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MAXSUS KO'RINISHDAGI INTEGRAL CHEGARALANISHLI QUVISH VA QOCHISH MASALASI

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Ibaydullayev To'lanboy Tursunboyevich
F.-m.f.n., Dotsent, Andijon davlat universiteti
Xusanboeva Nodiraxon Rafiqjon qizi
Magistr, Farg'ona davlat universiteti
Tursunova Dildoraxon To'lanboy qizi
Bakalavr, Andijon davlat universiteti

Annotatsiya. *Mazkur maqolada quvlovchining boshqaruviga maxsus integral ko'rinishdagi chegaralanish qo'yilgan quvish va qochish masalalari qaralgan. Bu kabi chegaralanishlar ilk bor A.Azamov tomonidan qo'yilgan. Fizikaviy nuqtai nazardan quvlovchining boshqaruviga qo'yilgan bu shartlar ularga berilgan resurslarni uzluksiz tiklanib borishini anqlatadi. Qochuvchining boshqaruviga esa odatdagi geometrik chegaralanish qo'yilgan. Shunday shartlar keltirilganki, ularning bajarilishi quvlovchi uchun yutuqli, aks holda esa qochuvchi uchun yutuqli bo'ladi.*

Kalit so'zlar. *Differensial o'yinar, quvish masalasi, qochish masalasi,* Π*-strategiya, A.Azamov chegaralanishlari, geometrik chegaralanish.*

KIRISH

1. Masalaning qo'yilishi

Aytaylik R^n fazosida x boshqariluvchi obyekt y boshqariluvchi obyektni quvlayotgan bo'lsin. x orqali quvlovchining, y orqali esa qochuvchining R^n fazosidagi holatini belgilaymiz.

Mazkur ishda quvlovchi va qochuvchining harakatlari mos ravishda quyidagi dinamik sistemalar bilan ifodalangan differensial o'yin qaraladi:

$$\dot{x} = u, \quad x(0) = x_0, \qquad (1)$$

$$\dot{y} = v, \quad y(0) = y_0, \qquad (2)$$

bu yerda, $x, y, u, v \in R^n, n \geq 1$; x_0, y_0-obyektlarning boshlang'ich holatlari, $x_0 \neq y_0$ deb hisoblanadi, u, v-tezlik vektorlari, ular boshqaruv parametrlari bo'lib xizmat qiladi. Shu bilan birga u vektorning vaqt bo'yicha o'zgarishlari $u(\cdot): [0, \infty) \longrightarrow R^n$ o'lchovli funksiya bo'lishi lozim. Unga quyidagi maxsus ko'rinishdagi inegral chegaralanishlar qo'yiladi

$$\begin{cases} \int_0^t |u(s)|^2 \, ds \leq \rho^2, \text{при } 0 \leq t \leq \alpha, \\ \int_{t-\alpha}^t |u(s)|^2 \, ds \leq \rho^2, \text{при } t \geq \alpha. \end{cases} \qquad (3)$$

Bunday ko'rinishdagi chegaralanishlar ilk bor A.Azamov tomonidan qo'yilgan chegaralanishlarning xususiy holi bo'lib, u fizikaviy nuqtai nazardan quvlovchi energiyasining t vaqt o'tishi bilan uzluksiz tiklanuvchanligini bildiradi, bu yerda α-biror musbat son, ρ esa-nomanfiy son.

Shu kabi v vektorning vaqt bo'yicha o'zgarishlari $v(\cdot):[0,\infty)\longrightarrow R^n$ o'lchovli funksiya bo'lishi lozim. Unga odatdagi geometrik chegaralanish qo'yilgan bo'lsin

$$|v(t)|\le\frac{\sigma}{\sqrt{\alpha}}, t\ge 0\,, \qquad (4)$$

bu yerda, σ- nomanfiy son. Bunda maxraj simmetriya maqsadida qo'yilgan.

x obyektning maqsadi qochuvchini tutish, ya'ni chekli t vaqtda

$$x(t)=y(t) \qquad (5)$$

tenglikka erishish, bu yerda $x(t),y(t)$-o'yin jarayonida vujudga keladigan trayektoriyalar. Qochuvchi quvlovchi bilan uchrashmaslikka harakat qiladi, buning ilojisi bo'lmagan taqdirda xech bo'lmasa uchrashish vaqtini mumkin qadar cho'zishga harakat qiladi. Albatta, masalaning bunday qo'yilishi qo'shimcha izohlar talab qiladi.

Ushbu ishda quvlovchining bohqaruviga (3) ko'rinishdagi chegaralanish qo'yilgan differensial o'yin ilk bor o'rganilgan. U $0\le t\le\alpha$ vaqt oralig'ida odatdagi integral chegaralanish bilan ustma-ust tushadi. Bundan buyon (3) va (4) chegaralanishlarni birgalikda AG-chegaralanishlar, ular bilan berilgan (1) va (2) o'yinni AG-o'yin deb yuritamiz

1-ta'rif:

$$\begin{cases}\int_0^t|u(s)|^2\,ds\le\rho^2, \text{при } 0\le t\le\alpha\,,\\ \int_{t-\alpha}^t|u(s)|^2\,ds\le\rho^2, \text{при } t\ge\alpha\end{cases} \qquad (3)$$

va

$$|v(t)|\le\frac{\sigma}{\sqrt{\alpha}}, t\ge 0\,, \qquad (4)$$

chegaralanishlarni qanoatlantiradigan $u(t)$ va $v(t)$, $t\ge 0$ o'lchovli funksiyalar mos ravishda quvlovchi x va qochuvchi y ning joiz boshqaruvlari deyiladi, bu yerda α-biror musbat son, ρ va σ esa- nomanfiy sonlar.

Quvlovchining joiz boshqaruvlari sinfini, ya'ni (3) chegaralanishlarni qanoatlantiruvchi o'lchovli funksiyalar sinfini U orqali, qochuvchining joiz boshqaruvlari sinfini, ya'ni (4) chegaralanishni qanoatlantiruvchi o'lchovli funksiyalar sinfini esa V orqali belgilaymiz.

Quvlovchi va qochuvchining $u(\cdot)\in U$ va $v(\cdot)\in V$ joiz boshqaruvlarga mos trayektoriyalari mos ravishda quyidagi tenglamalar bilan aniqlanadi

$$x(t)=x_0+\int_0^t u(s)\,ds, y(t)=y_0+\int_0^t v(s)\,ds.$$

Bizga quyidagi lemma kerak bo'ladi.

1-lemma. Agar o'lchovli $u(t)$, $t\ge 0$ funksiya uchun

$$\int_{t-\alpha}^t|u(s)|^2\,ds\le\rho^2, \text{при } t\ge\alpha \qquad (3')$$

ko'rinishdagi A-chegaralanish o'rinli bo'lsa, u holda unung uchun quyidagi

$$\int_{t-\alpha}^t|u(s)|\;ds\le\sqrt{\alpha}\,\rho, t\ge\alpha, \qquad (5)$$

chegaralanish ham o'rinli bo'ladi, bu yerda α-biror musbat son, ρ va σ esa- nomanfiy sonlar.

Aytish joizki, (5) dan (3') har doim ham kelib chiqavermaydi. Lemmaning isboti Koshi-Bunyakovskiy tengsizligidan oson kelib chiqadi.

$\int_{t-\alpha}^{t}|u(s)|\ ds = \int_{t-\alpha}^{t}1\cdot|u(s)|\,ds \le \sqrt{\alpha}\cdot\sqrt{\int_{t-\alpha}^{t}|u(s)|^2\,ds} \le \sqrt{\alpha}\,\rho$, ya'ni,

$$\int_{t-\alpha}^{t}|u(s)|\ ds \le \sqrt{\alpha}\,\rho. \qquad (5)$$

2-lemma. Agar o'lchovli $v(t),\, t \ge 0$ funksiya uchun

$$|v(t)| \le \frac{\sigma}{\sqrt{\alpha}},\, t \ge 0\,, \qquad (4)$$

ko'rinishdagi geometrik chegaralanish o'rinli bo'lsa, u holda unung uchun quyidagi

$$\int_{t-\alpha}^{t}|v(s)|\ ds \le \sqrt{\alpha}\,\sigma,\, t \ge \alpha, \qquad (6)$$

va

$$\int_{t-\alpha}^{t}|v(s)|^2\,ds \le \sigma^2, \text{при } t \ge \alpha \qquad (6')$$

chegaralanishlar ham o'rinli bo'ladi, bu yerda α-biror musbat son, ρ va σ esa- nomanfiy sonlar.

Aytish joizki, (6) va (6') lardan (4) har doim ham kelib chiqavermaydi. Lemmaning isboti (4) ni $[t-\alpha; t]$ oraliqda integrallashdan oson kelib chiqadi.

Endi o'yinchilrning strategiyalariga ta'rif beramiz.

2. Strategiyalar ta'riflari

Quvish masalasini yechish uchun quvlovchining strategiyasi ta'rifini keltiramiz [1-8].

2-ta'rif. $u: V \longrightarrow U$ akslantirish quyidagi shartlar bajarilganda quvlovchining strategiyasi deyiladi:

1. Joizlik sharti. Ixtiyoriy $v(\cdot) \in V$ uchun biror $[0;T]$ oraliqda $u(\cdot) = u\big(v(\cdot)\big) \in U$ munosabat bajarilsin, bunda $u(t) = u\big(v(t)\big), t \ge 0$ funksiya $u\big(v(\cdot)\big), v \in V$ strategiyani amalga oshirish funksiyasi deyiladi.

2. Volterra sharti. Agar $v_1(\cdot), v_2(\cdot) \in V$ uchun $[0;T]$ da $v_1(t) = v_2(t)$ tenglik deyarli bajarilsa, u holda $[0;T]$ da $u_1(t) = u_2(t)$ tenglik deyarli bajarilsin, bu yerda $u_i(\cdot) = u\big(v_i(\cdot)\big),\ i = 1,2.$

3-ta'rif: $z(t) = x(t) - y(t),\ z_0 = x_0 - y_0$ bo'lsin. U holda agar ixtiyoriy $v(\cdot) \in V$ uchun

$$\dot z = u\big(v(t)\big) - v(t),\ z(0) = z_0 \qquad (7)$$

Koshi masalasining yechimini

$$z(t) = \Lambda\big(t, v(\cdot)\big)z_0, \Lambda\big(0, v(\cdot)\big) = 1 \qquad (8)$$

ko'rinishda tasvirlash mumkin bo'lsa, u holda u strategiyani parallel quvish strategiyasi yoki П-strategiya deb ataladi, bu yerda $\Lambda\big(t, v(\cdot)\big)$- $t, t \ge 0$ ning biror skalyar funksiyasi bo'lib, uni odatda quvish masalasida yaqinlashish funksiyasi deb ataladi.

4-ta'rif. Agar ixtiyoriy $v(\cdot) \in V$ uchun

a) vaqtning shunday $t^* \in [0;T]$ momenti mavjud bo'lsaki, uning uchun $z(t^*) = 0$ bo'lsa;

b) $[0;t^*]$ vaqt oralig'ida $u(v(\cdot)) \in U$ bo'lsa, u holda Π-strategiya quvlovchi uchun $[0;T]$ vaqt oralig'ida yutuqli deyiladi.

Endi o'yinni qochuvchi nuqtai nazari bilan qaraymiz.

5-ta'rif. Agar ixtiyoriy $u(\cdot) \in U$ uchun

$\dot{z} = u(t) - v^*(t), \qquad z(0) = z_0$

Koshi masalasining yechimi $z(t)$ barcha $t \geq 0$ larda noldan farqli bo'lsa, ya'ni $z(t) \neq 0$, barcha $t \geq 0$ uchun, u holda $v^*(\cdot) \in V$ boshqaruv qochuvchi uchun yutuqli deyiladi.

NATIJALAR

3. Quvish va qochish masalasining yechimi

Mazkur bandda qaralyotgan AG-o'yinda quvish va qochish masalasini yechish shartlari keltiriladi.

Teorema. Agar A-o'yinda $\rho > \sigma$ bo'lsa, u holda o'yin quvlovchi uchun yutuqli bo'ladi va kafolatlangan tutish vaqti $T = \sqrt{\alpha}\frac{|z_0|}{\rho-\sigma}$ bo'ladi. Agar $\rho \leq \sigma$ bo'lsa, u holda o'yin qochuvchi uchun yutuqli bo'ladi.

Isboti: 1. Dastavval o'yinchilarning erishish sohalarini aniqlaymiz.

(1) ni $t - \alpha$ dan t gacha oraliqda integrallaymiz:

$$x(t) - x(t-\alpha) = \int_{t-\alpha}^{t} u(s)\,ds. \qquad (9)$$

(2) ni esa 0 dan t gacha oraliqda integrallaymiz:

$$y(t) - y_0 = \int_0^t v(s)\,ds. \qquad (10)$$

(9) dan va 1-lemmadan, ya'ni (5) dan

$|x(t) - x(t-\alpha)| \leq \sqrt{\alpha}\,\rho$

hosil qilamiz.

$t = \alpha$ da

$|x(\alpha) - x_0| \leq \sqrt{\alpha}\,\rho$

bo'ladi.

$t = 2\alpha$ da

$|x(2\alpha) - x(\alpha)| \leq \sqrt{\alpha}\,\rho$

bo'ladi. Va hokazo $t = m\alpha$

$|x(m\alpha) - x((m-1)\alpha)| \leq \sqrt{\alpha}\,\rho$

ga ega bo'lamiz. Bulardan

$|x(m\alpha) - x_0| \leq |x(m\alpha) - x\big((m-1)\alpha\big) + x\big((m-1)\alpha\big) - x\big((m-2)\alpha\big) +$

$+x\big((m-2)\alpha\big) - \cdots + x(\alpha) - x_0| \leq$

$\leq |x(m\alpha) - x((m-1)\alpha)| + \cdots + |x(2\alpha) - x(\alpha)| + |x(\alpha) - x_0| \leq$

$\leq m\sqrt{\alpha}\,\rho$, ya'ni

$$|x(m\alpha) - x_0| \leq m\sqrt{\alpha}\,\rho. \qquad (11)$$

ni topamiz.

(10) dan esa (4) ga asosan

$$|y(t) - y_0| \leq \frac{\sigma}{\sqrt{\alpha}} t$$

ni topamiz. Bundan $t = m\alpha$ da

$$|y(m\alpha) - y_0| \leq m\sqrt{\alpha}\,\sigma \qquad (12)$$

topamiz.

Agar $S_{m\sqrt{\alpha}\,\rho}(x_0)$ orqali markazi x_0 nuqtada, radiusi $m\sqrt{\alpha}\,\rho$ teng bo'lgan sharni belgilasak, u holda (11) tengsizlikdan quvlovchininq erishish sohasi aynan $S_{m\sqrt{\alpha}\,\rho}(x_0)$ shar bo'lishi, agar $S_{m\sqrt{\alpha}\,\sigma}(y_0)$ orqali markazi y_0 nuqtada, radiusi $m\sqrt{\alpha}\,\sigma$ teng bo'lgan sharni belgilasak, u holda (12) tengsizlikdan qochuvchininq erishish sohasi aynan $S_{m\sqrt{\alpha}\,\sigma}(y_0)$ shar bo'lishi kelib chiqadi.

Isbotlash mumkinki, $\rho > \sigma$ bo'lganda, shunday m natural soni topiladiki, natijada

$$|z_0| + m\sqrt{\alpha}\,\sigma \leq m\sqrt{\alpha}\,\rho. \qquad (13)$$

bo'ladi, ya'ni quvlovchining erishish sohasi (shari), qochuvchininq erishish sohasini (sharini) to'liq o'z ichiga oladi:

$$S_{m\sqrt{\alpha}\,\sigma}(y_0) \subset S_{m\sqrt{\alpha}\,\rho}(x_0).$$

(13) dan m ning bu qiymati

$$m \geq \frac{|z_0|}{\sqrt{\alpha}(\rho - \sigma)} \qquad (14)$$

tengsizlikni qanoatlantiradigan eng kichik natural son bo'ladi.

2. Quvish masalasi.

Dastlab $0 \leq t \leq \alpha$ oraliqni qaraymiz.

Bu oraliqda quvlovchining boshqaruvi odatdagi integral chegaralanish bilan ifodalanadi: $\int_0^t |u(s)|^2\, ds \leq \rho^2, 0 \leq t \leq \alpha$. Ravshanki qochuvchining boshqaruvi doimiy bir xil gemetrik ko'rinishda bo'ladi: $|v(t)| \leq \frac{\sigma}{\sqrt{\alpha}}, t \geq 0$. П-strategiyaning klassik modelini qo'llab, quvlovchining strategiyasini quyidagicha quramiz:

$$u(t) = v(t) - \left(\langle v(t), e\rangle + \sqrt{\langle v(t), e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}} \right) e\,, 0 \leq t \leq \alpha, \qquad (15)$$

bu yerda $e = \frac{z_0}{|z_0|}$ – birlik vektor.

Ko'rsatish mumkinki, (15) strategiya joiz bo'ladi. Haqiqatan, (15) ning har ikki tomonini kvadratga ko'taramiz:

$$|u(t)|^2 = |v(t)|^2 - 2\langle v(t),e\rangle\left(\langle v(t),e\rangle + \sqrt{\langle v(t),e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}\right) +$$

$$+\langle v(t),e\rangle^2 + 2\langle v(t),e\rangle\sqrt{\langle v(t),e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}} + \langle v(t),e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha} =$$

$$= |v(t)|^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha} \le \frac{\sigma^2}{\alpha} + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha} = \frac{\rho^2}{\alpha}.$$

Buni 0 dan t gacha oraliqda integrallaymiz: $\int_0^t |u(t)|^2\, ds \le \frac{\rho^2}{\alpha} t$ va $0 \le t \le \alpha$ da $\int_0^t |u(t)|^2\, ds \le \rho^2$ bo'lishini topamiz.

(15) ni (7) ga ya'ni $\dot{z} = u(t) - v(t), z(0) = z_0$ sistemaga olib borib qo'yib, 0 dan t gacha oraliqda integrallaymiz:

$$z(t) = z_0 - \int_0^t \left(\langle v(s),e\rangle + \sqrt{\langle v(s),e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}\right) ds \cdot e\,. \qquad (16)$$

(16) dan

$$z(t) = \left(|z_0| - \int_0^t \left(\langle v(s),e\rangle + \sqrt{\langle v(s),e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}\right) ds\right) \cdot e$$

ni topamiz. Keyin λ orqali integral ostidagi qavsni belgilaymiz va uni v bo'yicha minimumga tekshiramiz:

$$\lambda(v(t)) = \langle v(t),e\rangle + \sqrt{\langle v(t),e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}\,.$$

Buning uchun $\langle v(t),e\rangle = x$ belgilash kiritib, $f(x) = x + \sqrt{x^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}$ funksiyani $-\frac{\sigma}{\sqrt{\alpha}} \le x \le \frac{\sigma}{\sqrt{\alpha}}$ oraliqda minimumga tekshiramiz: $f'(x) = 1 + \frac{x}{\sqrt{x^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}} \ge 0$, ya'ni $f(x)$ monoton o'suvchi funksiya, shuning uchun

$$\min_{-\frac{\sigma}{\sqrt{\alpha}} \le x \le \frac{\sigma}{\sqrt{\alpha}}} f(x) = f\left(-\frac{\sigma}{\sqrt{\alpha}}\right) = \frac{\rho - \sigma}{\sqrt{\alpha}}\,.$$

Demak, $\min_{|v(t)| \le \frac{\sigma}{\sqrt{\alpha}}} \lambda(v(t)) = \frac{\rho - \sigma}{\sqrt{\alpha}}$. Bundan xal qiluvchi (8) funksiya uchun quyidagi bahoni topamiz:

$$\Lambda(v(t)) = |z_0| - \int_0^t \left(\langle v(s),e\rangle + \sqrt{\langle v(s),e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}\right) ds,$$

$$|\Lambda(v(t))| \le |z_0| - \frac{\rho - \sigma}{\sqrt{\alpha}} t,\ \ 0 \le t \le \alpha.$$

Bundan ko'rinadiki, agar $\sqrt{\alpha}\frac{|z_0|}{\rho - \sigma} \le \alpha$ bo'lsa, u holda o'yin $\theta = \sqrt{\alpha}\frac{|z_0|}{\rho - \sigma}$ vaqtga qadar quvlovchining foydasiga hal bo'ladi.

Aytaylik, $\sqrt{\alpha}\frac{|z_0|}{\rho-\sigma} > \alpha$, bo'lsin, ya'ni $|z_0| > \sqrt{\alpha}(\rho-\sigma)$. Unda quvlovchining (15) strategiyasi $0 \le t \le \alpha$ oraliqda uning uchun yutuqli bo'lmaydi va shuning uchun o'yinni $t \ge \alpha$ da qaraymiz. Bunda quvlovchining strategiyasini yana (15) ko'rinishda tanlaymiz:

$$u(t) = v(t) - \left(\langle v(t), e\rangle + \sqrt{\langle v(t), e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}\right)e\,,\ t \ge \alpha,$$

bu yerda $e = \frac{z_0}{|z_0|}$ - birlik vektor.

(15) strategiyaning joizligini $t \ge \alpha$ uchun ham ko'rsatish mumkin. Haqiqatan, (15) ning har ikki tomonini yana kvadratga ko'taramiz:

$$|u(t)|^2 = |v(t)|^2 - 2\langle v(t), e\rangle\left(\langle v(t), e\rangle + \sqrt{\langle v(t), e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}\right) +$$

$$+\langle v(t), e\rangle^2 + 2\langle v(t), e\rangle\sqrt{\langle v(t), e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}} + \langle v(t), e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha} =$$

$$= |v(t)|^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha} \le \frac{\sigma^2}{\alpha} + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha} = \frac{\rho^2}{\alpha}.$$

Keyin buning har ikki tomonini $t-\alpha$ dan t gacha oraliqda integrallasak,

$$\int_{t-\alpha}^{t} |u(s)|^2\, ds \le \int_{t-\alpha}^{t} \frac{\rho^2}{\alpha}\, ds = \rho^2$$

ni hosil qilamiz.

(15) ni yana $\dot{z} = u(t) - v(t), z(0) = z_0$ sistemaga olib borib qo'yib, $t-\alpha$ dan t gacha oraliqda integrallaymiz:

$$z(t) = z(t-\alpha) - \int_{t-\alpha}^{t}\left(\langle v(s), e\rangle + \sqrt{\langle v(s), e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}\right) ds\, e\,. \qquad (17)$$

Ravshanki, $t = \alpha$ paytdan to o'yin oxirigacha $z(t-\alpha) \uparrow\uparrow e$ bo'ladi. Haqiqatan, $t \ge \alpha$ da

$$z(t-\alpha) = z_0 - \int_{0}^{t-\alpha}\left(\langle v(s), e\rangle + \sqrt{\langle v(s), e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}\right) ds\, e =$$

$$= \left(|z_0| - \int_{0}^{t-\alpha}\left(\langle v(s), e\rangle + \sqrt{\langle v(s), e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}\right) ds\right) e\,.$$

(17) dan

$$z(t) = \left(|z(t-\alpha)| - \int_{t-\alpha}^{t}\left(\langle v(s), e\rangle + \sqrt{\langle v(s), e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}\right) ds\right) e \qquad (18)$$

ni hosil qilamiz. Yana λ orqali integral ostidagi qavsni belgilaymiz va uni v bo'yicha minimumga tekshiramiz:

$$\lambda(v(t)) = \langle v(t), e\rangle + \sqrt{\langle v(t), e\rangle^2 + \frac{\rho^2}{\alpha} - \frac{\sigma^2}{\alpha}}\,.$$

$\min_{|v(t)|\leq\frac{\sigma}{\sqrt{\alpha}}}\lambda(v(t))=\frac{\rho-\sigma}{\sqrt{\alpha}}$. Natijada, xal qiluvchi funksiya uchun quyidagi bahoni topamiz:

$$\Lambda(v(t))=|z_0|-\int_{t-\alpha}^{t}\left(\langle v(s),e\rangle+\sqrt{\langle v(s),e\rangle^2+\frac{\rho^2}{\alpha}-\frac{\sigma^2}{\alpha}}\right)ds,$$

$$|\Lambda(v(t))|\leq|z(t-\alpha)|-\sqrt{\alpha}(\rho-\sigma),\ t\geq\alpha.$$

Bundan va (18) dan $t\geq\alpha$ da

$$|z(t)|\leq|z(t-\alpha)|-\sqrt{\alpha}\,(\rho-\sigma)$$

bo'lishini topamiz.

$t=\alpha$ da

$$|z(\alpha)|\leq|z_0|-\sqrt{\alpha}\,(\rho-\sigma),$$

$t=2\alpha$ da

$$|z(2\alpha)|\leq|z(\alpha)|-\sqrt{\alpha}\,(\rho-\sigma),$$

va hokazo $t=m\alpha$ da

$$|z(m\alpha)|\leq|z((m-1)\alpha)|-\sqrt{\alpha}\,(\rho-\sigma)$$

ga ega bo'lamiz. Bulardan

$$|z(m\alpha)|\leq|z((m-1)\alpha)|-\sqrt{\alpha}\,(\rho-\sigma)\leq\cdots\leq|z_0|-m\sqrt{\alpha}\,(\rho-\sigma)$$

ni topamiz.

Agar $|z_0|-m\sqrt{\alpha}\,(\rho-\sigma)\leq 0$ bo'lsa, u holda $|z(m\alpha)|\leq 0$ bo'lib, o'yinning $t=m\alpha$ vaqtga qadar quvlovchi foydasiga xal bo'lishi kelib chiqadi. m natural sonining qiymatini $|z_0|-m\sqrt{\alpha}\,(\rho-\sigma)\leq 0$ tengsizlikdan aniqlaymiz va (13) ni topamiz:

$$m\geq\frac{|z_0|}{\sqrt{\alpha}\,(\rho-\sigma)}. \qquad (19)$$

Yuqorida $|z_0|>\sqrt{\alpha}(\rho-\sigma)$ shartga ega bo'lganimizni e'tiborga olsak, m ning 1 dan qat'iy katta natural son ekani ma'lum bo'ladi. Agar (19) ning o'ng tomoni natural bo'lsa, u holda m sifatida aynan o'sha o'ng tomondagi ifodani olamiz. Bulardan kafolatlangan tutish vaqtini topamiz:

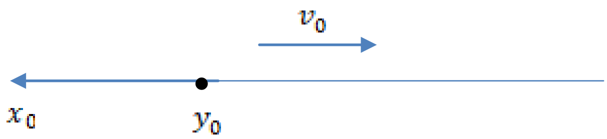
$$T=\sqrt{\alpha}\frac{|z_0|}{\rho-\sigma}. \qquad (20)$$

Ma'lum bo'ldiki, o'yin $0\leq t\leq\alpha$ oraliqda quvlovchining foydasiga xal bo'lganda ham, $t\geq\alpha$ oraliqda shunday bo'lganda ham kafolatlangan tutish vaqti (20) ko'rinishda bo'ladi.

3. Qochish masalasi.

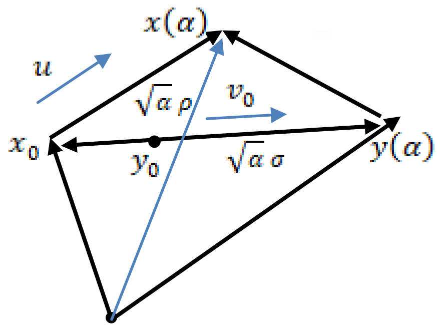
Aytaylik endi $\rho\leq\sigma$ bo'lsin. Dastavval $0\leq t\leq\alpha$ vaqt oralig'ini qaraymiz. Quvlovchining AG-chegaralanishli boshqaruv parametri u ni ixtiyoriy bo'lsin deb, qochuvchining boshqaruv parametrini $v_0=-\frac{\sigma}{\sqrt{\alpha}}\frac{z_0}{|z_0|}$ ko'rinishda tanlaymiz, bu yerda $z_0=x_0-y_0$ (1-rasm). U joiz:

$$|v_0|^2 = \left|-\frac{\sigma}{\sqrt{\alpha}}\frac{z_0}{|z_0|}\right|^2 = \frac{\sigma^2}{\alpha}.$$



1-rasm

Shu bilan birga bizga (11) va (12) lardan ma'lumki, $0 \le t \le \alpha$ vaqt oralig'ida quvlovchining erishish sohasi $|x(\alpha) - x_0| \le \sqrt{\alpha}\,\rho$ bo'ladi, ya'ni $S_{\sqrt{\alpha}\,\rho}(x_0)$ shar, qochuvchiniki esa, $|y(\alpha) - y_0| \le \sqrt{\alpha}\,\sigma$ shar. Shartga ko'ra $\sigma \ge \rho$ bo'lgani uchun $t = \alpha$ paytga qadar quvlovchining erishish sohasi qochuvchining erishish sohasini o'z ichiga olmaydi, ya'ni barcha $t \in [0;\alpha]$ uchun $S_{\sqrt{\alpha}\,\sigma}(y_0) \not\subset S_{\sqrt{\alpha}\,\rho}(x_0)$. $t = \alpha$ paytda o'yinchilar orasidagi masofani quyidagicha topamiz.



2-rasm

$\Delta x_0 x(\alpha) y(\alpha)$ uchburchakda (2-rasm) uchburchak tengsizligidan barcha $t \in [0;\alpha]$ uchun

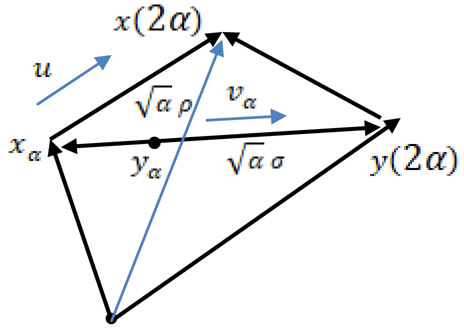
$$|x(\alpha) - y(\alpha)| \ge (|x_0 - y_0| + |y(\alpha) - y_0|) - |x(\alpha) - x_0| \ge$$
$$\ge \left(|x_0 - y_0| + \sqrt{\alpha}\sigma\right) - \sqrt{\alpha}\rho = |z_0| + \sqrt{\alpha}(\sigma - \rho) \ge |z_0|$$ ni topamiz.

Demak, $t = \alpha$ paytga qadar o'yinchilar orasidagi boshlang'ich masofa saqlanadi.

Endi $\alpha \le t \le 2\alpha$ vaqt oralig'ini qaraymiz. $t = \alpha$ paytda o'yinchilar yangi x_α va y_α boshlang'ich holatlarni egallaydi. Yana quvlovchining AG-chegaralanishli boshqaruv parametri u ni ixtiyoriy bo'lsin deb, qochuvchining boshqaruv parametrini $v_\alpha = -\frac{\sigma}{\sqrt{\alpha}}\frac{z_\alpha}{|z_\alpha|}$ ko'rinishda tanlaymiz, bu yerda $z_\alpha = x_\alpha - y_\alpha$ (3-rasm).

U ham joiz:

$$|v_\alpha|^2 = \left|-\frac{\sigma}{\sqrt{\alpha}}\frac{z_\alpha}{|z_\alpha|}\right|^2 = \frac{\sigma^2}{\alpha}.$$



3-rasm

Bunda $\alpha \le t \le 2\alpha$ oraliqda quvlovchining erishish sohasi $|x(2\alpha) - x_\alpha| \le \sqrt{\alpha}\,\rho$ bo'ladi, ya'ni $S_{\sqrt{\alpha}\,\rho}(x_\alpha)$ shar. Qochuvchining erishish sohasi esa

$|y(2\alpha) - y_\alpha| \le \sqrt{\alpha}\,\sigma$ bo'ladi, ya'ni $S_{\sqrt{\alpha}\,\sigma}(y_\alpha)$ shar. Shartga ko'ra $\sigma \ge \rho$ bo'lgani uchun $t = 2\alpha$ paytga qadar quvlovchining erishish sohasi qochuvchining erishish sohasini o'z ichiga olmaydi, ya'ni barcha $t \in [\alpha; 2\alpha]$ uchun $S_{\sqrt{\alpha}\,\sigma}(y_\alpha) \not\subset S_{\sqrt{\alpha}\,\rho}(x_\alpha)$. $t = 2\alpha$ paytda o'yinchilar orasidagi masofani quyidagicha topamiz.

$\Delta x_\alpha x(2\alpha) y(2\alpha)$ uchburchakda (3-rasm) uchburchak tengsizligidan barcha $t \in [\alpha; 2\alpha]$ uchun

$|x(2\alpha) - y(2\alpha)| \ge (|x_\alpha - y_\alpha| + |y(2\alpha) - y_\alpha|) - |x(2\alpha) - x_\alpha| \ge$
$\ge (|x_\alpha - y_\alpha| + \sqrt{\alpha}\sigma) - \sqrt{\alpha}\rho = |z_\alpha| + \sqrt{\alpha}(\sigma - \rho) \ge |z_\alpha|$ ni topamiz.

Demak, $t = 2\alpha$ paytga qadar o'yinchilar orasidagi yangi boshlang'ich masofa saqlanadi, bu masofa o'z navbatida dastlabki boshlang'ich masofadan kichik bo'lmasligini oldingi qadamda ko'rdik: $|z_\alpha| \ge |z_0|$.

Bu jarayonni davom ettirib, m-qadamda barcha $t \in [(m-1)\alpha; m\alpha]$ uchun

$$|x(m\alpha) - y(m\alpha)| \ge |x_{(m-1)\alpha} - y_{(m-1)\alpha}| + \sqrt{\alpha}\,\sigma - \sqrt{\alpha}\,\rho \ge |z_{(m-1)\alpha}|$$

ni hosil qilamiz.

Demak, ma'lum bo'ldiki, $t = m\alpha$ paytga qadar o'yinchilar orasidagi yangi boshlang'ich masofa $|z_{(m-1)\alpha}|$ saqlanadi, bu masofa o'z navbatida avvalgi qadamdagi boshlang'ich masofadan kichik bo'lmaydi: $|z_{(m-1)\alpha}| \ge |z_{(m-2)\alpha}|$ va hokazo, ya'ni

$$|x(m\alpha) - y(m\alpha)| \ge |z_{(m-1)\alpha}| \ge |z_{(m-2)\alpha}| \ge \cdots \ge |z_0|.$$

Bundan $m \longrightarrow \infty$ da barcha $t \in [0; +\infty)$ uchun $|x(t) - y(t)| \ge |z_0|$ hosil qilamiz.

Teorema to'la isbotlandi.

XULOSA

Ushbu maqolada biz ilk bor Akademik A.Azamov tomonidan qo'yilgan maxsus ko'rinishdagi integral chegaralanishli quvish va qoshish o'yinini qaradik. Bunda aytilgan chegaralanish faqat quvlovchiga qo'yildi. Qochuvchiga esa oddiy geometrik chegaralanish qo'yildi. O'yinning qanday shartlarda quvlovchining foydasiga xal bo'lishi va qanday shartlarda qochuvchining foydasiga xal bo'lishi batafsil ko'rsatildi. Albatta, yuqorida aytilgan maxsus chegaralanishlarni qochuvchi uchun ham qo'yish mumkin. Yana bu shartlarning umumiy hollari ham mavjudki, ularni o'rganish juda ham dolzarb hisoblanadi. Bulardan tashqari olingan natijalarni ko'p quvlovchili o'yinlar uchun ham qo'llash mumkin bo'ladi.

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