IKKINCHI TARTIBLI XUSUSIY HOSILALI BUZILADIGAN DIFFERENSIAL TENGLAMA UCHUN CHEGARAVIY MASALA

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Annatotsiya: *Ushbu maqolada buziladigan ikkinchi tartibli tenglama uchun boshlang`ich-chegaraviy malasa bayon qilingan va tadqiq etilgan. Malasaning yechimining yagonali energiya integrallari usulidan foydalanib isbotlangan. Malasa yechimining mavjud ekanligi esa o`zgaruvchilarni ajratish usuli yordamida ko`rsatilgan.*

Kalit so‘zlar: *buziladigan differensial tenglamalar, chegaraviy masala, energiya integrallari usuli, o`zgaruvchilarni ajratish usuli.*

Annotation: *In this article, the initial-boundary problem for the degenerative second-order equation is described and researched. The solution of the problem is proved using the method of energy integrals. The existence of a solution to the problem is shown using the method of separation of variables.*

Key words: *Degenerative differential equations, boundary value problem, method of energy integrals, method of separation of variables.*

Аннотация: *В данной статье описана и исследована начально-краевая задача для вырождающегося уравнения второго порядка. Решение задачи доказывается методом интегралов энергии. Существование решения задачи показано с помощью метода разделения переменных.*

Ключевые слова: *Вырождающиеся дифференциальные уравнения, краевая задача, метод интегралов энергии, метод разделения переменных.*

I. Kirish. Masalaning qo`yilishi. O‘tgan asrdan boshlab buziladigan ikkinchi tartibli xususiy hosilali differensial tenglamalar uchun chegaraviy masalalar o‘rganish boshlangan va anchagina boy tarixga ega. Bu tadqiqotlar ko‘plab olib borilishiga qaramasdan so‘ngi yillarda ham giperbolik tipdagi buziladigan ikkinchi tur tenglamalar uchun boshlang‘ich va chegaraviy masalalar o‘rganib kelinmoqda(masalan [1]-[9]).

So‘ngi yillarda esa kasr tartibli to‘lqin tenglamalari uchun chegaraviy masalalarni o‘rganishga bo‘lgan qiziqish ortgan. Shu sababli biz ushbu ishda ikkinchi tartibli xususiy hosilali buziladigan differensial tenglamalar uchun chegaraviy masalani ko‘rib chiqamiz.

$\Omega=\left\{(x,t):0<x<1, 0<t<T\right\}$ sohada

$$ {}_C D_{0t}^{\alpha} u(x,t)=\left[x^{\beta}u_x(x,t)\right]_x \quad (1)$$

tenglamani qaraylik, bu erda ${}_C D_{0t}^{\alpha}$ - Kaputo ma’nosidagi kasr tartibili operator [7]

$$ {}_C D_{0t}^{\alpha} u(x,t)=\frac{1}{\Gamma(2-\alpha)}\int_0^t \frac{u_{zz}(x,z)}{(t-z)^{\alpha-1}}dz ,$$

α,β,T - o‘zgarmas haqiqiy sonlar bo‘lib, $0\le\beta<1$, $1<\alpha<2$, $T>0$.

B masala. Shunday $u(x,t)$ funksiya topilsinki u quydagi xossalarga ega bo‘lsin:

1) $u(x,t)$, $x^{\beta}u_x(x,t)\in C(\overline{\Omega})$, $\left[x^{\beta}u_x(x,t)\right]_x$, ${}_C D_{0t}^{\alpha}u(x,t)\in C(\Omega)$ sinfga tegishli;

2) Ω soha (1) tenglamani qanoatlantiradi.

3) Ω soha chegarasida esa ushbu

$$\lim_{x\to 0} x^{\beta}u_x(x,t)=0,\ u(1,t)=0,\ \ t\in[0,T] \qquad (2)$$

$$u(x,0)=\varphi_1(x),\ u_t(x,0)=\varphi_2(x),\ \ x\in[0,1] \qquad (3)$$

chegaraviy va boshlang’ich shartlarni qanoatlantiradi, bu erda $\varphi_1(x)$, $\varphi_2(x)$ - berilgan funksiyalar.

Endi masalaning yechimini mavjudligini ko‘rsatish maqsadida (1) tenglamaning yechimini

$$u(x,t)=X(x)T(t) \qquad (4)$$

ko‘rinishda qidirib, uni (1) tenglamaga qo‘yib, ba’zi soddalashtirishlarni amalga oshirib,

$$\frac{{}_C D_{0t}^{\alpha}T(t)}{T(t)}=\frac{\left[x^{\beta}X'(x)\right]'}{X(x)} \qquad (5)$$

tenglamani hosil qilamiz.

(5) tenglamani uning o‘ng tomoni t ga, chap tomoni esa x ga bog‘liq bo‘lmagani uchun uni o‘zgarmas $(-\lambda)$ soniga tenglab,

$${}_C D_{0t}^{\alpha}T(t)+\lambda T(t)=0,\ 0<t<T; \qquad (6)$$

$$\left[x^{\beta}X'(x)\right]'+\lambda X(x)=0,\ 0<x<1; \qquad (7)$$

ko‘rinishidagi oddiy differensial tenglamalarni hosil qilamiz. (5) tenglikka asosan (3) shartlardan $X(x)$ funksiya uchun

$$\lim_{x\to 0} x^{\beta}X'(x)=0,\ X(1)=0 \qquad (8)$$

shartlar kelib chiqadi.

$\{(7),(8)\}$ spektral masalaning xos sonlarini topish uchun (7) tenglamani $X(x)$ ga ko‘paytirib $[0,1]$ oraliqda integrallaymiz:

$$\int_0^1\left[x^{\beta}X'(x)\right]_x X(x)dx+\lambda\int_0^1 X^2(x)dx=0. \qquad (9)$$

(9) ni bir marta bo‘laklab integrallab, (8) shartni e’tiborga olsak, u holda

$$\int_0^1 x^{\beta}\left[X'(x)\right]^2dx=\lambda\int_0^1 X^2(x)dx \qquad (10)$$

tenglikni hosil qilamiz, undan esa $\lambda\geq 0$ ekanligi kelib chiqadi.

Dastlab, $\lambda=0$ bo‘lsin (8) ga ko‘ra $\left[x^{\beta}X'(x)\right]'=0$ bo‘lishidan $X(x)=C_1\dfrac{x^{1-\beta}}{1-\beta}+C_2$ kelib chiqadi, uni (8) shartga bo‘ysundirsak, $X(x)\equiv 0$ ekanligi kelib chiqadi. Demak $\lambda=0$ xos son emas.

$\lambda > 0$ bo‘lsin. U holda $X(x)=V(z)$

$$z=\frac{x^{2-\beta}}{(2-\beta)^2} \qquad (11)$$

belgilash kiritib olamiz. (11) dan kerakli hosilalarni olib, (7) ga qo‘llab, ba’zi soddalashtirishlarni amalga oshirib,

$$zV''(z)+\frac{V'(z)}{2-\beta}+\lambda V(z)=0 \qquad (12)$$

tenglamani hosil qilamiz.

(12) tenglama Bessel tenglamasiga keltrilgan tenglama bo‘lib, uning yechimi

$$V(z)=\left(2\sqrt{z}\right)^{\frac{1-\beta}{2-\beta}}\left[C_1J_\nu\left(2\sqrt{z\lambda}\right)+C_2J_{-\nu}\left(2\sqrt{z\lambda}\right)\right]$$

ko‘rinishda bo‘ladi, bu yerda C_1, C_2 va $\nu-$o‘zgarmas sonlar, $\nu=(1-\beta)/(2-\beta)$ $J_\nu(z)-$birinchi tur Bessel funksiyasi.

(11) belgilashga ko‘ra (8) tenglamaning umumiy echimi

$$X(x)=\left(\frac{2}{2-\beta}\right)^{\frac{1-\beta}{2-\beta}}x^{\frac{1-\beta}{2}}\left[C_1J_\nu\left(2\sqrt{\lambda}\frac{x^{\frac{2-\beta}{2}}}{2-\beta}\right)+C_2J_{-\nu}\left(2\sqrt{\lambda}\frac{x^{\frac{2-\beta}{2}}}{2-\beta}\right)\right] \qquad (13)$$

Ko‘rinishida bo‘ladi, (13) dan hosila olib x^β ga ko‘paytirib, (8) chegaraviy shartlarni birinchisiga bo‘ysindirib, $C_1=0$ ekanligini topamiz.

$X(1)=0$ dan esa $C_2J_{-\nu}\left(\frac{2\sqrt{\lambda}}{2-\beta}\right)=0$ tenglamani hosil qilamiz. $C_2\neq 0$ deb, $J_{-\nu}\left(\frac{2\sqrt{\lambda}}{2-\beta}\right)=0$ tenglamani yechamiz. $\nu>0$ bo‘lganligi uchun $J_{-\nu}\left(\frac{2\sqrt{\lambda}}{2-\beta}\right)=0$ tenglama absalyut qiymati bo‘yicha cheksiz kattalashib boruvchi sanoqli sondagi haqiqiy yechimlarga ega [8]. Uning $n-$musbat yechimini θ_n bilan belgilasak $\{(7),(8)\}$ masalaning sanoqli sondagi

$$\lambda_n=\left[\frac{2-\beta}{2}\theta_n\right]^2,\ n=1,2,...$$

xos qiymatlariga ega bo‘lamiz, unga mos keluvchi xos funksiyalar esa

$$X_n(x)=x^{\frac{1-\beta}{2}}J_{-\nu}\left[\theta_n x^{\frac{2-\beta}{2}}\right],\ n=1,2,... \qquad (14)$$

ko‘rinishida bo‘ladi.

1-lemma. *(14) formula bilan aniqlanuvchi $X_n(x)$, $n\in N$ funksiyalar $(0,1)$ kesmada ortonormal va to‘la sistema tashkil qiladi.* [10]

Yuqoridagilarga asosan B masalaning formal yechimi

$$u(x,t)=\sum_{n=1}^{+\infty} x^{\frac{1-\beta}{2}} J_{-\nu}\left[\theta_n x^{\frac{2-\beta}{2}}\right]\left[\varphi_{1n}E_{\alpha,1}\left(-\lambda_n t^{\alpha}\right)+\varphi_{2n}E_{\alpha,2}\left(-\lambda_n t^{\alpha}\right)\right] \qquad (15)$$

ko'rinishda bo'ladi, bu erda $\nu=(1-\beta)/(2-\beta)$, $\varphi_{1n}=\frac{1}{\mu_n}\int_0^1\varphi_1(x)X_n(x)dx$,

$\varphi_{2n}=\frac{1}{\mu_n}\int_0^1\varphi_2(x)X_n(x)dx$, $\mu_n=\int\limits_0^1 X_n^2(x)dx=\left[1/(2-\beta)\right]J_{-\nu+1}^2(\theta_n)$, $\theta_n=\frac{2}{2-\beta}\sqrt{\lambda_n}$,

$E_{\alpha,\beta}(z)=\sum\limits_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)}$, $(z,\beta\in\square;\Re(\alpha)>0)$ - Mittag-Leffler funksiyasi.

1-Teorema. *Agar* $\varphi_j(x)$, $x^{\beta}\varphi_j'(x)$, $\left[x^{\beta}\varphi_j'(x)\right]'\in C(0,1)$, $x^{\frac{\beta}{2}}\left[x^{\beta}\varphi_j'(x)\right]''\in C(0,1)\cap L_2(0,1)$, $\varphi_j(1)=0$, $\lim\limits_{x\to0}x^{\beta}\varphi_j{}'(x)=0$ *va* $\lim\limits_{x\to0}\left[\varphi_j'(x)x^{\beta}\right]'=0$, $\lim\limits_{x\to1}\left[\varphi_j'(x)x^{\beta}\right]'=0$ *bo'lsa, u holda B masalaniung yechimi mavjud va yagona bo'lib, u (15) qator bilan aniqlanadi.*

Isbot. Teoremani isbotlash uchun (15) va $x^{\beta}u_x(x,t)$, $\left[x^{\beta}u_x(x,t)\right]_x$, ${}_CD_{0t}^{\alpha}u(x,t)$ ga mos keluvchi qatorlarni tekis yaqinlashuvchi ekanligini isboltash yetarli.

Dastlab (15) qatorning tekis yaqinlashishini ko'rsatish maqsadida qatorni baholaymiz:

$$|u(x,t)|\le\sum_{n=1}^{\infty}|X_n(x)||T_n(t)|=$$

$$\sum_{n=1}^{\infty}\left|x^{\frac{1-\beta}{2}}J_{\nu}\left(\theta_n x^{\frac{2-\beta}{2}}\right)\right|\left|\varphi_{1n}E_{\alpha}\left(-\lambda_n t^{\alpha}\right)+\varphi_{2n}E_{\alpha,2}\left(-\lambda_n t^{\alpha}\right)\right| \qquad (16)$$

$X_n(x)$ ni Bessel-Klifford funksiyasidan foydalanib,

$$X_n(x)=C_3\left(\sqrt{\lambda_n}\right)^{-\frac{1-\beta}{2-\beta}}\bar{J}_{-\nu}\left(\frac{2\sqrt{\lambda_n}}{2-\beta}x^{\frac{2-\beta}{2}}\right) \qquad (17)$$

ko'rinishda yozib olamiz, bu yerda

$$\bar{J}_{-\nu}\left(\theta_n x^{\frac{2-\beta}{2}}\right)=\left(\frac{\theta_n}{2}\right)^{\nu}\Gamma(-\nu+1)x^{\frac{\beta-1}{2}}J_{-\nu}\left(\theta_n x^{\frac{2-\beta}{2}}\right)$$

$\left|\bar{J}_{-\nu}(z)\right|\le1$ va $\left|E_{\alpha,1}(-z)\right|,\left|E_{\alpha,2}(-z)\right|<C$ tengsizliklardan foydalanib,

$$|u(x,t)|\le C_4\sum_{n=1}^{+\infty}\left|\sqrt{\lambda_n}\varphi_{1n}\right|\left|\sqrt{\lambda_n}\right|^{-(3-2\beta)/(2-\beta)}+C_5\sum_{n=1}^{+\infty}\left|\sqrt{\lambda_n}\varphi_{2n}\right|\left|\sqrt{\lambda_n}\right|^{-(3-2\beta)/(2-\beta)}$$

tengsizlikni hosil qilamiz.

Koshi-Bunyakoviskiy tengsizligini qo'llab,

$$\left|u(x,t)\right| \le C_4\left(\sum_{n=1}^{+\infty}\lambda_n\varphi_{1n}^2\sum_{n=1}^{+\infty}\lambda_n^{(-2)/(2-\beta)}\right)^{1/2} + C_5\left(\sum_{n=1}^{+\infty}\lambda_n\varphi_{2n}^2\sum_{n=1}^{+\infty}\lambda_n^{(-2)/(2-\beta)}\right)^{1/2} \qquad (18)$$

tengsizlikni hosil qilamiz.

Endi ushbu tenglikni qaraymiz:

$$\varphi_{jn} = \frac{2-\beta}{\overline{J}_{-v+1}^2\left(\theta_n\right)}\int_0^1 \varphi_j(x)X_n(x)dx\,. \qquad (19)$$

(7) tenglamadan foydalanib, (19) dan

$$\sqrt{\lambda_n}\varphi_{jn} = \frac{\beta-2}{\overline{J}_{-v+1}^2\left(\theta_n\right)}\int_0^1 \varphi_j(x)\frac{\left[x^\beta X_n'(x)\right]}{\sqrt{\lambda_n}}dx \qquad (20)$$

tenglikni hosil qilamiz. (20) dagi integralni bo‘laklab integrallab

$$\sqrt{\lambda_n}\varphi_{jn} = \frac{2-\beta}{\overline{J}_{-v+1}^2\left(\theta_n\right)}\left(\left.\frac{\varphi_j(x)X'(x)x^\beta}{\sqrt{\lambda_n}}\right|_0^1 - \int_0^1\varphi_j'(x)x^{\frac{\beta}{2}}\left(\frac{X_n'(x)x^{\frac{\beta}{2}}}{\sqrt{\lambda_n}}\right)dx\right)$$

tenglikni hosil qilamiz, bu yerda $\varphi_j(1)=0$ bo‘lganligi uchun

$$\sqrt{\lambda_n}\varphi_{jn} = \frac{\beta-2}{\overline{J}_{-v+1}^2\left(\theta_n\right)}\int_0^1\varphi_j'(x)x^{\frac{\beta}{2}}\left(\frac{X_n'(x)x^{\frac{\beta}{2}}}{\sqrt{\lambda_n}}\right)dx \qquad (21)$$

ekanli kelib chiqadi oxirigi ifoda $\sqrt{\lambda_n}\varphi_{jn}$ ni $\varphi_j(x)x^{\frac{\beta}{2}}$ funksiyaning $\left\{\frac{x^{\frac{\beta}{2}}X_n'(x)}{\sqrt{\lambda_n}}\right\}$ sistema bo‘yicha Furye koeffitsenti bo‘ladi, u holda Bessel tensizligiga ko‘ra

$$\sum_{n=0}^{\infty}\left|\lambda_n\varphi_{jn}^{\ 2}\right| \le C_6\int_0^1\left(\varphi_j'(x)\right)^2x^\beta dx \le M \qquad (22)$$

tengsizlik o‘rinli bo‘ladi, bu erda $C_6 = \left(\frac{\beta-2}{\overline{J}_{-v+1}^2\left(\theta_n\right)}\right)^2$.

(22) ga ko‘ra (18) tengsizlikdagi $\sum_{n=1}^{+\infty}\lambda_n\varphi_{jn}^2$, $j=\overline{1,2}$ qatorlar yaqinlashuvchi bo‘ladi, $\beta\in(0,1)$ bo‘lgani uchun $\sum_{n=1}^{\infty}\lambda_n^{-\frac{3-2\beta}{2-\beta}}$ qator umumlashgan garmonik bo‘lib u yaqinlashuvchi bo‘ladi.

Bularga ko‘ra (18) yoki (15) qator tekis yaqinlashuvchi ekanligi kelib chiqadi.

Endi $\left[x^\beta u_x(x,t)\right]_x$ funksiya mos qatorni tekis yaqinlashuvchi ekanligini ko‘rsatish maqsadida

$$\left[x^{\beta}u_x(x,t)\right]_x=\sum_{n=1}^{\infty}\left[x^{\beta}X_n'(x)\right]'\left[\varphi_{1n}E_{\alpha}\left(-\lambda_n t^{a}\right)+\varphi_{2n}tE_{\alpha,2}\left(-\lambda_n t^{\alpha}\right)\right]\quad(23)$$

tenglikni qaraymiz. Bu tenglikda (7) tenglamadan foydalanib,

$$\left[x^{\beta}u_x(x,t)\right]_x=\sum_{n=1}^{\infty}(-\lambda_n)X_n(x)\left[\varphi_{1n}E_{\alpha}\left(-\lambda_n t^{a}\right)+\varphi_{2n}tE_{\alpha,2}\left(-\lambda_n t^{\alpha}\right)\right]\qquad(24)$$

tenglikni hosil qilamiz.

Yuqoridagi tengsizliklarga asoslanib,

$$\left|\left[x^{\beta}u_x(x,t)\right]_x\right|\le C_4\sum_{n=1}^{+\infty}\left|\lambda_n\varphi_{1n}\right|\left|\sqrt{\lambda_n}^{\,-(1-\beta)/(2-\beta)}\right|+C_5\sum_{n=1}^{+\infty}\left|\lambda_n\varphi_{2n}\right|\left|\sqrt{\lambda_n}^{\,-(1-\beta)/(2-\beta)}\right|$$

tengsizlikni hosil qilamiz.

Oxirigi tengsizlikka Koshi-Bunyakoviskiy tengsizligini qo‘lab,

$$\left|x^{\beta}u_x(x,t)\right|\le C_4\left(\sum_{n=1}^{+\infty}\left|\lambda_n^3\varphi_{1n}^2\right|\sum_{n=1}^{+\infty}\left|\lambda_n^{\frac{-(3-2\beta)}{(2-\beta)}}\right|\right)^{1/2}+C_5\left(\sum_{n=1}^{+\infty}\left|\lambda_n^3\varphi_{2n}^2\right|\sum_{n=1}^{+\infty}\left|\lambda_n^{\frac{-(3-2\beta)}{(2-\beta)}}\right|\right)^{1/2}\qquad(25)$$

tengsizlikni hosil qilamiz.

Endi (7) dan foydalanib, (19) dan

$$\lambda_n\sqrt{\lambda_n}\varphi_{jn}=\frac{\beta-2}{\bar{J}_{-v+1}^{2}(\theta_n)}\sqrt{\lambda_n}\int_0^1\varphi_j(x)\left[x^{\beta}X_n'(x)\right]'dx\qquad(26)$$

tenglikni hosil qilamiz, (26) dagi integralni ikki martta bo‘laklab integrallab

$$\lambda_n\sqrt{\lambda_n}\varphi_{jn}=\frac{\beta-2}{\bar{J}_{-v+1}^{2}(\theta_n)}\sqrt{\lambda_n}\left[\varphi_j(x)x^{\beta}X_n'(x)\Big|_{x=0}^{x=1}-\varphi_j'(x)x^{\beta}X_n'(x)\Big|_{x=0}^{x=1}+\int_0^1\left[x^{\beta}\varphi_n'(x)\right]'X_n'(x)dx\right]$$

tenglikni hosil qilamiz, bu yerda $\varphi_j(1)=0$ va $\lim\limits_{x\to 0}x^{\beta}\varphi_j'(x)=0$ bo‘lganligi uchun

$$\lambda_n\sqrt{\lambda_n}\varphi_{jn}=\frac{\beta-2}{\bar{J}_{-v+1}^{2}(\theta_n)}\sqrt{\lambda_n}\int_0^1\left[x^{\beta}\varphi_n'(x)\right]'X_n'(x)dx\qquad(27)$$

(7) dan foydalanib (27) ni

$$\lambda_n\sqrt{\lambda_n}\varphi_{jn}=\frac{2-\beta}{\bar{J}_{-v+1}^{2}(\theta_n)}\int_0^1\left[x^{\beta}\varphi_n'(x)\right]'\frac{\left[x^{\beta}X_n'(x)\right]'}{\sqrt{\lambda_n}}dx\qquad(28)$$

ko‘rinishida yozib olamiz, (28) dagi integralni bo‘laklab integrallab

$$\lambda_n\sqrt{\lambda_n}\varphi_{jn}=\frac{2-\beta}{\bar{J}_{-v+1}^{2}(\theta_n)}\left[\left[x^{\beta}\varphi_j'(x)\right]'\frac{\left[x^{\beta}X_n'(x)\right]}{\sqrt{\lambda_n}}\Bigg|_{x=0}^{x=1}-\int_0^1\left[x^{\beta}\varphi_n'(x)\right]''x^{\frac{\beta}{2}}\frac{x^{\frac{\beta}{2}}X_n'(x)}{\sqrt{\lambda_n}}dx\right]$$

tenglikni hosil qilamiz. Bu yerda $\lim\limits_{x\to 1}\left[x^{\beta}\varphi_j'(x)\right]'=0$, $x^{\frac{\beta}{2}}\left[x^{\beta}\varphi_j'(x)\right]''\in L_2(0,1)$ bo‘lganligi uchun

$$\lambda_n\sqrt{\lambda_n}\varphi_{jn}=\frac{\beta-2}{\overline{J}^2_{-\nu+1}(\theta_n)}\int_0^1\left[x^\beta\varphi_n'(x)\right]''x^{\frac{\beta}{2}}\frac{x^{\frac{\beta}{2}}X_n'(x)}{\sqrt{\lambda_n}}dx$$

ekanligi kelib chiqadi oxirgi ifoda $\lambda_n\sqrt{\lambda_n}\varphi_{jn}$ ni $\left[x^\beta\varphi_n'(x)\right]''x^{\frac{\beta}{2}}$ funksiyaning $\left\{\frac{x^{\frac{\beta}{2}}X_n'(x)}{\sqrt{\lambda_n}}\right\}$ sistema bo‘yicha Furye koeffitsenti bo‘ladi, u holda Bessel tensizligiga ko‘ra

$$\sum_{n=1}^{\infty}\lambda_n^3\varphi_{jn}^2\le C_6\int_0^1\left(\left[\varphi_j'(x)x^\beta\right]''\right)^2x^\beta dx\le M \qquad (29)$$

ifodani hosil qilamiz, bu erda $C_6=\left(\frac{\beta-2}{J^2_{-\nu+1}(\theta_n)}\right)^2$

(29) ga ko‘ra (25) tengsizlikdagi birinchi qator yaqinlashuvchi, $\beta\in(0,1)$ bo‘lgani uchun $\sum_{n=1}^{\infty}\lambda_n^{-\frac{3-2\beta}{2-\beta}}$ qator umumlashgan garmonik qator bo‘lib u yaqinlashuvchi bo‘ladi.

Bularga ko‘ra (25) yoki (23) qator tekis yaqinlashuvchi bo‘lishi kelib chiqadi.

Faraz qilaylk, $V_1(x,t)$ va $V_2(x,t)$ yechimlarga ega bo‘lsin. Undan $V(x,t)=V_1(x,t)-V_2(x,t)$ funksiya Ω sohada (1) tenglamaga mos bir jinsli tenglamani, uning chegarasida esa $V(x,0)=0$, $\lim\limits_{x\to0}x^\beta V_x(x,t)=0$, $V(1,t)=0$, $V(x,0)=0$tengliklarni qanoatlantiradi.

Quyidagi funksiyani qaraylik:

$$v_n(t)=\int_0^1V(x,t)X_n(x)dx \qquad (30)$$

(30) dan foydalanib

$$_CD_{0t}^\alpha v_n(t)=\int_0^1{}_CD_{0t}^\alpha V(x,t)X_n(x)dx \qquad (31)$$

(31) ni topamiz.

(1) tenglamadan foydalanib (31) ni

$$_CD_{0t}^\alpha v_n(t)=\int_0^1\left[x^\beta V_x(x,t)\right]_xX_n(x)dx \qquad (33)$$

ko‘rinishiga yozamiz. (33) ni ikki martta bo‘laklab integrallab (2) va (8) dan foydalanib,

$$_CD_{0t}^\alpha v_n(t)=\int_0^1\left[x^\beta X_n'(x)\right]'V(x,t)dx \qquad (34)$$

(30), (34) va $V(x,0)=0$, $V_t(x,1)=0$ shartlardan foydalanib,

$$\begin{aligned}&_CD_{0t}^\alpha v_n(t)+\lambda_nv_n(t)=0\\&v_n(0)=0,\ v_n'(0)=0\end{aligned} \qquad (35)$$

masalani hosil qilamiz. (35) masalaning yechimi $v_n(t)\equiv0$ ekanligini ko‘rish qiyin emas.

(30) ga ko‘ra

$$\int_0^1 V(x,t) X_n(x) dx = 0$$

bo‘ladi. $\{X_n(x)\}$ sistema to‘la bo‘lgani uchun $V(x,t) \equiv 0$ bo‘ladi. Budan esa $V_1(x,t) = V_2(x,t)$ kelib chiqadi. Bundan esa B masalaning yechimi yagona ekanligi kelib chiqadi.

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