# APPLICATION OF A LAGRANGE FUNCTION

**Teshaeva Madina**
*3rd year student of the Finnish-Uzbek Pedagogical Institute*
**Kuyanbaeva Karina**
*1st year student of Samarkand State University named after Sharof Rashidov*

**Abstract;** *It is easy to find the extrema of functions of many variables. This thesis explores the properties of the largest surface and volume forms that are useful in optimization problems using the conditional extremum of multivariate functions.*

**Keywords:** *conditional extremum, Lagrange function, partial derivative, surface, volume.*

It is known that the problem of finding the extrema of functions of one and many variables is relevant in practice. Also, the extremum of a multivariable function found under certain conditions is important in solving problems of optimization and its practical application. Below we will look at the concepts of conditional extremum and the problems solved by this concept.

**Definition.** Suppose that the function $f(x)$ with $n$ variables is defined around a point $a$. If we find that the circumference of point $a$ is such that the inequality $f\ (a)\ \geq f\ (x)\ (f\ (a)\ \leq f\ (x))$ holds for any point $x$ taken from this circumference, then the function $f$ is the *local maximum (local minimum)* at point $a \in \mathbb{R}^n$.

We now study the problem of finding the extreme values of a given function when some additional conditions are met. In this case, the additional conditions are usually given in the form of limiting the values of the variables.

For example, the arguments for the maximum value of the function $f(x, y, z)$ with three variables

$$g(x, y, z) = 0$$

let us consider the problem of finding when an additional condition is satisfied. In this case, the function of three variables $g(x, y, z)$ is a function that is sufficiently differentiable. In this case, the maximum value is called the *conditional maximum*, the minimum value is called the *conditional minimum*. Conditional maximum and conditional minimum together are called *conditional extremum*.

The Lagrange multiplier method is mainly used to find conditional extrema. That is, by selecting such a number $\mu$, the following Lagrange function is created:

$$L(x, y, z, \mu) = f(x, y, z) - \mu g(x, y, z)$$

Then the following system of equations, formed by equalizing all partial derivatives of the Lagrange function to zero, is solved and the number $\mu$ is found:

$$\begin{cases} \dfrac{\partial L(x,y,z,\mu)}{\partial x} = 0 \\ \dfrac{\partial L(x,y,z,\mu)}{\partial y} = 0 \\ \dfrac{\partial L(x,y,z,\mu)}{\partial z} = 0 \end{cases}$$

Putting the found number $\mu$ in the system of equations, we find the relationship between the variables, conditioning the found relations $g(x,y,z) = 0$, which gives the function $f(x,y,z)$ to the conditional extremum $(x,y,z)$ point is found. The value of the found point in the function $f(x,y,z)$ is the conditional extremum of the function.

Here are some problems with conditional extremum.

**Problem 1. The problem of finding a rectangle with the largest surface area.**

Suppose we are given a rectangle with sides $x$ and $y$, the perimeter of which is equal to a certain number $P$. Then find the ratio of $x$ and $y$ to the largest surface of this rectangle.

**Solution.** It is known from the problem condition that we need to find the maximum value of the function of two variables $f(x,y) = xy$ on the basis of the following condition $g(x,y) = P - 2(x+y) = 0$. Let's create a Lagrange function:

$$L(x,y,\mu) = xy - \mu(2x + 2y - P)$$

We create the following system of equations by equaling the partial derivatives of the generated Lagrange function to zero:

$$\begin{cases} \dfrac{\partial L}{\partial x} = y - 2\mu = 0 \\ \dfrac{\partial L}{\partial y} = x - 2\mu = 0 \end{cases}.$$

From this system of equations, we find the equation $\mu = \frac{x}{2} = \frac{y}{2}$, and from it the equation $x = y$. So the square with the largest surface is the quadrate.

**Problem 2. The problem of finding a right prism with the largest volume at the base of a rectangle.**

Let us be given a rectangular prism with base $x, y$, height $h$, and the solid surface equal to define $S_t$. Under what conditions will this prism have the largest volume?

**Solution.** According to the condition of the problem, it is required to find the maximum value of the function $f(x,y,h) = xyh$ on the basis of the condition $g(x,y,h) = 2xy + 2xh + 2yh - S_t = 0$.

We construct the Lagrange function and the system of equations:

$$L(x,y,h,\mu) = xyh - \mu(2xy + 2xh + 2yh - S_t),$$

$$\begin{cases} \dfrac{\partial L}{\partial x} = yh - \mu(2y + 2h) = 0 \\ \dfrac{\partial L}{\partial y} = xh - \mu(2x + 2h) = 0, \\ \dfrac{\partial L}{\partial h} = xy - \mu(2x + 2y) = 0 \end{cases}$$

Using the above system, we find the following equations $\mu = \frac{yh}{2(y+h)} = \frac{xh}{2(x+h)} = \frac{xy}{2(x+y)}$. Based on the resulting equations, we find the relation $x = y = h$. Hence, a straight rectangle with sides $x, y$ and a straight prism of height $h$ when the equation $x = y = h$ is valid, or when the prism is exactly a cube, then reach the largest volume.

**Problem 3. The problem is to find the right cylinder with the largest volume.**

Let us be given a straight cylinder with radius $R$ of base and height $H$, as well as th solid surface equal to $S_t$. Under what conditions will this cylinder have the largest volume?

**Solution.** Based on the condition of the problem, it was required to find the maximum value of the function $f(R,H) = \pi R^2 H$ on the basis of the following condition $g(R,H) = 2\pi R^2 + 2\pi RH - S_t = 0$.

We construct the Lagrange function and the system of equations:

$$L(R,H,\mu) = \pi R^2 H - \mu(2\pi R^2 + 2\pi RH - S_t),$$

$$\begin{cases} \frac{\partial L}{\partial R} = 2\pi RH - \mu(4\pi R + 2\pi H) = 0 \\ \frac{\partial L}{\partial H} = \pi R^2 - \mu 2\pi R = 0 \end{cases},$$

From this system of equations we find the equations $\mu = \frac{R}{2} = \frac{2\pi Rh}{4\pi R + 2\pi H}$. From these equations we find the following relation $H = 2R$. This means that the volume of a straight circular cylinder reaches its maximum when the height of the cylinder is equal to the diameter of the base.

**Problem 4. The problem of finding the largest volume cone.**

Suppose that the generatrix $l$ and the radius are given by $R$, and that the perimeter of the cone (the length of the spread) is given by a certain number $P$. Find the conditions under which this cone will have the largest volume.

**Solution.** Find the maximum value of the function $f(R,l) = \frac{1}{3}\pi R^2\sqrt{l^2 - R^2}$ on the basis of the condition $g(R,l) = 2\pi R + 2l - P = 0$ required.

We construct the Lagrange function and the system of equations:

$$L(R,l,\mu) = \frac{1}{3}\pi R^2\sqrt{l^2 - R^2} - \mu(2\pi R + 2l - P),$$

$$\begin{cases} \frac{\partial L}{\partial R} = \frac{2\pi R}{3}\sqrt{l^2 - R^2} - \frac{\pi R^3}{3} \cdot \frac{1}{\sqrt{l^2 - R^2}} - 2\pi\mu = 0 \\ \frac{\partial L}{\partial l} = \frac{\pi R^2 l}{3\sqrt{l^2 - R^2}} - 2\mu = 0 \end{cases}$$

From this system of equations $\mu = \frac{\pi R^2 l}{6\sqrt{l^2 - R^2}} = \frac{2\pi R(l^2 - R^2) - \pi R^3}{6\pi\sqrt{l^2 - R^2}}$. We find the equations $2l^2 - \pi Rl - 3R^2 = 0$. From the equations found,

$$l = \frac{\pi + \sqrt{\pi^2 + 24}}{2} \cdot R$$

Using the formula $H = \sqrt{l^2 - R^2}$ for the height of the cone, the relationship between the height of the cone and the radius of the base can also be determined. This means that the cone will have the largest volume when the above equation holds.

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