UCH O‘ZGARUVCHI GIPERGEOMETRIK FUNKSIYASINI REKURSIYA FORMULALARI

Mo‘minova Nozima Isoqjon qizi
Farg‘ona davlat universiteti magistranti

Annotatsiya: *Avvalroq ikki o’zgaruvchili Appell gipergeometrik funksiyalari uchun rekursiv formulalar X.Wang [1] tomonidan topilgan edi. X.Wangning ishlaridan ruhlanib, uch o’zgaruvchili $F_A^{(3)}$ Laurichella gipergeometrik funksiyasi sonli parametrlaridan birortasining o’zgarishini huddi shu funksiyalarning chekli yig’indisi orqali ifodalash imkonini beradigan rekursiv formulalar topilgan.*

Kalit so’zlar: *Laurichella funksiyasi; rekursiv formula;Poxgammer simvoli*

Annotation: *Inspired by the recent work of X.Wang [1], who gave the recursion formulas for Appell’s functions in two variables, we establish the recursion formulas for Laurichella function $F_A^{(3)}$ by the contiguous relations of hypergeometric series.*

Keywords: *Lauchella function; recursion formulas Poxgammer symbol*

Аннотация: *Вдохновляясь недавней работой Х. Ванга [1, 422], который дал рекурсивные формулы для функций Аппеля от двух переменных, мы устанавливаем рекурсивные формулы для функции Лауричелла $F_A^{(3)}$ по смежным соотношениям гипергеометрических рядов.*

Ключевые слова: *функция Лаучеллы; рекурсивные формулы; символ Поксгаммера.*

Quyidagi uch o‘lchovli Laruchella gipergeometrik funksiyani qaraylik.

$$F_D^{(3)}\left[\begin{matrix} a; b_1, b_2, b_3; \\ c; \end{matrix} X\right] = \sum_{m,n,p=0}^{\infty} \frac{(a)_{m+n+p}(b_1)_m (b_2)_n (b_3)_p}{(c)_{m+n+p}} \frac{x^m y^n z^p}{m!n!p!}, \qquad (1)$$

bu yerda $X := (x, y, z)$ uch o’lchovli funksiyalar $a, b_1, b_2, b_3, c;$ berilgan sonli paramertlar, $(\lambda)_0 = 1;$ $(\lambda)_\nu = \lambda(\lambda+1)(\lambda+2)...(\lambda+\nu-1);$ $\lambda = a, b, c;$ $\nu = 1, 2, 3....$

Laruchella gipergeometrik funksiyasi $F_A^{(3)}$ ning sonli parametrlaridan birortasining o’zgarishini xuddi shu funksiyalarning chekli yig’indisi orqali ifodalash imkonini beradigan rekursiv formulalar topilgan.

Biz Laruchella funksiyasining ikkinchi parametr c bo’yicha rekursiya formulalarini o’rganamiz.

2-teorema. Quyidagi tenglik o‘rinli.

$$F_D^{(3)}\left[\begin{matrix} a; b_1, b_2, b_3 \\ c-n \end{matrix} X\right] = F_D^{(3)}\left[\begin{matrix} a; b_1, b_2, b_3 \\ c \end{matrix} X\right] +$$

$$+ab_1 x \sum_{k=1}^{n} \frac{F_D^{(3)}[a+1; b_1+1, b_2, b_3; c+2-k; X]}{(c-k)(c-k+1)} +$$

$$+ab_2y\sum_{k=1}^{n}\frac{F_D^{(3)}\left[a+1;b_1,b_2+1,b_3;c+2-k;X\right]}{(c-k)(c-k+1)}+$$

$$+ab_3z\sum_{k=1}^{n}\frac{F_D^{(3)}\left[a+1;b_1,b_2,b_3+1;c+2-k;X\right]}{(c-k)(c-k+1)}; \qquad (2)$$

Isbot. Dastlab, (6) ni $n=1$ uchun isbotlaylik. Bu holda Laruchella funksiyasining (1) ta’rifi va

$$(c-1)_{m+n+p}=\frac{(1-c)(c)_{m+n+p}}{(1-c-m-n-p)};$$

$$\frac{1}{(c-1)_{m+n+p}}=\frac{1}{(c)_{m+n+p}}\left(1+\frac{m}{(c-1)}+\frac{n}{(c-1)}+\frac{p}{(c-1)}\right);$$

tenglikni e’tiborga olib, quyidagiga ega bo’lamiz:

$$F_D^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3\\ c-1\end{matrix}X\right]=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c-1)_{m+n+p}}\frac{x^my^nz^p}{m!n!p!}=$$

$$\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c)_{m+n+p}}\left(1+\frac{m}{c-1}+\frac{n}{c-1}+\frac{p}{c-1}\right)\frac{x^my^nz^p}{m!n!p!}=$$

$$=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c)_{m+n+p}}\frac{x^my^nz^p}{m!n!p!}+$$

$$+\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p\,m}{(c)_{m+n+p}(c-1)}\frac{x^my^nz^p}{m!n!p!}+$$

$$+\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p\,n}{(c)_{m+n+p}(c-1)}\frac{x^my^nz^p}{m!n!p!}+$$

$$+\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p\,p}{(c)_{m+n+p}(c-1)}\frac{x^my^nz^p}{m!n!p!}=F_A^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\ c;\end{matrix}X\right]+$$

$$+\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p\,\cancel{m}}{(c)_{m+n+p}(c-1)}\frac{x^my^nz^p}{\cancel{m}!n!p!}+$$

$$+\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p\,\cancel{n}}{(c)_{m+n+p}(c-1)}\frac{x^my^nz^p}{m!\cancel{n}!p!}+$$

$$+\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p\,\cancel{p}}{(c)_{m+n+p}(c-1)}\frac{x^my^nz^p}{m!n!\cancel{p}!}=$$

$$=F_D^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\c;\end{matrix}X\right]+\sum_{\substack{n,p=0\\m=1}}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c)_{m+n+p}(c-1)}\frac{x^m y^n z^p}{(m-1)!n!p!}+$$

$$+\sum_{\substack{m,p=0\\n=1}}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c)_{m+n+p}(c-1)}\frac{x^m y^n z^p}{m!(n-1)!p!}+$$

$$+\sum_{\substack{m,n=0\\p=1}}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c)_{m+n+p}(c-1)}\frac{x^m y^n z^p}{m!n!(p-1)!};$$

$m-1$ ni m bilan, $n-1$ ni n va $p-1$ni p almashtirib, (2) formulaning $n=1$ uchun isbotiga ega bo'lamiz:

$$F_D^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\c;\end{matrix}X\right]+\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+1+n+p}(b_1)_{m+1}(b_2)_n(b_3)_p}{(c)_{m+1+n+p}(c-1)}\frac{x^{m+1} y^n z^p}{m!n!p!}+$$

$$+\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+1+p}(b_1)_m(b_2)_{n+1}(b_3)_p}{(c)_{m+n+1+p}(c-1)}\frac{x^m y^{n+1} z^p}{m!n!p!}+$$

$$+\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p+1}(b_1)_m(b_2)_n(b_3)_{p+1}}{(c)_{m+n+p+1}(c-1)}\frac{x^m y^n z^{p+1}}{m!n!p!}=$$

$$F_D^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\c;\end{matrix}X\right]+$$

$$+\sum_{m,n,p=0}^{\infty}\frac{a(a+1)\ldots(a+m+n+p)b_1(b_1+1)\ldots(b_1+m)(b_2)_n(b_3)_p}{c(c+1)\ldots(c+m+n+p)(c-1)}\frac{x^{m+1} y^n z^p}{m!n!p!}+$$

$$+\sum_{m,n,p=0}^{\infty}\frac{a(a+1)\ldots(a+m+n+p)(b_1)_m b_2(b_2+1)\ldots(b_2+n)(b_3)_p}{c(c+1)\ldots(c+m+n+p)(c-1)}\frac{x^m y^{n+1} z^p}{m!n!p!}+$$

$$+\sum_{m,n,p=0}^{\infty}\frac{a(a+1)\ldots(a+m+n+p)(b_1)_m(b_2)_n b_3(b_3+1)\ldots(b_3+p)}{c(c+1)\ldots(c+m+n+p)(c-1)}\frac{x^m y^n z^{p+1}}{m!n!p!}=$$

$$F_D^{(3)}\left[\begin{matrix}a;b_1,b_2,b_3;\\c;\end{matrix}X\right]+$$

$$+\frac{ab_1x}{c(c-1)}F_D^{(3)}\left[\begin{matrix}a+1;b_1+1,b_2,b_3;\\c+1;\end{matrix}X\right]+\frac{ab_2y}{c(c-1)}F_D^{(3)}\left[\begin{matrix}a+1;b_1,b_2+1,b_3;\\c+1;\end{matrix}X\right]+$$

$$+\frac{ab_3z}{c(c-1)}F_D^{(3)}\left[\begin{matrix}a+1;b_1,b_2,b_3+1;\\c+1;\end{matrix}X\right]; \qquad (3)$$

Endi (2) munosabatni $n=2$ uchun ko'rsatamiz. (3) tenglikni e'tiborga olsak, quyidagiga ega bo'lamiz:

$$F^{(3)}{}_D\begin{bmatrix} a;b_1,b_2,b_3 \\ c-2 \end{bmatrix} X = F^{(3)}{}_D\begin{bmatrix} a;b_1,b_2,b_3 \\ c \end{bmatrix} X +$$

$$+\frac{ab_1x}{(c-2)(c-1)}F_D^{(3)}\begin{bmatrix} a+1;b_1+1,b_2,b_3; \\ c; \end{bmatrix} X + \frac{ab_2y}{(c-2)(c-1)}F_D^{(3)}\begin{bmatrix} a+1;b_1,b_2+1,b_3; \\ c; \end{bmatrix} X +$$

$$+\frac{ab_3y}{(c-2)(c-1)}F_D^{(3)}\begin{bmatrix} a+1;b_1,b_2,b_3+1; \\ c; \end{bmatrix} X;$$

Bu munosabatni $F_A^{(3)}$ Loruchella funksiyani parametrni $c-n$ ni (3) munosabati bilan n marta qo'llab, (2) rekursiya formulani olamiz.Demak

$$F_D^{(3)}\begin{bmatrix} a;b_1,b_2,b_3 \\ c-n \end{bmatrix} X = F_D^{(3)}\begin{bmatrix} a;b_1,b_2,b_3 \\ c \end{bmatrix} X +$$

$$+ab_1x\left[\frac{F_D^{(3)}[a+1;b_1+1,b_2,b_3;c+1;X]}{(c-1)c}+\frac{F_D^{(3)}[a+1;b_1+1,b_2,b_3;c;X]}{(c-2)(c-1)}+\dots\frac{F_D^{(3)}[a+1;b_1+1,b_2,b_3;c+2-k;X]}{(c-k)(c-k+1)}\right]+$$

$$+ab_2y\left[\frac{F_D^{(3)}[a+1;b_1,b_2+1,b_3;c+1;X]}{(c-1)c}+\frac{F_D^{(3)}[a+1;b_1,b_2+1,b_3;c;X]}{(c-2)(c-1)}+\dots\frac{F_D^{(3)}[a+1;b_1,b_2+1,b_3;c+2-k;X]}{(c-k)(c-k+1)}\right]+$$

$$+ab_1x\left[\frac{F_D^{(3)}[a+1;b_1,b_2,b_3+1;c+1;X]}{(c-1)c}+\frac{F_D^{(3)}[a+1;b_1,b_2,b_3;c+1;X]}{(c-2)(c-1)}+\dots\frac{F_D^{(3)}[a+1;b_1,b_2,b_3+1;c+2-k;X]}{(c-k)(c-k+1)}\right]=$$

$$=F_D^{(3)}\begin{bmatrix} a;b_1,b_2,b_3 \\ c \end{bmatrix} X +$$

$$+ab_1x\sum_{k=1}^{n}\frac{F_D^{(3)}[a+1;b_1+1,b_2,b_3;c+2-k;X]}{(c-k)(c-k+1)}+$$

$$+ab_2y\sum_{k=1}^{n}\frac{F_D^{(3)}[a+1;b_1,b_2+1,b_3;c+2-k;X]}{(c-k)(c-k+1)}+$$

$$+ab_3z\sum_{k=1}^{n}\frac{F_D^{(3)}[a+1;b_1,b_2,b_3+1;c+2-k;X]}{(c-k)(c-k+1)};$$

Foydalanilgan adabiyotlar.

1.Wang X. Recursion formulas for Appell functions. Integral transforms and special functions, 2012, 23(6), p.421-433.