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FUNKSIYALARNI DARAJALI QATORGA YOYISHNING BIR USULI

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Annotatsiya: *Ushbu tezisda funksiyalarni darajali qatoriga yoyishning yangicha usuli keltirilgan bo`lib, bu usul funksional qatorlarni differensiallab rekurrent formulalarga keltirish orqali amalga oshiriladi.*

Kalit so`zlar: *darajali qatori, differensiallash, rekurrent formula, analitik funksiya.*

Hayotiy jarayonlarni matematik modelini tuzish davomida ma`lum funksiyalarni darajali qatorga yoyish va bu orqali funksiya ustida bajarilishi kerak bo`lgan amallarni uning darajali qatori orqali bajarish muhim ahamiyatga ega. Shu bilan birga berilgan funksiyani uning hosilalarini ketma-ket olish yordamida darajali qatorga yoyish bir muncha murakkab va noqulay hisoblanadi. Tezisda darajali qatorlarni hadlab differensiyallash va integrallash haqidagi teoremalarga asosan ayrim sinfdagi funksiyalarning darajali qatorini topish usuli keltirilgan. Ushbu usul amaliyotda qo`llash uchun qulay va tushunarlidir.

1-misol: $f(x) = e^{x^2}$ funksiyani $x = 0$ nuqta atrofida Darajali qatoriga yoying.

Yechimi: $f(x) = e^{x^2}$ funksiya $x = 0$ nuqta atrofida cheksiz marta differensiallanuvchi. Demak, $f(x) = e^{x^2}$ funksiyani $x = 0$ nuqta atrofida darajali qatorga yoyish mumkin. Faraz qilaylik, ushbu funksiya darajali qatoriga quyidagicha ko'rinishda yoyilgan bo'lsin:

$$e^{x^2} = \sum_{n=0}^{\infty} \frac{a_n x^n}{n!} \qquad (1)$$

Bunda a_n –darajali qator koeffitsiyentlari va $a_0 = f(0) = 1, \quad a_1 = f'(0) = 0.$

Ushbu funksional qatorni $x = 0$ nuqta atrofida hadlab differensiallash mumkin:

$$2xe^{x^2} = \left(e^{x^2}\right)' = \frac{d}{dx}\sum_{n=0}^{\infty} \frac{a_n x^n}{n!} = \sum_{n=1}^{\infty} \frac{a_n x^{n-1}}{(n-1)!} \qquad (2)$$

Agar (2) tenglikdagi e^{x^2} had o'rniga uning (1) ko`rinishini olib borib qo'ysak,

$$\sum_{n=0}^{\infty}\frac{2a_nx^{n+1}}{n!}=\sum_{n=0}^{\infty}\frac{a_{n+1}x^n}{n!}=\sum_{n=-1}^{\infty}\frac{a_{n+2}x^{n+1}}{(n+1)!}=a_1+\sum_{n=0}^{\infty}\frac{a_{n+2}x^{n+1}}{(n+1)!}$$
$$=\sum_{n=0}^{\infty}\frac{a_{n+2}x^{n+1}}{(n+1)!},$$

tenglik o'rinli bo'ladi. Bu ikki darajali qator teng bo'lishi uchun x ning mos darajalari oldidagi koeffitsiyentlari teng bo`lishi kerak. Demak,

$$\frac{2a_nx^{n+1}}{n!}=\frac{a_{n+2}x^{n+1}}{(n+1)!},$$
$$a_n=2(n-1)a_{n-2}.$$

tenglik kelib chiqadi.

Hosil bo`lgan rekurrent ketma-ketlikni soddalashtiramiz:

$a_0=1,\ a_1=0$ tengliklar bizga ma`lum, u holda

$a_2=2(2-1)a_0=2,\qquad a_3=3(3-1)a_1=0,\qquad a_4=4(4-1)a_0=12,$ $a_5=5(5-1)a_1=0,\ldots$

Demak, ketma-ketlikning toq nomerli hadlari 0 ga teng. Bundan esa rekurrent ketma-ketlikning umumiy hadi quyidagi ko`rinishda bo`lishini topamiz:

$$a_n=2^{\frac{n}{2}}(n-1)(n-3)\ldots 1$$

Bundan foydalanib berilgan funksiyani darajali qator ko`rinishda yozishimiz mumkin:

$$e^{x^2}=\sum_{n=0}^{\infty}\frac{2^{\frac{n}{2}}(n-1)(n-3)\ldots 1}{n!}x^n.$$

2-misol: $f(x)=\sin(\mu\arcsin x)$ funksiyani $x=0$ nuqta atrofida darajali qatoriga yoying.

Yechimi: Faraz qilaylik $f(x)$ funksiyani darajali qatoriga yoyilmasi quyidagi ko'rinishda bo'lsin:

$$\sin(\mu\arcsin x)=\sum_{n=0}^{\infty}\frac{a_nx^n}{n!}\qquad(3)$$

$\{a_n\}$ biror sonli ketma-ketlik. Biz shu $\{a_n\}$ ketma-ketlikni umumiy ko'rinishini topish masalasini qaraymiz. Agar (3) tenglik o'rinli bo'lsa quyidagilar ham o'rinli bo'ladi:

$$f'(x)=\sum_{n=1}^{\infty}\frac{a_nx^{n-1}}{(n-1)!},\qquad f''(x)=\sum_{n=2}^{\infty}\frac{a_nx^{n-2}}{(n-2)!}\qquad(4)$$

Shuningdek,

$f'(x)=\frac{\mu}{\sqrt{1-x^2}}\cos(\mu\arcsin x),\qquad f''(x)=\frac{\mu x}{(1-x^2)\sqrt{1-x^2}}\cos(\mu\arcsin x)-\frac{\mu^2}{1-x^2}\sin(\mu\arcsin x).$

Demak, $x=0$ nuqtaning yetarlicha kichik atrofida

$$f''(x)=\frac{x}{1-x^2}f'(x)-\frac{\mu^2}{1-x^2}f(x).$$

tenglik o'rinli bo'ladi.

ushbu hosil qilingan tenglikka (4) tenglikdagi ifodalarni olib kelib qo`ysak, quyidagi tenglikka ega bo'lamiz:

$$\frac{1}{1-x^2}\sum_{n=1}^{\infty}\frac{a_n x^{n-1}}{(n-1)!}-\mu^2\sum_{n=0}^{\infty}\frac{a_n x^n}{(n)!}=\sum_{n=2}^{\infty}\frac{a_n x^{n-2}}{(n-2)!},$$

$$\sum_{n=1}^{\infty}\frac{a_n x^n}{(n-1)!}-\sum_{n=0}^{\infty}\frac{\mu^2 a_n x^n}{n!}=\sum_{n=2}^{\infty}\frac{a_n x^{n-2}}{(n-2)!}-\sum_{n=2}^{\infty}\frac{a_n x^n}{(n-2)!}.$$

Demak,

$$\sum_{n=0}^{\infty}\frac{a_{n+1}x^{n+1}}{(n)!}-\sum_{n=0}^{\infty}\frac{\mu^2 a_n x^n}{n!}=\sum_{n=0}^{\infty}\frac{\mu^2 a_{n+2}x^{n+2}}{n!}-\sum_{n=0}^{\infty}\frac{\mu^2 a_{n+2}x^n}{n!},$$

$$\sum_{n=0}^{\infty}x^n\left(\frac{a_n}{(n-1)!}-\frac{\mu^2 a_n}{n!}\right)=\sum_{n=0}^{\infty}\left(\frac{a_{n+2}}{(n)!}-\frac{a_n}{(n-2)!}\right)x^n.$$

Ikki darajali qator teng bo'lishi uchun ularning mos koeffitsiyentlari teng bo'lishi kerak:

$$\frac{a_n}{(n-1)!}-\frac{\mu^2 a_n}{n!}=\frac{a_{n+2}}{(n)!}-\frac{a_n}{(n-2)!},$$

$$a_{n+2}=a_n(n)+a_n(n)(n-1)-\mu^2 a_n,$$

$$a_{n+2}=a_n(n^2-\mu^2),$$

ekanligi kelib chiqadi. Bizga $a_0=0\ va\ a_1=\mu$ koeffitsiyentlar ma`lum. Demak,

$$a_2=(0-\mu^2)a_0=0,\quad a_3=(1-\mu^2)a_1=1-\mu^2,\ \ a_4=(4-\mu^2)a_2=0,\ldots$$

Bundan esa hosil qilingan rekurrent ketma-ketlikning juft nomerli hadlari nolga teng ekanligi kelib chiqadi. Ya'ni rekurrent ketma-ketligimizning umumiy hadi quyidagi ko`rinishda bo`ladi:

$$a_{2k+1}=((2k-1)^2-\mu^2)\ldots(1-\mu^2)\mu.$$

Demak, berilgan funksiyaning darajali qatori quyidagichi bo`ladi:

$$\sin(\mu\arcsin x)=\mu x+\sum_{k=1}^{\infty}\frac{((2k-1)^2-\mu^2)((2k-3)^2-\mu^2)\ldots(1-\mu^2)\mu}{(2k+1)!}x^{2k+1}.$$

3-misol: $f(x)=e^x\sin x$ funksiyani darajali qatoriga yoying.

Yechimi: Faraz qilaylik, $f(x)$ funksiyaning darajali qatorga yoyilmasi quyidagi ko'rinishda bo'lsin:

$$f(x)=e^x\sin x=\sum_{n=0}^{\infty}\frac{a_n x^n}{n!}\qquad(5)$$

Bunda a_n- koeffitsiyentlar va $a_0=f(0)=0,\ a_1=f'(0)=1.$

Ushbu funksional qatorni $x=0$ nuqta atrofida ikki marta hadlab differensiallab quyidagilarni hosil qilamiz:

$$e^x \sin x + e^x \cos x = (e^x \sin x)' = \frac{d}{dx}\sum_{n=0}^{\infty}\frac{a_n x^n}{n!}$$

$$= \sum_{n=1}^{\infty}\frac{a_n x^{n-1}}{(n-1)!} \qquad (6)$$

$$2e^x \cos x = (e^x \sin x)'' = \frac{d^2}{dx^2}\sum_{n=0}^{\infty}\frac{a_n x^n}{n!}$$

$$= \sum_{n=2}^{\infty}\frac{a_n x^n}{(n-2)!} \qquad (7)$$

Berilgan $f(x) = e^x \sin x$ funksiyadan I va II tartibli hosilalarini olib, quyidagi tenglikni hosil qilish mumkin:

$$f'(x) = f(x) + \frac{1}{2}f''(x) \qquad (8)$$

Endi (8) tenglikka (6) va (7) tengliklardagi ifodalarni qo’ysak, quyidagi tenglikka ega bo`lamiz:

$$\sum_{n=1}^{\infty}\frac{a_n x^{n-1}}{(n-1)!} = \sum_{n=0}^{\infty}\frac{a_n x^n}{n!} + \frac{1}{2}\sum_{n=2}^{\infty}\frac{a_n x^{n-2}}{(n-2)!},$$

$$\sum_{n=1}^{\infty}\frac{a_{n+1} x^n}{n!} = \sum_{n=0}^{\infty}\frac{a_n x^n}{n!} + \frac{1}{2}\sum_{n=2}^{\infty}\frac{a_{n+2} x^n}{n!}.$$

Bu ikki darajali qator teng bo’lishi uchun x ning mos darajalari oldidagi koeffitsiyentlari teng bo`lishi kerak. Demak,

$$\frac{a_{n+1}}{n!} = \frac{a_n}{n!} + \frac{1}{2}\frac{a_{n+2}}{n!} \Rightarrow a_n = 2a_{n-1} - 2a_{n-2} \qquad (9)$$

Rekurrent ketma-ketlikni topish formulasiga asosan (9) tenglikni quyidagicha ifodalash mumkin:

$$y^2 = 2y - 2 \qquad (10)$$

(10) tenglik ildizlari $y_1 = 1 + i,\ \ y_2 = 1 - i$. Ushbu ildizlardan foydalanib quyidagi tenglikni hosil qilamiz:

$$a_n = C_1(y_1)^n + C_2(y_2)^n \Rightarrow a_n = C_1(1+i)^n + C_2(1-i)^n \qquad (11)$$

Bizda $a_0 = f(0) = 0,\ a_1 = f'(0) = 1$ ma’lum. Bulardan foydalanib (11) tenglikdagi C_1 va C_2 topamiz:

$$C_1 = \frac{1}{2i}, \qquad C_2 = -\frac{1}{2i}.$$

Endi C_1 va C_2 larning qiymatlarini (11) tenglikka qo`yamiz va soddalashtiramiz:

$$a_n = \frac{1}{2i}(1+i)^n - \frac{1}{2i}(1-i)^n,$$

$$a_n = \frac{(\sqrt{2})^n}{2i}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^n - \frac{(\sqrt{2})^n}{2i}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)^n,$$

$$a_n = \frac{\left(\sqrt{2}\right)^n}{2i}\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4} - \cos\frac{7n\pi}{4} - i\sin\frac{7n\pi}{4}\right) \Rightarrow a_n = 2^{\frac{n}{2}}\sin\frac{n\pi}{4}.$$

Demak, berilgan funksiyaning darajali qatori quyidagi ko`rinishda ekan:

$$e^x \sin x = \sum_{n=0}^{\infty} \frac{2^{\frac{n}{2}}\sin\frac{n\pi}{4}}{n!} x^n.$$

4-misol. $f(x) = \frac{1}{x^2+x+1}$ funksiyani darajali qatorga yoying.

Yechimi: Faraz qilaylik $f(x)$ funksiyaning darajali qatorga yoyilmasi quyidagi ko'rinishda bo'lsin:

$$f(x) = \frac{1}{x^2+x+1} = \sum_{n=0}^{\infty} \frac{a_n x^n}{n!} \qquad (12)$$

Bunda $a_n -$ koeffitsiyentlar va $a_0 = f(0) = 1, \quad a_1 = f'(0) = -1.$

Ushbu funksional qatorni $x = 0$ nuqta atrofida hadlab differensiallash mumkin:

$$-\frac{2x+1}{(x^2+x+1)^2} = \left(\frac{1}{x^2+x+1}\right)' = \frac{d}{dx}\sum_{n=0}^{\infty}\frac{a_n x^n}{n!} = \sum_{n=1}^{\infty}\frac{a_n x^{n-1}}{(n-1)!} \qquad (13)$$

(13) ifodani soddalashtirsak,

$$-\frac{2x+1}{x^2+x+1} = (x^2+x+1)\sum_{n=1}^{\infty}\frac{a_n x^{n-1}}{(n-1)!} \qquad (14)$$

Agar (14) tenglikdagi $\frac{1}{x^2+x+1}$ had o'rniga uning (12) ko`rinishini olib borib qo'ysak,

$$-(2x+1)\sum_{n=0}^{\infty}\frac{a_n x^n}{n!} = (x^2+x+1)\sum_{n=1}^{\infty}\frac{a_n x^{n-1}}{(n-1)!},$$

$$\sum_{n=0}^{\infty} -\frac{2a_n x^{n+1}}{n!} - \frac{a_n x^n}{n!} = \sum_{n=1}^{\infty}\frac{a_n x^{n+1}}{(n-1)!} + \frac{a_n x^n}{(n-1)!} + \frac{a_n x^{n-1}}{(n-1)!},$$

$$\sum_{n=0}^{\infty}\left(-\frac{2a_n}{n!} - \frac{a_n}{(n+1)!}\right)x^{n+1} = \sum_{n=1}^{\infty}\left(\frac{a_n}{(n-1)!} + \frac{a_{n+1}}{n!} + \frac{a_{n+2}}{(n+1)!}\right)x^{n+1}.$$

tenglik o'rinli bo'ladi. Bu ikki darajali qator teng bo'lishi uchun x ning mos darajalari oldidagi koeffitsiyentlari teng bo`lishi kerak. Demak,

$$-\frac{2a_n}{n!} - \frac{a_n}{(n+1)!} = \frac{a_n}{(n-1)!} + \frac{a_{n+1}}{n!} + \frac{a_{n+2}}{(n+1)!},$$

$$-2a_n(n+1) - a_{n+1} = a_n(n^2+n) + a_{n+1}(n+1) + a_{n+2},$$

$$a_{n+2} = -(n+2)\big(a_{n+1} + a_n(n+1)\big).$$

Hosil bo`lgan rekurrent ketma-ketlikni soddalashtiramiz:

$a_0 = 1,\ \ a_1 = -1,\ \ a_2 = 2(-a_1 - a_0) = 0,\ \ a_3 = 3(-a_2 - 2a_1) = 6,$

$a_4 = 4(-a_3 - 3a_2) = -24, \qquad a_5 = 5(-a_4 - 4a_3) = 0, \ldots$

Demak, $\{a_n\}$ had ko'rinishi quyidagicha ko'rinishda:

$$a_n = \begin{cases} 0, & agar\ \ n = 3k - 1, \quad k \in N \\ n!\,, & agar\ n = 3k, \qquad\quad k \in N, \\ -n!\,, & agar\ n = 3k + 1, k \in N \end{cases}$$

yoki

$$a_n = \frac{2}{\sqrt{3}}(n!)\sin\frac{2\pi(n+1)}{3}.$$

Berilgan funksiyaning darajali qatorga yoyilmasi quyidagicha bo`ladi:

$$\frac{1}{x^2 + x + 1} = \sum_{0}^{\infty} \frac{2}{\sqrt{3}} \sin\frac{2\pi(n+1)}{3} x^n.$$

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