QUVURLARDAGI IKKI FAZALI MUHITDA VAQTINCHALIK HARAKAT DIFFERENSIAL TENGLAMALARI

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Annotatsiya: *Ushbu maqola filtrlash uchun suv yo'qotilishini hisobga olgan holda turbulent beqaror harakatni (ikki o'lchovli holat) tavsiflovchi differentsial tenglamalar tizimini yechish yo'llari yoritiladi. Variatsion-farq usuli qo'llaniladi va sonli natijalar keltiriladi.*

Kalit so'zlar: *kvazy bir o'lchovli vaqtinchalik harakat, quvurlardagi ikki fazali muhit, gidravlik qarshilik, quvurlar kesishgan maydoni, Laplas transformatsiyasi, qutblar.*

Аннотация: *В данной статье решаются системы дифференциальных уравнений, которые описывают турбулентное неустановившиеся движение (двумерный случай) с учетом потерь воды на фильтрацию. Применяется вариационно-разностный метод и приводятся численные результаты*

Ключевые слова: *квазиодномерные неустановившиеся движения, двухфазных среды в трубах, гидравлическое сопротивление, площадь сечения трубы, преобразование Лапласа, полюсы.*

Annotation: *This article solves systems of differential equations that describe turbulent unsteady motion (two-dimensional case), taking into account water losses due to filtration. The variational-difference method is applied and numerical results are presented.*

Keywords: *quasi-one-dimensional transient motion, two-phase media in pipes, hydraulic resistance, pipe cross-sectional area, Laplace transform, poles.*

Ushbu maqolada, ikki fazali muhitning kvaziyali bir o'lchovli beqaror harakatining tenglamalari gidravlik qarshilik (1) hisobga olingan holda kvazistatsion gipotezani qo'llash asosida olingan.

Silindrsimon trubadagi ikki fazali suyuqlikning harakat tenglamalari (2) shaklga ega ekanligi ma'lum.

$$\rho_1 \frac{\partial u_1}{\partial t} = f_1 \mu_1 \left(\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r} \frac{\partial u_1}{\partial r} \right) + k(u_2 - u_1) - f_1 \frac{\partial p}{\partial z}, \quad (1)$$

$$\rho_2 \frac{\partial u_2}{\partial t} = f_2 \mu_2 \left(\frac{\partial^2 u_2}{\partial r^2} + \frac{1}{r} \frac{\partial u_2}{\partial r} \right) + k(u_1 - u_2) - f_2 \frac{\partial p}{\partial z}, \quad (2)$$

Bu yerda $\mu_1, \mu_2, \rho_1, \rho_2 -$ birinchi va ikkinchi fazalarning yopishqoqligi va zichligi;

$f_1, f_2 -$g'ovakliligi; k -o'zaro ta'sir koeffitsienti.

(1) va (2) tengliklarni ko'paytirib $2\pi f_1 f_2 r dr$ va 0 dan R gacha integrallab, (2) dan foydalanib, oddiy o'zgarishlardan keyin biz quyidagi natijani olamiz

$$- f_1^2 f_2 \frac{\partial p}{\partial z} = f_2 \rho_1 \frac{\partial W_1}{\partial t} - f_1 f_2 \mu_1 A_1 W_1 - k(f_1 W_2 - f_2 W_1). \quad (3)$$

$$-f_2^2 f_1 \frac{\partial p}{\partial z} = f_1 \rho_2 \frac{\partial W_2}{\partial t} - f_1 f_2 \mu_2 A_2 W_2 - k(f_2 W_1 - f_1 W_2). \ (4)$$

Bu yerda,

$$A_1 = \frac{\alpha_1^0 \frac{mI_1(mR)}{I_0(mR)} - \frac{kR}{2 f_1 f_2 \mu_1 \mu_2}}{\alpha_1^0 \frac{I_1(mR)}{mI_0(mR)} + \left(\frac{kR^2}{8 f_1 f_2 \mu_1 \mu_2} - \alpha_1^0\right)}; \ A_2 = \frac{\alpha_2^0 \frac{mI_1(mR)}{I_0(mR)} - \frac{kR}{2 f_1 f_2 \mu_1 \mu_2}}{\alpha_2^0 \frac{I_1(mR)}{mI_0(mR)} + \left(\frac{kR^2}{8 f_1 f_2 \mu_1 \mu_2} - \alpha_2^0\right)};$$

$$\alpha_1^0 = \frac{1}{f_1 \mu_1 + f_2 \mu_2} - \frac{1}{\mu_1}; \ \alpha_2^0 = \frac{1}{f_1 \mu_1 + f_2 \mu_2} - \frac{1}{\mu_2};$$

W_1, W_2 – birinchi va ikkinchi fazalarning o'rtacha tezligi.

Ikki fazali muhit uchun doimiylik tenglamasi quyidagi shaklga ega

$$\frac{\partial p_i}{\partial t} + \frac{\partial(\rho_i u_i)}{\partial r} + \frac{\partial(\rho_i u_i)}{\partial z} + \frac{\rho_i u_i}{r} = 0, \ (i = 1,2). \ (5)$$

(5) ni $2\pi f_i r dr$ ga ko'paytirib va 0 dan R gacha integrallab, har bir tarkibiy qism uchun alohida, quyidagilarni yozamiz

$$f_1 s \frac{\partial \rho_1}{\partial t} + \frac{\partial M_1}{\partial z} = 0. \ (6), \ f_2 s \frac{\partial \rho_2}{\partial t} + \frac{\partial M_2}{\partial z} = 0. \ (7)$$

Bu yerda $M_i = 2\pi f_i \int_0^R \rho_i u_i r dr$ – massa xarajatlari, s- truba kesimining yuzi.

Aytaylik, aralashmaning o'rtacha zichligi va trubaning maydoni uchun Гук qonuni amal qiladi $\rho_{cm} = \rho_0 \left(1 + \frac{p - p_0}{K_{ж}}\right)$, $s = s_0 \left(1 + \alpha \frac{p - p_0}{E}\right)$,

Bu yerda ρ_0- p_0 bosim zichligi s_0- $p = p_0$. bo'lgandagi yuza. $K_{ж}$- suyuqlikni siqish moduli, E-elastiklik moduli, α- o'lchovsiz koeffitsient. (1) tenglamagako'ra $\alpha \frac{(p - p_0)^2}{K_{ж} E}$,

bundan, $\rho_{cm} s = s_0 \rho_0 \left[1 + \left(\frac{1}{K_{ж}} + \frac{a}{E}\right)(p - p_0)\right]$. (8)

(6) va (7) niqo'shib, (8) nihisobgaolganholdavaba'zio'zgarishlardankeyinquyidaginiyozamiz

$$-\frac{1}{c^2} \frac{\partial p}{\partial t} = \frac{\partial(\rho_1 W_1)}{\partial z} + \frac{\partial(\rho_2 W_2)}{\partial z}. (9)$$

bu yerda

$$c^2 = \frac{k}{\rho_0}, \ k = \frac{K_{ж}}{1 + a \frac{K_{ж}}{E}}$$

(3), (4) va (9) tenglamalar - buikkifazalimuhitningo'rtachatezligi, zichligivabosimikesishishlarigataalluqlikvazyozometrikharakat.

Olingan tenglamalar, gaz quvurining boshida va oxirida bosim yoki suyuqlik oqimining o'zgarishini o'rganish, shuningdek, ikki fazali muhit (k, μ, ρ va boshqalar) parametrlarini aniqlash uchun teskari muammolarni yechishda qiziqish uyg'otadi.

Quvurlardagi ikki fazali muhitning beqaror harakati masalasini yechishni ko'rib chiqamiz, buning uchun (3), (4) va (9) tenglamalarni quyidagicha yozamiz:

$$-\frac{\partial p}{\partial t}=\frac{\rho_1}{f_1^2}\left(\frac{\partial W_1}{\partial t}+a_1W_1+b_1W_2\right),\ -\frac{\partial p}{\partial t}=\frac{\rho_2}{f_2^2}\left(\frac{\partial W_2}{\partial t}+b_2W_1+a_2W_2\right),(10)$$

$$-\frac{\partial p}{\partial t}=c_2\left(\rho_1\frac{\partial W_1}{\partial z}+\rho_2\frac{\partial W_2}{\partial z}\right),\ \text{где}\quad a_1=\frac{k-f_1\mu_1A_1}{\rho_1},\ a_2=\frac{k-f_2\mu_2A_2}{\rho_2},$$

$$b_1=-\frac{kf_1}{\rho_1(1-f_1)},\ b_2=-\frac{kf_2}{\rho_2(1-f_2)}.$$

Biz boshlang'ich va chegara sharoitida (10) sistema uchun yechimni qidirmoqdamiz:

$$W_1(z,0)=W_2(z,0)=0,\ p(z,0)=0,\ p(0,t)=\varphi(t),\ W_1(l,t)=f(t),(11)$$

Bu yerda $\varphi(t)$, $f(t)$-ma'lum farqlanadigan funktsiyalar Laplas transformatsiyasi hisoblanadi.

t, c o'zgaruvchilari bo'yicha Laplas qoidasini (10) tenglamaga qo'llab (II) hisobga olib, biz quyidagini yozamiz

$$\left.\begin{aligned}&-\frac{\partial p}{\partial z}=\frac{\rho_1}{f_1^2}(s+a_1)W_1(z,s)+\frac{\rho_1b_1}{f_1^2}W_2(z,s)\\&-\frac{\partial p}{\partial z}=\frac{p_2b_2}{f_2^2}W_1(z,s)+\frac{p_2}{f_2^2}(s+a_2)W_2(z,s)\qquad(12)\\&-sp=c^2\left(\rho_1\frac{\partial W_1}{\partial z}+\rho_2\frac{\partial W_2}{\partial z}\right)\end{aligned}\right\}$$

$$p(0,c)=\Phi(s);\ W_1(\ell,s)=F(s).(13)$$

(12) sistemaning birinchi ikki tenglamasidan topamiz

$$W_1=-\frac{\partial p}{\partial z}.\frac{\Delta_1}{\Delta};\ W_2=-\frac{\partial p}{\partial z}.\frac{\Delta_2}{\Delta};(14)$$

bu yerda

$$\Delta(s)=\alpha_1s^2+\alpha_2s+\alpha_3;\ \Delta_1(s)=\beta_1s+\beta_2;\ \Delta_2(s)=c_1s+c_2,$$

$$\alpha_1=\frac{\rho_1\rho_2}{f_1^2f_2^2};\ \alpha_2=\alpha_1(a_1+a_2);\ \alpha_3=(a_1a_2-b_1b_2),\ \beta_1=\frac{\rho_2}{f_2^2};\ \beta_2=\frac{\rho_2a_2}{f_2^2}-\frac{\rho_1b_1}{f_1^2};\ c_1=\frac{\rho_1}{f_1^2};$$

$$c_2=\frac{\rho_1a_1}{f_1^2}-\frac{\rho_2b_2}{f_2^2}.$$

(14) sistemadan $W_2=\frac{\Delta_2}{\Delta_1}W_1=\frac{c_1s+c_2}{\beta_1s+\beta_2}W_1$ или $W_2=\frac{c_1}{\beta_1}W_1+\frac{c_2\beta_1-c_1\beta_2}{\beta_1^2}\cdot\frac{1}{s+\frac{\beta_2}{\beta_1}}.W_1.$

undan kelib chiqadiki, $W_1(z,t)$ va $W_2(z,t)$ asl nusxalar funktsional munosabatlar bilan bog'langan

$$W_2(z,t)=\frac{c_1}{\beta_1}W_1(z,t)+\frac{c_2\beta_1-c_1\beta_2}{\beta_1^2}\int_0^1e^{-\frac{\beta_2}{\beta_1}(t-\tau)}W_1(z,\tau)d\tau.\qquad(15)$$

(12) sistemaning uchinchi tenglamasini (14) ga qo'yamiz $\frac{\partial^2p}{\partial z^2}-\lambda^2p=0.$ u holda $P(z,s)=Ae^{\lambda z}+Be^{-\lambda z},\ ,W_1(z,s)=\frac{\lambda\Delta_1}{\Delta}\left(-Ae^{\lambda z}+Be^{-\lambda z}\right).$

Bu yerda $\lambda(s)=\frac{1}{c}\sqrt{\frac{\alpha_1 s^3+\alpha_2 s^2+\alpha_3 s}{\gamma_1 s+\gamma_2}}$, (16)

$\gamma_1=\rho_1\beta_1+\rho_2 c_1;\ \gamma_2=\rho_1\beta_2+\rho_2 c_2.$ (13) shart bizga Ava B o'zgarmaslarni topishga yordam beradi:

$$A+B=\Phi(s);\ \frac{\lambda\Delta_1}{\Delta}\left(-Ae^{\lambda\ell}+Be^{-\lambda\ell}\right)=F(s),\quad A=\Phi(s)-\Phi(s)\frac{e^{\lambda\ell}}{2ch\lambda\ell}-\frac{\Delta}{\lambda\Delta_1}\cdot\frac{1}{2ch\lambda\ell}F(s).$$

$$B=\Phi(s)\frac{e^{\lambda\ell}}{2ch\lambda\ell}+\frac{\Delta}{\lambda\Delta_1}\cdot\frac{1}{2ch\lambda\ell}F(s).$$

Topshiriqlarning yechimi quyidagi ko'rinishga ega:

$$P(z,s)=s\Phi(s)\frac{ch\lambda(\ell-z)}{sch\lambda\ell}-sF(s)\frac{\Delta}{\lambda\Delta_1}\frac{sh\lambda z}{sch\lambda\ell},$$

$$W_1(z,s)=s\Phi(s)\frac{\lambda\Delta_1}{\Delta}\frac{sh\lambda(\ell-z)}{sch\lambda\ell}+sF(s)\frac{sh\lambda z}{sch\lambda\ell},\ (17)$$

Tasvirlar uchun asl nusxalar $S\Phi(s)$ va $SF(s)$ vazifalari mavjud $\varphi'(t)$ va $f'(t)$, va tasvirlar mahsuloti asl nusxalarning yaxlit yig'ilishiga mos keladi.

(17) da turgan $S\Phi(s)$ va $SF(s)$ omillarni tahlil qilish kerak.

$\frac{ch\lambda(\ell-z)}{sch\lambda\ell},\ \frac{sh\lambda z}{sch\lambda\ell}$ funksiyalar tenglamaning ildizlariga mos keladigan oddiy S_η qutblarga ega

$ch[\lambda(s)\ell]=0$ (18)

$S_0=0,$ qutb va funktsiyalar $\frac{\Delta}{\lambda\Delta_1}\frac{sh\lambda z}{sch\lambda\ell},\ \frac{\lambda\Delta_1}{\Delta}\frac{sh\lambda(\ell-z)}{sch\lambda\ell},$ (19)

qutblardan tashqari mos ravishda $S=-\frac{\beta_2}{\beta_1}$ va $S=-\frac{\gamma_2}{\gamma_1}$

Darhaqiqat, $\frac{\lambda\Delta_1}{\Delta s}\frac{sh\lambda(\ell-z)}{ch\lambda\ell}=\frac{\lambda^2\Delta_1}{\Delta s}\frac{sh\lambda(\ell-z)}{\lambda ch\lambda\ell}=\frac{\beta_1 s+\beta_2}{\gamma_1 s+\gamma_2}\cdot\frac{sh\lambda(\ell-z)}{\lambda ch\lambda\ell}\cdot s\neq 0,$ qiymat buning uchun $\lambda(s)=0$ funktsiyasi uchun (19) qutb emas, chunki $\lim_{\lambda\to 0}\frac{sh\lambda z}{\lambda}=z.$ (18) dan esa $\cos i\lambda\ell=0.$

bundan $\lambda^2+\left(\frac{\pi}{2\ell}\right)^2(2n-1)^2=0;$ n=1,2,... (20)

Qutblar S_η (20) tenglama yechimlari bo'lib, bunda u (16) tenglik bilan aniqlanadi.

Ushbu funktsiyalarning qoldiqlarini hisoblab, konvolyutsiya teoremasini qo'llasak, masalaning yechimini asl nusxada topamiz.

$$p(z,t)=\varphi(t)-\frac{\alpha_3 z}{\beta_2}f(t)+\frac{\Delta\left(-\frac{\beta_2}{\beta_1}\right)sh\left[\lambda\left(-\frac{\beta_2}{\beta_1}\right)z\right]}{\beta_2\lambda\left(-\frac{\beta_2}{\beta_1}\right)ch\left[\lambda\left(-\frac{\beta_2}{\beta_1}\right)\ell\right]}\cdot\int_0^t f'(\tau)e^{-\frac{\beta_2}{\beta_1}(t-\tau)}d\tau+$$

$$+\sum_{k=1}^{\infty}\frac{(-1)^{k-1}i}{\ell\cdot S_k}\cdot\frac{\cos\left[\frac{\pi}{2}\left(1-\frac{z}{\ell}\right)(2k-1)\right]}{\frac{d\lambda(-S_k)}{ds}}\cdot\int_0^t\varphi'(\tau)e^{S_k(t-\tau)}d\tau-$$

$$-\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}i}{2k-1}\cdot\frac{\Delta(S_k)}{S_k\Delta_1(S_k)}\cdot\frac{\sin\left[\frac{\pi z}{2\ell}(2k-1)\right]}{\frac{d\lambda(S_k)}{ds}}\cdot\int_0^t f'(\tau)e^{S_k(t-\tau)}d\tau;$$

$$W_1(z,t)=f(t)+\frac{(\gamma_1\beta_2-\gamma_2\beta_2)sh\left[\lambda\left(-\frac{\gamma_2}{\gamma_1}\right)(\ell-z)\right]}{c^2\gamma_1^2\lambda\left(-\frac{\gamma_2}{\gamma_1}\right)ch\left[\lambda\left(-\frac{\gamma_2}{\gamma_1}\right)\ell\right]}\cdot\int_0^t\varphi'(\tau)e^{-\frac{\gamma_2}{\gamma_1}(t-\tau)}d\tau+$$

$$+\sum_{k=1}^{\infty}\frac{(-1)^{k-1}i\cos\left[\frac{\pi z}{2\ell}(2k-1)\right]}{\ell\cdot S_k\,\frac{\alpha\lambda(S_k)}{ds}}\int_0^t f'(\tau)e^{S_k(t-\tau)}dt+$$

$$+\frac{2}{\pi c^2}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{2k-1}\cdot\frac{\beta_1S_k+\beta_2}{\gamma_1S_k+\gamma_2}\sin\left[\frac{\pi}{2}\left(1-\frac{z}{\ell}\right)(2k-1)\right]\int_0^t\varphi'(\tau)e^{S_k(t-\tau)}d\tau;$$

Ikkinchi fazaning tezligi (15) tenglikdan aniqlanadi.

Muammoning aniq yechimidan bir qator amaliy masalalarni ko'rib chiqishda foydalanish mumkin, masalan, suv bosimi, quvurlardagi bosim o'zgarishi.

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