UCHINCHI TARTIBLI PARABOLO-GIPERBOLIK TENGLAMA UCHUN BIR NOLOKAL MASALA

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Annotatsiya: *Ushbu maqolada uchinchi tartibli parabolo-giperbolik tenglama uchun bir nolokal masala bayon qilingan va qo'yilgan masalaning bir qiymatli yechilishi o'rganilgan.*

Kalit so'zlar: *parabolo-giperbolik tenglama, nolokal masala, integral tenglama, Volterra tenglamasi.*

$D = D_1 \cup D_0 \cup D_2$ sohada ushbu

$$\frac{\partial}{\partial x}\left[u_{xx} - \frac{1}{2}(1 - signy)u_{yy} - \frac{1}{2}(1 - signy)u_y\right] = 0 \qquad (1)$$

uchinchi tartibli tenglamani qaraylik, bu yerda $D_1 = \{(x,y): 0 < x < 1, 0 < y \le 1\}$, $I = \{(x,0): 0 < x < 1\}$, $D_2 = \{(x,y): 0 < x < 1, (-y) < x < y+1\}$.

Dastlab, (1) tenglamani D_1 sohada qaraylik. Bu sohada $y > 0$ va $signy = 1$ bo'lib, tenglama parabolik tipga tegishli va

$$\frac{\partial}{\partial x}\left[u_{xx} - u_y\right] = 0 \qquad (2)$$

ko'rinishga ega bo'ladi. (2) tenglamani x bo'yicha integrallab,

$$u_{xx} - u_y = \omega_1(y),\ (x,y) \in D_1 \qquad (3)$$

ko'rinishda yozish mumkin bo'ladi, bu yerda $\omega_1(y)$ – noma'lum funksiya.

D_2 sohada $y < 0$ va $signy = -1$ bo'lib, (1) tenglama giperbolik tipga tegishli hamda

$$\frac{\partial}{\partial x}\left[u_{xx} - u_{yy}\right] = 0 \qquad (4)$$

ko'rinishga ega, bu tenglamani integrallab

$$u_{xx} - u_{yy} = \omega_2(y),\ (x,y) \in D_2 \qquad (5)$$

tenglamaga ega bo'lamiz, bu yerda $\omega_2(y)$ – noma'lum funksiya.

D sohada (1) tenglama uchun quyidagi masalani qaraylik.

1-masala. Shunday $u(x,y) \in C(\overline{D}) \cap C^{3,1}_{x,y}(D_1) \cap C^{3,2}_{x,y}(D_2)$ funksiya topilsinki, u D_1 va D_2 sohalarda (1) tenglamani hamda ushbu

$$u(0,y) = \varphi_1(y),\ u(1,y) = \alpha(y)u(0,y) + \varphi_2(y),\ 0 \le y \le 1; \qquad (6)$$

$$u_x(x,y)\big|_{x=0} = \varphi_3(y), \qquad 0 \le y \le 1; \qquad (7)$$

$$a(x)\frac{d}{dx}u\left(\frac{x}{2}, -\frac{x}{2}\right) + b(x)\frac{d}{dx}u\left(\frac{x-1}{2}, \frac{x-1}{2}\right) = c(x),\ 0 \le x \le 1; \qquad (8)$$

$$\left.\frac{\partial u}{\partial n}\right|_{y=-x}=\psi(x),\qquad 0\le x\le\frac{1}{2};\qquad (9)$$

$$u_y(x,+0)=u_y(x,-0),\qquad 0<x<1\qquad (10)$$

shartlarni qanoatlantirsin, bu yerda $a(x),b(x),c(x),\psi(x),\ \alpha(y),\varphi_1(y),\varphi_2(y),\varphi_3(y)$ - berilgan uzluksiz funksiyalar, n - $y=-x$ to‘g’ri chiziqqa o‘tkazilgan ichki normal.

Masalani tadqiq qilish uchun (10) shartga asosan quyidagi belgilashlarni kiritaylik:

$$u(x,0)=\tau(x),0\le x\le 1;\ u_y(x,0)=\nu(x),\ 0<x<1;$$

$$\tau(x)\in C[0,1]\cap C^2(0,1),\nu(x)\in C(0,1)\cap L(0,1).$$

Qo‘yilgan masalani D_2 sohada qaraylik. Yuqoridagi belgilashlarni e’tiborga olib, D_2 sohada (5) tenglama uchun Koshi masalasining yechimi

$$u(x,y)=\frac{1}{2}\left[\tau(x+y)+\tau(x-y)\right]+\frac{1}{2}\int\limits_{x-y}^{x+y}\nu(t)dt+\frac{1}{2}\int\limits_0^y\int\limits_{x-(y-t)}^{x+(y-t)}\omega_2(t)d\xi dt\qquad (11)$$

formula orqali ifodalanadi. (11) formulani (9) shartga bo‘ysundirsak,

$$\omega_2(y)=-\sqrt{2}\psi'(-y)\qquad (12)$$

ega bo‘lamiz. (12) ni (11) formulaga qo‘yib va hosil bo‘lgan $u(x,y)$ funksiyani (8) shartga bo`ysundirib, $\tau(x)$ va $\nu(x)$ noma’lum funksiyalar orasidagi

$$\left[a(x)+b(x)\right]\tau'(x)+\left[b(x)-a(x)\right]\nu(x)=2c(x)+$$

$$+a(x)\int\limits_0^{x/2}\omega_2(t)dt-b(x)\int\limits_0^{(x-1)/2}\omega_2(t)dt,\qquad 0<x<1\qquad (13)$$

munosabatni topamiz.

(12) tenglikni (13) munosabatga qo‘ysak

$$\left[a(x)+b(x)\right]\tau'(x)+\left[b(x)-a(x)\right]\nu(x)=$$

$$=2c(x)+a(x)\int\limits_0^{x/2}\left(-\sqrt{2}\psi'(-t)\right)dt-b(x)\int\limits_0^{(x-1)/2}\left(-\sqrt{2}\psi'(-t)\right)dt,\ 0<x<1\qquad (14)$$

hosil bo‘ladi. So‘nggi tenglikdagi oxirgi ikki integralda $-t=z$ almashtirish va hisoblashlarni bajarsak:

$$\int\limits_0^{x/2}\psi'(-t)dt=-\int\limits_0^{x/2}\psi'(-t)d(-t)=\{-t=z\}=$$

$$-\int\limits_0^{x/2}\psi'(z)d(z)=-\int\limits_0^{x/2}d(\psi(z))=-\psi\left(\frac{x}{2}\right)+\psi(0),$$

$$\int\limits_0^{x-1/2}\psi'(-y)dt=-\int\limits_0^{x-1/2}\psi'(-y)d(-y)=\{-y=z\}=$$

$$-\int_0^{x/2}\psi'(z)d(z)=-\int_0^{x/2}d(\psi(z))=-\psi\left(\frac{x-1}{2}\right)+\psi(0)$$

kelib chiqadi. Yuqoridagi hisoblashlarni e'tiborga olib, (14) munosabatdan

$$\left[a(x)+b(x)\right]\tau'(x)+\left[b(x)-a(x)\right]\nu(x)=f(x) \qquad (15)$$

tenglikka ega bo'lamiz, bu yerda

$$f(x)=2c(x)-\sqrt{2}\left(a(x)\left(-\psi\left(\frac{x}{2}\right)+\psi(0)\right)-b(x)\left(-\psi\left(\frac{x-1}{2}\right)+\psi(0)\right)\right).$$

D_1 sohada (3) tenglama va (6),(7) shartlarda $y\to+0$ da limitga o'tib,

$$\tau''(x)-\nu(x)=\omega_1(0),\ 0<x<1 \qquad (16)$$

$$\tau(0)=\varphi_1(0),\ \tau(1)=\alpha(0)\varphi_1(0)+\varphi_2(0),\ \ \tau'(0)=\varphi_3(0) \qquad (17)$$

munosabatlarga ega bo'lamiz.

Endi $\{(15),(16),(17)\}$ masalani qaraylik. Bu masalani tadqiq qilishda $a(x)+b(x)\equiv 0$, $a(x)=-b(x)\neq 0$ bo'lsin. U holda (15) tenglikdan

$$\nu(x)=\frac{f(x)}{2b(x)}$$

ekanligini topamiz. Buni e'tiborga olsak, (16) tenglama

$$\tau''(x)=\frac{f(x)}{2b(x)}+\omega_1(0)$$

ko'rinishda bo'ladi. Hosil bo'lgan tenglamani ikki marta x bo'yicha integrallab (17) shartlarning birinchi va uchunchi shartlarini e'tiborga olsak,

$$\tau(x)=\frac{1}{2}\int_0^x(x-z)\frac{f(z)}{b(z)}dz+\omega_1(0)\frac{x^2}{2}+\varphi_3(0)x+\varphi_1(0) \qquad (18)$$

ekanligini topamiz. Oxirgi tenglikni (17) ning ikkinchi shartiga bo'ysindirib,

$$\omega_1(0)=2\alpha(0)\varphi_1(0)+2\varphi_2(0)-2\varphi_1(0)-2\varphi_3(0)-\int_0^1(1-z)\frac{f(z)}{b(z)}dz$$

ega bo'lamiz. Buni e'tiborga olib, $\{(16),(17)\}$ masala yechimini berilganlar orqali

$$\tau(x)=\varphi_3(0)x+\varphi_1(0)+\frac{1}{2}\int_0^x(x-z)\frac{f(z)}{b(z)}dz+$$

$$+\left(2\alpha(0)\varphi_1(0)+2\varphi_2(0)-2\varphi_1(0)-2\varphi_3(0)-\int_0^1(1-z)\frac{f(z)}{b(z)}dz\right)\frac{x^2}{2}, \qquad (19)$$

$$\nu(x)=\frac{f(x)}{2b(x)} \qquad (20)$$

ko‘rinishda yoziladi. Demak, $\tau(x)$ funksiya (19), $\nu(x)$ funksiya esa (20) ko‘rinishda topilgandan so‘ng o‘rganilayotgan 1-masalaning yechimi D_2 sohada (11) formula bilan, D_1 sohada esa (5) tenglama uchun quyidagi 1'-masala yechimi ko‘rinishda yoziladi:

$$u_{xx}-u_y=\omega_1(y),\quad (x,y)\in D_1; \qquad (21)$$

$$u(x,0)=\tau(x),\ 0\le x\le 1;\ u(0,y)=\varphi_1(y),\ u(1,y)=\varphi_2(y),\ 0\le y\le 1; \qquad (22)$$

$$u_x(x,y)\big|_{x=0}=\varphi_3(y),\quad 0\le y\le 1. \qquad (23)$$

Agar $\omega_1(y)$ funksiyani vaqtincha ma’lum deb hisoblasak, (21) tenglamaning (22) shartlarni qanoatlantiruvchi yechimi $[10]$

$$u(x,y)=\int_0^y\varphi_1(\eta)G_{1\xi}(x,y;0,\eta)d\eta-\int_0^y\varphi_2(\eta)G_{1\xi}(x,y;1,\eta)d\eta+$$

$$+\int_0^1\tau(\xi)G_1(x,y;\xi,0)d\xi-\int_0^y\omega_1(\eta)d\eta\int_0^1 G_1(x,y;\xi,\eta)d\xi,\ (x,y)\in D_1 \qquad (24)$$

ko‘rinishida ifodalash mumkin, bu yerda $G(x,y;\xi,\eta)$ – Grin funksiyasi,

$$G_j(x,y;\xi,\eta)= \qquad (25)$$

$$=\frac{1}{2\sqrt{\pi(y-\eta)}}\sum_{m=-\infty}^{+\infty}\left\{\exp\left[-\frac{(x-\xi+2m)^2}{4(y-\eta)}\right]+(-1)^j\exp\left[-\frac{(x+\xi+2m)^2}{4(y-\eta)}\right]\right\},$$

$j=\overline{1,2}$.

Endi (24) ni (23) shartga bo‘ysindirish maqsadida x bo‘yicha differensiallasak,

$$u_x=\int_0^y\varphi_1(\eta)G_{1\xi x}(x,y;0,\eta)d\eta-\int_0^y\varphi_2(\eta)G_{1\xi x}(x,y;1,\eta)d\eta+$$

$$+\int_0^1\tau(\xi)G_{1x}(x,y;\xi,0)d\xi-\int_0^y\omega_1(\eta)d\eta\int_0^1 G_{1x}(x,y;\xi,\eta)d\xi$$

hosil bo‘ladi. Oxirgi tenglikni $u_x(0,y)=\varphi_3(y)$ shartga bo‘ysindirib, ba‘zi amallarni bajarsak, $\omega_1(y)$ funksiyaga nisbatan ushbu ko‘rinishdagi

$$\int_0^y\omega_1(\eta)M(y,\eta)d\eta=g_3(y),\qquad 0\le y\le 1, \qquad (26)$$

birinchi tur Volterra integral tenglamasiga ega bo‘lamiz, bu yerda

$$g_3(y)=\int_0^y\varphi_1(\eta)d\eta\int_0^1 G_{1\xi x}(x,y;0,\eta)dx-\int_0^y\varphi_2(\eta)d\eta\int_0^1 G_{1\xi x}(x,y;1,\eta)dx+$$

$$+\int_0^1\tau(\xi)d\xi\int_0^1 G_{1x}(x,y;\xi,0)dx-\varphi_3(y), \qquad (27)$$

$$M(y,\eta)=\int_0^1\int_0^1 G_{1x}(x,y;\xi,\eta)\,d\xi dx.$$

Ma’lumki $[10]$,

$$\lim_{\eta\to y}\int_0^1 G_{1x}(x,y;\xi,\eta)\,d\xi=1. \qquad (28)$$

(28) tenglikni e’tiborga olib, (26) differensial tenglamani y bo‘yicha differensiallab, ushbu

$$\omega_1(y)+\int_0^y \omega_1(\eta)(\partial/\partial y)M(y,\eta)\,d\eta=g_3'(y),\quad 0<y<1 \qquad (29)$$

ikkinchi tur Volterra integral tenglamasiga ega bo‘lamiz.

(25) va $(\partial/\partial y)G_1=\left(\partial^2/\partial y^2\right)G_1$, $(\partial/\partial x)G_1=-(\partial/\partial\xi)G_2$ munosabatlarni e’tiborga olib

$$\frac{\partial}{\partial y}\int_0^1\int_0^1 G_1(x,y;\xi,\eta)\,d\xi dx=-2\left[\pi(y-\eta)\right]^{-1/2}-2M_0(y,\eta),$$

$$M_0(y,\eta)=\frac{2}{\sqrt{\pi(y-\eta)}}\sum_{m=1}^{+\infty}\left\{\exp\left[-\frac{m^2}{y-\eta}\right]-\exp\left[-\frac{(2m-1)^2}{4(y-\eta)}\right]\right\}$$

ekanligini topamiz, bu erda $M_0(y,\eta)$ funksiya $\{(y,\eta):0\le\eta<y<1\}$ da uzliksiz differentsiallanuvchi hamda $\eta\to y$ da o'zi va uning hosilasi nolga intiladi. Agar buni hisobga olsak, u holda $(\partial/\partial y)M(y,\eta)$ funksiya, ya’ni (29) integral tenglamaning yadrosi $1/2$ tartibli maxsuslikka ega.

Endi (29) tenglamaning o'ng tomoni $g_3'(y)$ funksiyani tadqiq qilamiz. (25) va $(\partial/\partial\xi)G_1=-(\partial/\partial x)G_2$ munosabatlardan foydalanib,

$$\int_0^1 G_{1\xi}(x,y;0,\eta)\,dx=-\int_0^1 G_{1\xi}(x,y;1,\eta)\,dx=\left[\pi(y-\eta)\right]^{-1/2}+M_0(y,\eta)$$

ekanligini topamiz. Endi (27) tenglikning o‘ng tomonidagi birinchi va ikkinchi hadlar yig'indisini $g_4(y)$ bilan belgilaylik. So‘ngra, oxirgi tenglikni inobatga olib $g_4(y)$ funksiyani

$$g_4(y)=\int_0^y\left[\varphi_1(\eta)+\varphi_2(\eta)\right]\left\{\left[\pi(y-\eta)\right]^{-1/2}+M_0(y,\eta)\right\}d\eta$$

ko‘rinishda yozish mumkin. Bu tenglikni y bo‘yicha differensiallab, so‘ngra bo‘laklab integrallasak,

$$g_4'(y)=\frac{1}{\sqrt{\pi y}}\left[\varphi_1(0)+\varphi_2(0)\right]+$$

$$+\int_0^y\left[\frac{\partial}{\partial\eta}\{[\varphi_1(\eta)+\varphi_2(\eta)]\}\frac{1}{\sqrt{\pi(y-\eta)}}+[\varphi_1(\eta)+\varphi_2(\eta)]\frac{\partial}{\partial y}M_0(y,\eta)\right]d\eta \qquad (30)$$

Tenglikka ega bo'lamiz. (27) tenglikning o'ng tomonidagi uchinchi qo'shiluvchini $g_5(y)$ bilan belgilab, so'ngra (25) va

$$(\partial/\partial y)G_1=(\partial^2/\partial x^2)G_1,\ (\partial/\partial x)G_1=-(\partial/\partial\xi)G_2$$

munosabatlarni e'tiborga olsak,

$$\frac{\partial}{\partial y}\int_0^1 G_1(x,y;\xi,0)dx=\frac{\partial^2}{\partial x^2}\int_0^1 G_1(x,y;\xi,0)dx=\frac{\partial}{\partial\xi}[G_2(0,y;\xi,0)-G_2(1,y;\xi,0)].$$

kelib chiqadi. Oxirgi tenglikka asosan $g_5'(y)$ni

$$g_5'(y)=\int_0^1\tau(\xi)\frac{\partial}{\partial\xi}[G_2(1,y;\xi,0)-G_2(0,y;\xi,0)]d\xi.$$

ko'rinishda yozish mumkin. So'ngra oxirgi tenglikni bo'laklab integrallab va ushbu

$$G_2(1,y;1,0)-G_2(0,y;1,0)=G_2(0,y;0,0)-G_2(1,y;0,0)=\frac{1}{\sqrt{\pi y}}+M_0(y,0)$$

tenglikni e'tiborga olsak

$$g_5'(y)=[\tau(1)+\tau(0)]\left[(\pi y)^{-1/2}+M_0(y,0)\right]+$$

$$\int_0^1\tau'(\xi)[G_2(1,y;\xi,0)-G_2(0,y;\xi,0)]d\xi \qquad (31)$$

kelib chiqadi. (41) va (42) tengliklarni $g_3'(y)=g_4'(y)+g_5'(y)+\varphi_3'(y)$ ga olib borib qo'yib, $\tau(\xi),\ \varphi_j(y)\in C^1[0,1]$, $j=\overline{1,3}$; $\tau(0)=\varphi_1(0)$, $\tau(1)=\varphi_2(0)$ ekanligini, shuningdek, $M_0(y,\eta)$ va $G_2(x,y;\xi,\eta)$ funksiyalarning xossalarini e'tiborga olib, $g_5'(y)\in C[0,1]$ degan xulosaga kelamiz. $(\partial/\partial y)M(y,\eta)$ va $g_3'(y)$funksiyalarining xossalariga va ikkinchi tur Volterra integral tenglamalar nazariyasiga ko'ra, (29) integral tenglama yagona yechimga ega. $\omega_1(y)$ funksiya (29) tenglamadan topilgandan so'ng 1'-masala yechimi $u(x,y)$ funksiyani (24) formula bilan yozish mumkin bo'ladi. Qo'yilgan 1-masala to'liq tadqiq qilindi.

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