INTEGRO-DIFFERENSIAL TENGLAMALAR SISTEMASI UCHUN KOSHI MASALASI

https://doi.org/10.5281/zenodo.7799293

O.A.Abdubannopova

Annotatsiya: *Ushbu maqolada integro-differensial tenglamalar sistemasi uchun Koshi masalasi yechimi Dalamber usulidan foydalanib aniqlangan.*

Quyidagi

(1) $$\begin{cases} x'(t)=aI_{mt}^{\gamma}x(t)+bI_{mt}^{\gamma}y(t)+f_1(t), \\ y'(t)=cI_{mt}^{\gamma}x(t)+dI_{mt}^{\gamma}y(t)+f_2(t), \end{cases} \qquad t>m$$

sistemasining

(2) $$x(m)=x_0, y(m)=y_0$$

boshlang'ich shartlarni qanoatlantiruvchi yechimini topish masalasini qaraylik, bu yerda I_{mt}^{γ} kasr tartibli Riman-Liuvill ma'nosidagi integral operator bo'lib[1], u quyidagicha aniqlanadi:

$$I_{mt}^{\gamma}z(t)=\frac{1}{\Gamma(\gamma)}\int_m^t (t-s)^{\gamma-1}z(s)ds,$$

$\Gamma(z)$ – Eylerning gamma funksiyasi [2], γ,a,b,c,d,x_0,y_0 – berilgan haqiqiy sonlar bo'lib, $\gamma>0$; $f_1(t)$ va $f_2(t)$ berilgan funksiyalar, $x(t)$ va $y(t)$ lar esa noma'lum funksiyalar.

{(1),(2)} masala yechimini Dalamber usulidan foydalanib topamiz. [3],[4] ishlarda ham Dalamber usulidan foydalanib, ba'zi tenglamalar sistemasi uchun Koshi masalasining yechimi formulalari oshkor ko'rinishda topilgan.

{(1),(2)} Koshi masalasining yechimini topish bilan shug'ullanamiz. Shu maqsadda (1) ning ikkinchi tenglamasini λ songa ko'paytirib birinchi tenglamaga qo'shamiz. Natijada quyidagi tenglikka ega bo'lamiz:

(3) $$\frac{d}{dt}\{x(t)+\lambda y(t)\}=(a+\lambda c)I_{mt}^{\gamma}x(t)+(b+\lambda d)I_{mt}^{\gamma}y(t)+f_1(t)+\lambda f_2(t).$$

Oxirgi tenglikni quyidagi ko'rinishda qayta yozib olamiz:

(4) $$\frac{d}{dt}\{x(t)+\lambda y(t)\}=(a+\lambda c)I_{mt}^{\gamma}\left[x(t)+\frac{b+\lambda d}{a+\lambda c}y(t)\right]+f_1(t)+\lambda f_2(t).$$

λ sonni shunday tanlaylikki, $\frac{b+\lambda d}{a+\lambda c}=\lambda$ tenglamani qanoatlantirsin. Faraz qilaylik, λ_1 va λ_2 lar oxirgi tenglamaning ildizlari bo'lib, $\lambda_1 \neq \lambda_2$ bo'lsin. U holda bu sonlarni (4) ga qo'yib, ushbu tenglikni hosil qilamiz:

(5) $$\frac{d}{dt}\left\{x(t)+\lambda_i y(t)\right\}=(a+\lambda_i c)I_{mt}^{\gamma}\left[x(t)+\lambda_i y(t)\right]+f_1(t)+\lambda_i f_2(t) \qquad i=1,2.$$

Soddalik maqsadida quyidagi belgilashlarni kiritaylik:

(6) $x(t)+\lambda_i y(t)=z(t)$, $a+\lambda_i c=\tilde{\lambda}_i$, $f_1(t)+\lambda_i f_2(t)=f_{3,i}(t)$, $i=1,2$.

U holda (5) quyidagi ko‘rinishni oladi:

(7) $$\frac{d}{dt}z(t)=\tilde{\lambda}_i I_{mt}^{\gamma}z(t)+f_{3,i}(t) \qquad i=1,2.$$

(7) tenglamada t ni s ga almashtirib, so‘ngra hosil bo‘lgan tenglikni s bo‘yicha $[m,t]$ oraliqda integrallaymiz. Natijada

(8) $$z(t)-\tilde{\lambda}_i\int_m^t \frac{(t-s)^{\gamma}}{\Gamma(\gamma+1)}f_{4,i}(s)ds=f_{4,i}(t), \qquad i=1,2.$$

ko‘rinishdagi 2-tur Volterra integral tenglamasiga ega bo‘lamiz, bu yerda

$$f_{4,i}(t)=\int_m^t f_{3,i}(t)dt+z(m).$$

Oxirgi tenglama yechimini ketma-ket yaqinlashish usulidan foydalanib topamiz. Nolinchi yaqinlashish sifatida

$$z_0(t)=f_{4,i}(t)$$

ni qabul qilamiz. Birinchi va ikkinchi yaqinlashishlarni mos holda quyidagi formulalar bo‘yicha aniqlaymiz:

$$z_1(t)=f_{4,i}(t)+\tilde{\lambda}_i\int_m^t K(t,s)f_{4,i}(s)ds,$$

$$z_2(t)=f_{4,i}(t)+\tilde{\lambda}_i\int_m^t K(t,\tau)z_1(\tau)d\tau=$$

$$=f_{4,i}(t)+\tilde{\lambda}_i\int_m^t K(t,\tau)\left[f_{4,i}(\tau)+\tilde{\lambda}_i\int_m^{\tau}K(\tau,s)f_{4,i}(s)ds\right]d\tau=$$

$$=f_{4,i}(t)+\tilde{\lambda}_i\int_m^t K(t,s)f_{4,i}(s)ds+\tilde{\lambda}_i^2\int_m^t K_2(t,s)f_{4,i}(s)ds.$$

n-yaqinlashishni esa

$$z_n(t)=f_{4,i}(t)+\tilde{\lambda}_i\int_m^t\left[\sum_{j=0}^{n}\tilde{\lambda}_i^{j-1}K_j(t,s)\right]f_{4,i}(s)ds$$

formuladan foydalanib topamiz. Bu yerda $K_j(t,s)$ –iteratsiyalangan yadrolar bo‘lib,

$$K_1(t,s)=K(t,s)=(t-s)^{\gamma}\Gamma^{-1}(\gamma+1),$$

$$K_j(t,s)=\int_s^t K(t,\tau)K_{j-1}(\tau,s)d\tau, \quad j=2,3,\ldots.$$

Dastlab, $K_2(t,s)$ni hisoblaylik:

$$K_2(t,s)=\int_s^t K(t,\tau)K(\tau,s)d\tau=\int_s^t \frac{(t-\tau)^\gamma(\tau-s)^\gamma}{\Gamma^2(\gamma+1)}d\tau .$$

Ushbu almashtirishni bajaraylik: $\tau=(t-s)\eta+s$. U holda beta va gamma funksiya xossalaridan foydalansak,

$$K_2(t,s)=\int_0^1 \frac{(t-s)^{2\gamma+1}\eta^{\gamma+1-1}(1-\eta)^{\gamma+1-1}}{\Gamma^2(\gamma+1)}d\eta=\frac{(t-s)^{2\gamma+1}}{\Gamma^2(\gamma+1)}\int_0^1 \eta^{\gamma+1-1}(1-\eta)^{\gamma+1-1}d\eta=$$

$$=\frac{(t-s)^{2\gamma+1}}{\Gamma^2(\gamma+1)}B(\gamma+1,\gamma+1)=\frac{(t-s)^{2(\gamma+1)-1}}{\Gamma(2(\gamma+1))}.$$

Matematik induksiya usulidan foydalanib ko‘rsatish qiyin emaski, $K_j(t,s)$ uchun quyidagi o‘rinli:

$$K_j(t,s)=(t-s)^{j(\gamma+1)-1}\Gamma^{-1}\left[j(\gamma+1)\right], \qquad j=1,2,3,\dots .$$

$K_j(t,s)$dan foydalanib, $R(t,s;\tilde{\lambda}_i)$ rezolventani tuzamiz:

$$R(t,s;\tilde{\lambda}_i)=\sum_{j=1}^{\infty}\tilde{\lambda}_i^{j-1}K_j(t,s)=\sum_{j=1}^{\infty}\frac{\tilde{\lambda}_i^{j-1}(t-s)^{j(\gamma+1)-1}}{\Gamma\left[j(\gamma+1)\right]}.$$

Mittag-Leffler funksiyasining yoyilmasidan foydalansak[5], rezolventani quyidagicha yozishimiz mumkin:

$$R(t,s;\tilde{\lambda}_i)=(t-s)^{\gamma-1}E_{\gamma+1,\gamma+1}\left[\tilde{\lambda}_i(t-s)^{\gamma+1}\right].$$

U holda 2-tur Volterra integral tenglamalari nazariyasiga [6] asosan (8) tenglamani yechimi ushbu formula orqali ifodalanadi:

(9) $$z(t)=f_{4,i}(t)+\tilde{\lambda}_i\int_m^t (t-s)^{\gamma-1}E_{\gamma+1,\gamma+1}\left[\tilde{\lambda}_i(t-s)^{\gamma+1}\right]f_{4,i}(s)ds .$$

Endi (6) va (9) tengliklarga asosan quyidagi tenglamalar sistemasini tuzamiz:

$$x(t)+\lambda_i y(t)=$$

$$=f_{4,i}(t)+(a+\lambda_i c)\int_m^t (t-s)^{\gamma-1}E_{\gamma+1,\gamma+1}\left[(a+\lambda_i c)(t-s)^{\gamma+1}\right]f_{4,i}(s)ds,\ \ i=1,2.$$

Oxirgi tenglamalar sistemasini algebraik qo‘shish usulidan foydalanib, $x(t)$ va $x(t)$noma’lum funksiyalarni quyidagicha bir qiymatli aniqlaymiz:

$$x(t)=f_{4,1}(t)+(a+\lambda_1 c)\int_m^t (t-s)^{\gamma-1}E_{\gamma+1,\gamma+1}\left[(a+\lambda_1 c)(t-s)^{\gamma+1}\right]f_{4,1}(s)ds-\lambda_1 y(t);$$

$$y(t)=\frac{1}{\lambda_1-\lambda_2}\left\{f_{4,1}(t)-f_{4,2}(t)+(a+\lambda_1 c)\int_m^t (t-s)^{\gamma-1}E_{\gamma+1,\gamma+1}\left[(a+\lambda_1 c)(t-s)^{\gamma+1}\right]f_{4,1}(s)ds-\right.$$

$$-(a+\lambda_2 c)\int_m^t (t-s)^{\gamma-1} E_{\gamma+1,\gamma+1}\left[(a+\lambda_2 c)(t-s)^{\gamma+1}\right] f_{4,2}(s)ds\Bigg\}.$$

FOYDALANILGAN ADABIYOTLAR:

1. A.A.Kilbas, H.M.Srivastava and Trujilo. Theory and Applications of Fractional Differential equations. North Holland Mathematics Studies, 204p.

2. A.Q. O‘rinov. Maxsus funksiyalar va maxsus operatorlar. Farg‘ona: “Farg‘ona” nashriyoti, 2012, -112 bet.

3. A.O.Mamanazarov, O.A.Abdubannopova. Kasr tartibli differensial operatorni o‘z ichiga oluvchi chiziqli differensial tenglamalar sistemasini yechishning bir usuli haqida. O‘zbekistonda fanlararo innovatsiyalar va ilmiy tadqiqotlar jurnali. 14-son. 464-467-betlar. 2022.

4. O.A.Abdubannopova. Differensial tenglamalar sistemasi uchun Koshi masalasi. O‘zbekistonda fanlararo innovatsiyalar va ilmiy tadqiqotlar jurnali. 15-son. 757-761-betlar. 2023.

5. Podlubniy I. Fractional Differential Equations: An introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their solution, Mathematics in Science and Engineering. Vol.198 Diego: Academic Press, 1999.

6. M.S. Salohiddinov. Integral tenglamalar. Toshkent: Yangiyul polygraph Service, 2007, -256 bet.