IKKINCHI TARTIBLI INTEGRO-DIFFERENSIAL TENGLAMA UCHUN NOLOKAL SHARTLI TESKARI MASALA

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Annotatsiya: *Ushbu ishda ikkinchi tartibli integro-differensial tenglama uchun bir nolokal teskari masala bayon qilingan va tadqiq etilgan.*

Kalit so'zlar: *ikkinchi tartibli integro-differensial tenglama, Riman-Liuvill ma'nosidagi kasr tartibli integral, teskari masala.*

НЕЛОКАЛЬНАЯ УСЛОВНАЯ ОБРАТНАЯ ЗАДАЧА ДЛЯ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ВТОРОГО ПОРЯДКА

Аннотация: *В этой работе была сформулирована и исследована задача об одной нелокальной обратной задаче для Интегро-дифференциального уравнения второго порядка.*

Ключевые слова: *Интегро-дифференциальное уравнение второго порядка, Интеграл Римана-Лювилля в смысле дроби, обратная задача.*

A NON-LOCAL CONDITIONAL INVERSE PROBLEM FOR A SECOND-ORDER INTEGRO-DIFFERENTIAL EQUATION

Annotation: *In this paper, the problem of a non-local inverse problem for a second-order Integro-differential equation was formulated and investigated.*

Keywords: *Integro-differential equation of the second order, Riemann-Liouville integral in the sense of (fractions), inverse problem.*

I Kirish. So'ngi vaqtlarda noma'lum manbali differensial tengalamalar bilan shug'illanishga bo'lgan qiziqish ortib bormoqda. Bunga sabab ko'plab issiqlik taqalish va diffuziya jarayonlarini matematik modelini tuzish noma'lum manbali differensial tenglama uchun qo`yiladigan masalalarga keltiriladi. Bunday differensial tenglamalar uchun teskari masalalar ko'plab tadqiqotchilar tomonidan o'rganilgan (masalan, ushbu $[1]-[11]$ ishlarga qaralsin).

II Masalani qo'yilishi.

(0, 1) oraliqda ushbu

$$y''(x)-\lambda I_{0x}^{\gamma}y(x)=f(x) \qquad (1)$$

ikkinchi tartibli integro-differensial tenglamani qaraylik, bu yerda $y(x)-$ noma'lum funksiya; $f(x)-$ berilgan funksiya; λ,γ - o'zgarmas haqiqiy sonlar bo'lib; $I_{0x}^{\gamma}y(x)$ - Riman-Liuvill ma'nosida γ (kasr) tartibli integral [12]

$$I_{0x}^{\gamma}y(x)=\frac{1}{\Gamma(\gamma)}\int_0^x (x-t)^{\gamma-1}y(t)dt .$$

T-masala. Shunday $y(x)$ funksiya topilsinki, u quyidagi xossalarga ega bo'lsin:

1) (0, 1) oraliqda (1) tenglamani qanoatlantirsin;

2) $C^1[0,1]\cap C^2(0,1)$ sinfga tegishli bo'lsin;

3) $x=0$, $x=1$ nuqtalarda esa

$$py(0)=qy(1),\ qy'(0)=py'(1) \qquad (2)$$

lokal shartlarni qanoatlantisin, bu yerda $p,q-$berilgan o'zgarmas haqiqiy sonlar.

(1)tenglamani

$$y(0)=A_1,\ y'(0)=A_2$$

chegaraviy shartlarni qanoatlantiruvchi yechimini

$$y(x)=A_1E_{\beta,1}(\lambda x^{\beta})+A_2xE_{\beta,2}(\lambda x^{\beta})+\int_0^x (x-z)E_{\beta,4}\left[\lambda(x-z)^{\beta}\right]f(z)dz \qquad (3)$$

ko'rinishda yozib olamiz, [11] ishga qaralsin.

A_1, A_2-noma'lum sonlarni (2) shartdan foydalanib,

$$A_1=\frac{\left[q^2-qpE_{\beta,1}(\lambda)\right]\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^{\beta}\right]f(z)dz+pqE_{\beta,2}(\lambda)\int_0^1 E_{\beta,1}\left[\lambda(1-z)^{\beta}\right]f(z)dz}{qp\left[1+E_{\beta,1}^2(\lambda)-E_{\beta,2}(\lambda)E_{\beta,\beta}(\lambda)\right]+q^2\left[E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)\right]-p^2E_{\beta,1}(\lambda)}$$

$$A_2=\frac{\left[p^2-qpE_{\beta,1}(\lambda)\right]\int_0^1 E_{\beta,1}\left[\lambda(1-z)^{\beta}\right]f(z)dz-\left[q^2-qpE_{\beta,\beta}(\lambda)\right]\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^{\beta}\right]f(z)dz}{qp\left[1+E_{\beta,1}(\lambda)-E_{\beta,2}(\lambda)E_{\beta,\beta}(\lambda)\right]+q^2\left[E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)\right]-p^2E_{\beta,1}(\lambda)}$$

ko'rinishda topamiz. Topilgan A_1, A_2 ni (3) yechimga qo'yib, (1) tenglama yechimini

$$y(x)=E_{\beta,1}(\lambda x^{\beta})\left[\frac{\left[q^2-qpE_{\beta,1}(\lambda)\right]\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^{\beta}\right]f(z)dz+pqE_{\beta,2}(\lambda)\int_0^1 E_{\beta,1}\left[\lambda(1-z)^{\beta}\right]f(z)dz}{qp\left[1+E_{\beta,1}^2(\lambda)-E_{\beta,2}(\lambda)E_{\beta,\beta}(\lambda)\right]+q^2\left[E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)\right]-p^2E_{\beta,1}(\lambda)}\right]+$$

$$+xE_{\beta,2}(\lambda x^{\beta})\left[\frac{\left[p^2-qpE_{\beta,1}(\lambda)\right]\int_0^1 E_{\beta,1}\left[\lambda(1-z)^{\beta}\right]f(z)dz-\left[q^2-qpE_{\beta,\beta}(\lambda)\right]\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^{\beta}\right]f(z)dz}{qp\left[1+E_{\beta,1}^2(\lambda)-E_{\beta,2}(\lambda)E_{\beta,\beta}(\lambda)\right]+q^2\left[E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)\right]-p^2E_{\beta,1}(\lambda)}\right]+$$

$$+\int_0^x (x-z)E_{\beta,2}\left[\lambda(x-z)^{\beta}\right]f(z)dz \qquad (4)$$

ko'rinishda yozib olamiz, bu yerda $E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)}-$ Mittag-Leffler funksiyasi [13].

1-teorema. Agar

$qp\left[1+E_{\beta,1}^{2}(\lambda)-E_{\beta,2}(\lambda)E_{\beta,\beta}(\lambda)\right]+q^{2}\left[E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)\right]\neq p^{2}E_{\beta,1}(\lambda)$ bo‘lsa, u holda T masala yagona yechimga ega bo‘ladi va (4) formula bilan aniqlanadi.

Endi,

$$y''(x)-\lambda I_{0x}^{\gamma}y(x)=kf(x) \qquad (5)$$

tenglamani $(0,1)$ oraliqda qaraylik, bu yerda $y(x)$-noma’lum funksiya, λ,γ-o‘zgarmas haqiqiy sonlar, $f(x)$-berilgan funksiya, k-noma’lum son.

T₁ masala Shunday $y(x)$-funksiya va k sonni topilsinki u quyidagi xossalarga ega bo‘lsin:

1) (0, 1) oraliqda (7) tenglamani qanoatlantirsin;

2) $C^{1}[0,1]\cap C^{2}(0,1)$ sinfga tegishli bo‘lsin;

3) $x=0$, $x=1$ nuqtalarda esa (2) shartni va

$$y(\xi_0)=B_1 \qquad (6)$$

shartni qanoatlantirsin, bu yerda B_1- berilgan o'zgarmas haqiqiy son.

T₁ masala yechimini (4) formuladan foydalanib,

$$y(x)=kE_{\beta,1}(\lambda x^{\beta})\left[\frac{\left[q^{2}-qpE_{\beta,1}(\lambda)\right]\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^{\beta}\right]f(z)dz+pqE_{\beta,2}(\lambda)\int_0^1E_{\beta,1}\left[\lambda(1-z)^{\beta}\right]f(z)dz}{qp\left[1+E_{\beta,1}^{2}(\lambda)-E_{\beta,2}(\lambda)E_{\beta,\beta}(\lambda)\right]+q^{2}\left[E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)\right]-p^{2}E_{\beta,1}(\lambda)}\right]+$$

$$+xkE_{\beta,2}(\lambda x^{\beta})\left[\frac{\left[p^{2}-qpE_{\beta,1}(\lambda)\right]\int_0^1E_{\beta,1}\left[\lambda(1-z)^{\beta}\right]f(z)dz-\left[q^{2}-qpE_{\beta,\beta}(\lambda)\right]\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^{\beta}\right]f(z)dz}{qp\left[1+E_{\beta,1}(\lambda)-E_{\beta,2}(\lambda)E_{\beta,\beta}(\lambda)\right]+q^{2}\left[E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)\right]-p^{2}E_{\beta,1}(\lambda)}\right]+$$

$$+k\int_0^x(x-z)E_{\beta,2}\left[\lambda(x-z)^{\beta}\right]f(z)dz \qquad (7)$$

ko‘rinishda yozib olamiz.

(7) formulada

$$M_1=\frac{\left[q^{2}-qpE_{\beta,1}(\lambda)\right]\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^{\beta}\right]f(z)dz+pqE_{\beta,2}(\lambda)\int_0^1E_{\beta,1}\left[\lambda(1-z)^{\beta}\right]f(z)dz}{qp\left[1+E_{\beta,1}^{2}(\lambda)-E_{\beta,2}(\lambda)E_{\beta,\beta}(\lambda)\right]+q^{2}\left[E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)\right]-p^{2}E_{\beta,1}(\lambda)}$$

,

$$M_2=\frac{\left[p^{2}-qpE_{\beta,1}(\lambda)\right]\int_0^1E_{\beta,1}\left[\lambda(1-z)^{\beta}\right]f(z)dz-\left[q^{2}-qpE_{\beta,\beta}(\lambda)\right]\int_0^1(1-z)E_{\beta,2}\left[\lambda(1-z)^{\beta}\right]f(z)dz}{qp\left[1+E_{\beta,1}(\lambda)-E_{\beta,2}(\lambda)E_{\beta,\beta}(\lambda)\right]+q^{2}\left[E_{\beta,2}(\lambda)-E_{\beta,1}(\lambda)\right]-p^{2}E_{\beta,1}(\lambda)}$$

Belgilashlarni kiritamiz.

Endi, $y(\xi_0)=B_1$ nolokal shartdan k ni

$$k = B_1\left[M_1E_{\beta,1}\left(\lambda\xi_0^{\beta}\right)+\xi_0M_2E_{\beta,2}\left(\lambda\xi_0^{\beta}\right)+\int_0^{\xi_0}\left(\xi_0-z\right)E_{\beta,2}\left[\lambda\left(\xi_0-z\right)^{\beta}\right]f\left(z\right)dz\right]^{-1} \qquad (8)$$

ko‘rinishda topamiz.

Topilgan k ni (7) ga qo‘yib, T_1 masalaning yechimi hosil qilamiz.

2- teorema. Agar $M_1E_{\beta,1}\left(\lambda\xi_0^{\beta}\right)+\xi_0M_2E_{\beta,2}\left(\lambda\xi_0^{\beta}\right)+\int_0^{\xi_0}\left(\xi_0-z\right)E_{\beta,2}\left[\lambda\left(\xi_0-z\right)^{\beta}\right]f\left(z\right)dz\neq 0$ bo‘lsa, u holda T_1 masala yagona yechimga ega bo‘ladi va (7), (8) formulalar bilan aniqlanadi.

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