left[x^\beta u_x(x,t)\right]_x=\sum_{n=0}^{\infty}\left[x^\beta X_n'(x)\right]'\left[\frac{\left(\varphi_{2n}-\varphi_{1n}\right)}{E_{\alpha,1}\left(-\lambda_n T^\alpha\right)}\left(E_{\alpha,1}\left(-\lambda_n t^\alpha\right)-1\right)+\varphi_{1n}\right]$$

tenglikni qaraymiz. (5) tenglamadan foydalanib,

$$\left[x^{\beta}u_x(x,t)\right]_x = \sum_{n=0}^{\infty} -\lambda_n X_n(x)\left[\frac{(\varphi_{2n}-\varphi_{1n})}{E_{\alpha,1}(-\lambda_n T^{\alpha})}\left(E_{\alpha,1}(-\lambda_n t^{\alpha})-1\right)+\varphi_{1n}\right]$$

(26)

tenglikni hosil qilamiz.

Yuqoridagi tengsizliklarga ko‘ra

$$\left|\left[x^{\beta}u_x(x,t)\right]_x\right| \le C_4\sum_{n=0}^{\infty}\left|\lambda_n\varphi_{1n}\right|\left|\sqrt{\lambda_n}^{(1-\beta)/(2-\beta)}\right| + C_4\sum_{n=0}^{\infty}\left|\lambda_n\varphi_{2n}\right|\left|\sqrt{\lambda_n}^{(1-\beta)/(2-\beta)}\right|$$

(27)

tengsizlikka ega bo‘lamiz.

Oxiridagi tengsizlikning birinchi qo’shiluvchisiga Koshi-Bunyakovskiy tengsizligini qo’llab,

$$\left|\left[x^{\beta}u_x(x,t)\right]_x\right| \le C_4\left(\sum_{n=0}^{\infty}\left|\lambda_n^3\varphi_{1n}^2\right|\sum_{n=0}^{\infty}\left|\sqrt{\lambda_n}^{(-2)(2-\beta)}\right|\right)^{\frac{1}{2}}$$

(28)

tengsizlikni hosil qilamiz. (24) tengsizlikni integrallab,

$$\lambda_n\sqrt{\lambda_n}\varphi_{1n} = \frac{\beta-2}{J_{\nu+1}^2(\theta_n)}\int_0^1 x^{\frac{\beta}{2}}\left[\varphi_1'(x)x^{\beta}\right]'' dx$$

tenglikka kelamiz. $\left[\varphi'(x)x^{\beta}\right]'' \in L_2(0,1)$ ekanligidan yuqoridagi tenglik Furye koeffitsienti bo’ladi.

Bessel tengsizligiga ko‘ra,

$$\lambda_n^3\varphi_{1n}^2 \le C_5\int_0^1\left(\left[\tau'(x)x^{\beta}\right]''\right)^2 x^{\beta}dx \le M$$

(29)

ifodaga kelamiz, bu yerda $C_5 = \left(\frac{\beta-2}{J_{\nu+1}^2(\theta_n)}\right)^2$.

(29) ga ko’ra (27) tengsizlikdagi birinchi qator yaqinlashuvchi, $\beta\in(0,1)$ ekanligidan $\sum_{n=0}^{+\infty}\lambda_n^{(-2)/(2-\beta)}$ qator umumlashgan garmonik qator ekanligi kelib chiqadi. Bundan ma’lumki ikkinchi qator ham yaqinlashuvchi. Xuddi shu ishlarni (27) tengsizlikdagi ikkinchi qo‘shiluvchi uchun ham bajaramiz. Bunga ko’ra (27) qator tekis yaqinlashuvchi.

YECHIMNING YAGONALIGI

Faraz qilaylik, $u_1(x,t)$ va $u_2(x,t)$ yechimlarga ega bo‘lsin. Undan $u(x,t)=u_1(x,t)-u_2(x,t)$ funksiya Ω sohada (1) tenglamaga mos bir jinsli tenglamani, uning chegarasida esa $u(0,t)=0$, $u(1,t)=0$, $u(x,0)=\varphi_1(x)$, $u(x,T)=\varphi_2(x)$ tengliklarni qanoatlantiradi.

Quyidagi funksiyani ko‘raylik,

$$u_n(t)=\int_0^1 u(x,t)X_n(x)dx. \quad (30)$$

Bundan foydalanib,

$$D_{0t}^{\alpha}u_n(t)=\int_0^1 D_{0t}^{\alpha}u(x,t)\,X_n(x)dx \qquad (31)$$

tenglikni yozishimiz mumkin.

(1) ga asosan (31) tenglikni

$$D_{0t}^{\alpha}u_n(t)=\int_0^1\left\{\left[x^{\beta}u_x(x,t)\right]_x+f(x)\right\}X_n(x)dx \qquad (32)$$

ko‘rinishida yozamiz. Ma’lum bir soddalashtirishlarni bajarib,

$$D_{0t}^{\alpha}u_n(t)=\int_0^1\left[x^{\beta}u_x(x,t)\right]_x X_n(x)dx+f_n \qquad (33)$$

tenglikka ega bo‘lamiz. Bu tenglikning birinchi qismini ikki marta integrallab,

$$D_{0t}^{\alpha}u_n(t)=\int_0^1\left[x^{\beta}X_n'(x)\right]_x u(x,t)dx+f_n$$

ifodani hosil qilamiz. (2) va (14) lardan foydalansak, $f_n\equiv 0$, $u_n(t)\equiv 0$ kelib chiqadi.

$\{X_n(x)\}$ sistema to‘la bo‘lganligi uchun, $u(x,t)\equiv 0$, $f(x)\equiv 0$ bo‘ladi. Bundan $u_1(x,t)=u_2(x,t)$ va $f_1(x)=f_2(x)$ kelib chiqadi. Bundan esa A masalaning yagona yechimga ega ekanligi kelib chiqadi.

FOYDALANILGAN ADABIYOTLAR:

1. Терсенов С.А.О задаче Коши с данными на линии вырождения типа для гиперболического уровнения // Диффер. Уравн., 1966. Т.2, №1. С 125-130.

2.Терсенов С.А К теории гиперболических уровнений с данными на линии вырождения типа //Сиб. Матем. Журн., 1961. Т.2, № 6 С. 913-935.

3. Терсенов С. А. Введение в теорию уровнений, вырождающихся на границе. Новосибрск: : НГУ, 1937. 144 с.

4.Смирнов М.М. Вырождающиеся гиперболические уравнения. Минск: Высш.шк., 1977. 157 с.

5.Хайруллин Р. С. Задача Трикоми для уравнения второго рода с сильным вырождением. Казань: Казанск. унив., 2015. 336 с. END: UWLDMB.

6.Мамадалиев Н. К.О представлении решения видоизмененной задачи Коши // Сиб. матем. журн., 2000. Т. 41, №5. С. 1087-1097.

7. Уринов А. К., Окбоев А. Б. Видоизмененная задача Коши для одного вырождающе гося гиперболического уравнения второго рода // Укр. матем. журн., 2020.Т. 72, № 1 С. 100-118 11

8. Urinov A. K. Okboev A. B. On a Cauchy type problem for a second kind degenerating hyperbolic equation // Lobachevskit J. Math., 2022. vol. 43, no. 3. pp. 793-803. EDN: QPEVQB. DOI: https://doi.org/10.1134/S199508022206032.

9. Уринов А. К., Усмонов Д. А. О видоизменной задаче Коши для одного вырождаю щегося гиперболического уравнения второго рода // Бюл. Инст . mam ., 2021. Т.4, № 1. С. 46-63

10. Ватсон Г.Н., Теория бесселевых функий.Часть первая. -М.: Изд-во ИЛ,1949,789 с.

11. Usmonov D. A. Problem with shift condition for a second kind degenerated equation of hyperbolic type // Scientific Jurnal of the Fergana State University. 2020. № 6. pp.6-10.

12. D.A. Usmonov. A Cauchy-Goursat problem for a second kind degenerated equation of hyperbolic type // Scientific Bulletin. Physical and mathematical Research Vol.3 Iss.1 2021. pp. 76-83

13. O‘rinov A.Q. Usmonov D.A. A Cauchy-Goursat problem for a second kind degenerated equation of hyperbolic type // Scientific Jurnal of the Fergana State University.2021/ №5. pp. 6-16

14. O‘rinov A.Q. Usmonov D.A. A mixed problem for Secondary of the hyperbolic type with two fault lines the type equation // Scientific Bullettin of NamSU. 2022 / №5. Pp. 119-129

15. Уринов А. К., Усмонов Д. А. Initial boundary value problem for a hyperbolic equation with three lines of degeneracy of the second kind // Scientific Bulletin. Physical and mathematical Research Vol.4 Iss.1 2022. Pp. 56-59